Using the “Black Box” Features on a CAS Device to teach radicals

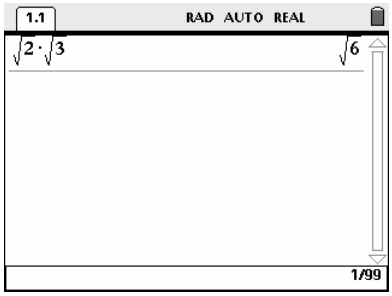
In the article “White box vs. Black Box – Systems of  Equations”, we looked at using a Computer Algebra System device as a “white box” where the steps that students do are visible. In this article we start to look at using the CAS devices as a black box. The black box approach occurs when the student enters a command and the answer pops out. This is the approach that concerns teachers who consider the development of skills to be of paramount importance in teaching mathematics – and with good reason. If the device does everything for the student, what have they learned? The answer is obvious – nothing.

So, why use it in this way? The answer lies in how we present it to our students. Keep in mind that we are facing children who are technologically savvy and who tend to think that a tech device must always be right. In fact, if a handheld calculator says that the answer is such and such, then to them, it is such and such. It does not matter that you said that same thing five minutes ago and presented one of the best lessons of your career. Your word is not worth as much as the tech device. What I try to do is to leverage their attitude to advantage – ours and theirs.

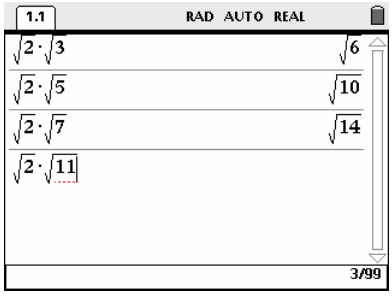
In teaching with a CAS device, I took advantage of this many times. I use a cycle of activities in the classroom. Using just one device at the front of the classroom, I present one example and ask “what happened”. If someone sees the pattern immediately, great. If not, I can do a few more hoping that the pattern will become more obvious. Once it has been explained, I put one more on the screen and ask the class to predict the answer. Finally, I have them use their devices to create a few more examples of the same type.

In this article, we will look at using the CAS device to develop skills in the area of radicals.

Take a look at the first screen. What I want students to think about is “just how did the device get from this question to the answer, because you won’t always have this device with you and you have to understand how this works”. Only the most intuitive student is going to see something with just one example, so let’s look at a couple more.

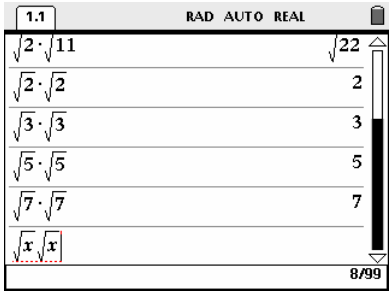



What I have done in the past is to present a few examples and then enter one more while asking if they can predict the answer. Then I ask them to explain what they’ve done. Notice that the examples all used prime numbers as the second factor. Hopefully, some student will ask about this. It’s not important but I like the fact that someone notices. However, I have avoided second factors that are even so that I don’t spoil what is coming later. At this

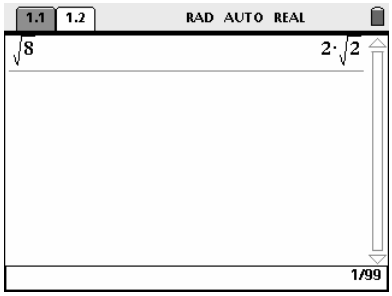


point, I want my students to make up a few more examples.

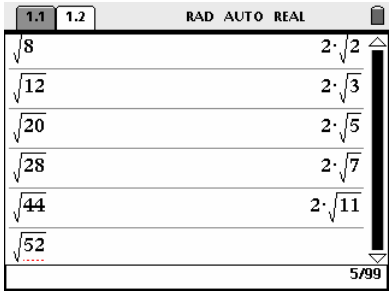
So, that's part one of my lesson. Now, let's move on to part two. I want to develop the idea that the square root of a value multiplied by itself is the value – that's a mouthful. Notice again that the final line has the question ready but no answer. I want my students to predict the answer and explain it. In other words, I turn this into a communication exercise.



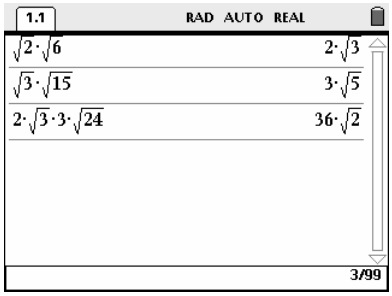

For part three, I want to start exploring problems where a pure radical is converted to a mixed radical. The device does the work of converting the form of the answer but to leave it at this does not contribute to students' understanding. Some more examples are needed.



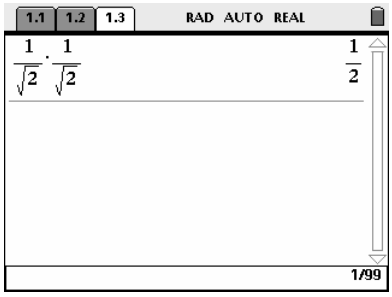

In the screen to the right, I am hoping that someone in the class notices that all of the arguments are multiples of 4. Once one student gets it, I'm also hoping that he/she is going to explain it to everyone in the class for me. Once that happens, they can predict the answer for the last problem posed on the last line. Once they have the idea, I want them make up some more examples on their own, this time without have a leading coefficient of 2. They should use their CAS device to test their hypothesis.



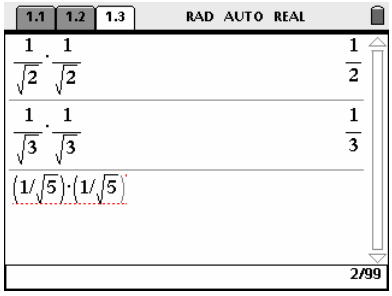

At this point, we can go back to two concepts omitted in the first section. What happens if we multiply two radicals which have common factors and what happens if we multiply mixed radicals? With the two sections completed above, most students will be able to follow the algebra completed by the device.



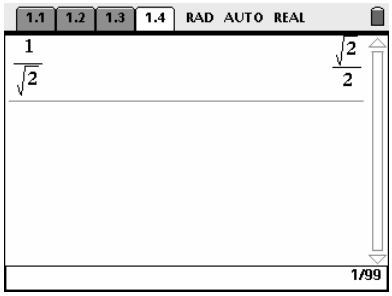
Moving on we can use the CAS to explore the concept of rationalizing denominators. Although it may be cumbersome to enter the expression to the right, it displays quite nicely. I want to know if the answer makes sense based upon the question.



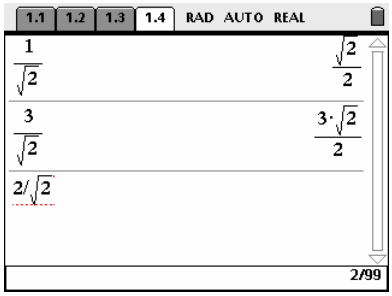

Doing another example and posing a third should help to solidify the concept. Notice how the expression is entered compared to how it is displayed. However, this has not completely addressed the idea of rationalizing the denominator. Once again, I want them to make up a few on their own and test them out on their own CAS device.



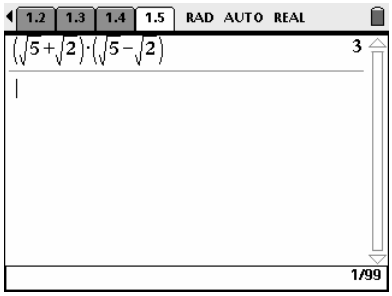

I prefer developing the concept in this manner so that the class can move on to rationalizing the denominator skills.



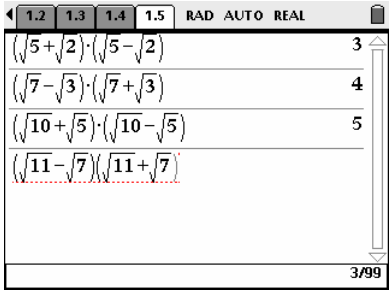

There may be a few students who can put the pieces together after one example, but most will need to see more. The really nice feature about using a CAS is that a large number of examples can be worked out in a short period of time, allowing students to pick up patterns. One problem that I have found in the past is that I haven't explained the idea of "rationalizing" well enough for them to understand what the result should look like. This is a good time to do that.



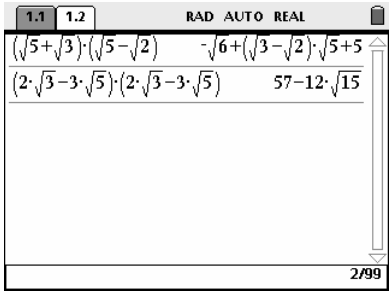

Of course, we may want to explore the products of conjugate radical expressions. I have found that this example catches the interest of students immediately. Almost everyone sees the pattern here and it's a good opportunity to use the CAS device to go back to a review of factoring the difference of squares. Hopefully, it's review and not new material - sigh.



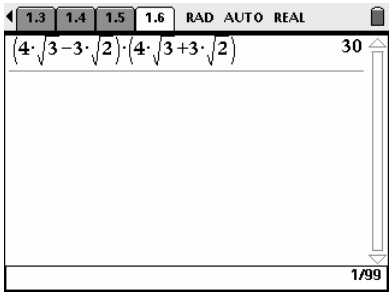
Just to ensure that the concept has been understood, we present a few more examples. A final example has been left unsolved and I want them to predict the answer. I also want to clarify that the product of these radical binomials produces a whole number or integer result. Afterwards, they are to make up a few of their own and test them.



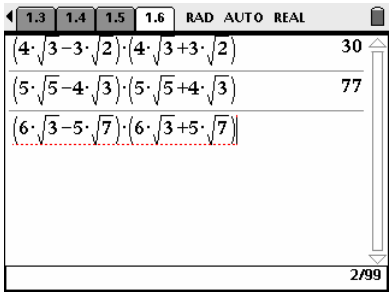

Further, the CAS allows us to quickly explore what happens if the binomials are not conjugates. The second product is a very common error made by students. This example shows clearly that the factors used do not produce an integer result. Students should verify this result with a few examples of their own.



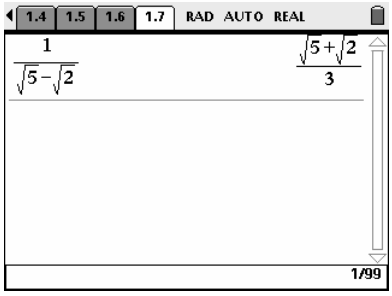

Hopefully, someone in the class is going to extend the idea. If no one does, I can present an example such as this one to see how the pattern extends. This is going to take a bit of explaining, but I'm hoping that students working in groups will come up with an explanation long before I have to. I haven't been disappointed yet.



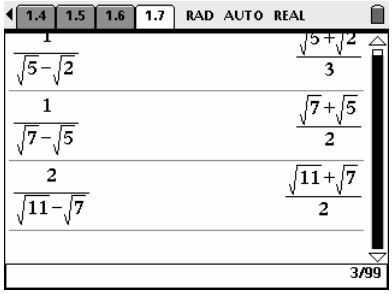

If this concept can be explained, then another example is needed to test that explanation. A third example is there for them to work on by themselves. Once again, it is an easy matter to try out examples where the factors are not conjugates.




Finally, we move to expressions where the CAS is used to rationalize denominators of expressions where the denominator is a binomial. Not too many can see all of the steps that occur between the question and the answer, but normally someone is able to put a solution on the board with the missing steps included.



As always, more examples are helpful. What I want students to do at this point is to make up more examples and then try them without the CAS.




The black box capability of the device can be a very helpful teaching tool if used wisely. Allowing students to use it without guidance can be dangerous. That is not to say that students shouldn't be allowed to explore on their own. However, guided discovery allows us to use the device to explore and create a situation where students are required to explain the missing steps.