Math Objectives

- Students will recognize the effect of a horizontal and vertical translation on the graph of a function.
- Students will relate the transformation of a graph $y = f(x)$ by a horizontal and vertical translation to the symbolic representation of the transformation: that is $y = f(x) + k$ shifts the graph up or down and $y = f(x - h)$ shifts the graph right or left.
- Students will combine translations to describe the graph of the function $y = f(x - h) + k$ in terms of $y = f(x)$.
- Students will look for and make use of structure (CCSS Mathematical Practice).
- Students will use appropriate tools strategically (CCSS Mathematical Practice).

Vocabulary

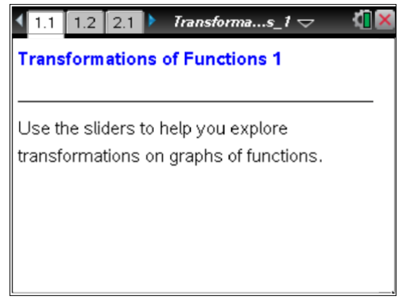
- vertical translation
- horizontal translation
- vertex of an absolute value function

About the Lesson

- This lesson involves investigating vertical and horizontal translations of a function.
- As a result, students will:
  - Recognize how transformations of the form $y = f(x) + k$ and $y = f(x - h)$ change the graph of $y = f(x)$.

TI-Nspire™ Navigator™ System

- Use Quick Polls to check student understanding.
- Use Screen Capture to examine patterns that emerge.
- Use Live Presenter to engage and focus students.
- Use Teacher Edition computer software to review student documents.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages

Tech Tips:

- Make sure the font size on your TI-Nspire handheld is set to Medium.
- You can hide the function entry line by pressing ctrl G.

Lesson Materials:

Student Activity

Transformations_of_Functions_1_Student.pdf

Transformations_of_Functions_1_Student.doc

TI-Nspire document

Transformations_of_Functions_1.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

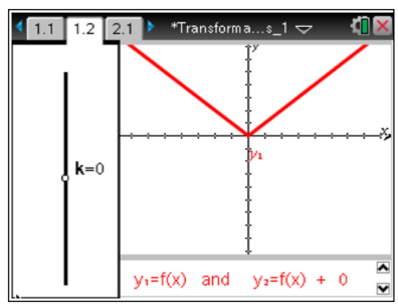
Discussion Points and Possible Answers

Tech Tip: If students experience difficulty grabbing and dragging the point, check to make sure that they have moved the arrow until it becomes a hand ( ) getting ready to grab the point. Press ctrl to grab the point and close the hand ( ). When finished moving the point, press esc to release the point.

Teacher Tip: You might want to break this activity into two different lessons, with the vertical translations in questions 1–3 as the first lesson and horizontal translations questions 4–7 as the second lesson.

It is important that students recognize that each tick on the horizontal and vertical axis represents one unit. They should also note that the basic function is represented by $y_1 = f(x)$, and the functions obtained by the translations are labeled $y_2 = f(x) + k$, where the subscripts are used to denote related but different functions.

Move to page 1.2.



Part One: *y* = *f*(*x*) + *k*.

1. Drag the point to change the value of *k*.
   a. What happens to the graph of $y_2 = f(x) + k$ as the value of *k* changes?

   <u>Answer:</u> As *k* increases, the graph shifts up. As *k* decreases, the graph shifts down.

   b. Move the point so that $k > 0$. Where is the graph of $y_2 = f(x) + k$ compared to $y_1 = f(x)$?

   <u>Answer:</u> For $k > 0$, the graph of $y_2 = f(x) + k$ is above the graph of $y_1 = f(x)$.

   c. Move the point so that $k < 0$. Where is the graph of $y_2 = f(x) + k$ compared to $y_1 = f(x)$?

   <u>Answer:</u> For $k < 0$, the graph of $y_2 = f(x) + k$ is below the graph of $y_1 = f(x)$.

d. Move the point so that $k = 0$. Where is the graph of $y_2 = f(x) + k$ compared to $y_1 = f(x)$?

Answer: For $k = 0$, the graph of $y_2 = f(x) + k$ is on the graph of $y_1 = f(x)$—the two graphs coincide.

TI-Nspire Navigator Opportunity: *Screen Capture*
See Note 1 at the end of this lesson.

2. For each of the following statements, indicate if it is True or False, and explain why you think so.

a. When k is negative, the graph of $y_2 = f(x) + k$ is below the graph of $y_1 = f(x)$.

Answer: True. Each point on the graph $y_1 = f(x)$ is (x, y) and each point on $y_2 = f(x) + k$ is (x, $y + k$). When k is negative, you are actually subtracting from the y-value, so the translated point is lower than the original point. This is true for all points on the graph.

b. When k is positive, the graph of $y_2 = f(x) + k$ is below the graph of $y_1 = f(x)$.

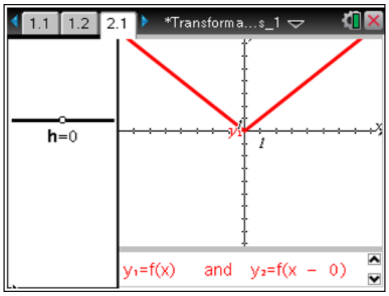
Answer: False because adding a positive value to $f(x)$ shifts the graph up by that amount.

c. There is a value of k that will make part of the graph of $y_2 = f(x) + k$ above the graph $y_1 = f(x)$ and the rest below the graph of $y_1 = f(x)$.

Answer: False because the way k affects one point is the way it will affect all of the points. The whole graph moves together for any value of k.

TI-Nspire Navigator Opportunity: *Quick Polls*
See Note 2 at the end of this lesson.

Teacher Tip: You might want to point out to students that a vertical shift is called a vertical translation.



Move to page 2.1.

Part Two: *y* = *f*(*x* – *h*)

4. Drag the point to change the value of *h*.
   a. What happens to the graph of $y_2 = f(x-h)$ as the value of *h* changes?

   Answer: As *h* increases, the graph shifts right. As *h* decreases, the graph shifts left.

   b. Move the point so that *h* > 0. Where is the graph of $y_2 = f(x-h)$ compared to $y_1 = f(x)$?

   Answer: For *h* > 0, the graph of $y_2 = f(x-h)$ is to the right of the graph of $y_1 = f(x)$.

   c. Move the point so that *h* < 0. Where is the graph of $y_2 = f(x-h)$ compared to $y_1 = f(x)$?

   Answer: For *h* < 0, the graph of $y_2 = f(x-h)$ is to the left of the graph of $y_1 = f(x)$.

   d. Move the point so that *h* = 0. Where is the graph of $y_2 = f(x-h)$ compared to $y_1 = f(x)$?

   Answer: For *h* = 0, the graph of $y_2 = f(x-h)$ is on the graph of $y_1 = f(x)$—the two graphs coincide.

5. The graph of $y_2 = f(x-h)$, when $h \neq 0$, is a *horizontal shift* of the graph of $y_1 = f(x)$. Why does the graph shift horizontally?

   Answer: A transformation of the form $x \to x - h$ shifts all of the points in the original function either right or left because you are subtracting a constant from each *x*-value.

> **Teacher Tip:** This is a very complicated concept, and one that is counter-intuitive. Encourage students to think about what happens to points such as the zeros of the function or the minimum values of a transformed function under the horizontal shift to see what would be necessary in order to return to the base (parent) function. You might want to tell students that a transformation of this form is called a horizontal translation.

6. All of the functions on pages 1.2 through 3.1 were of the absolute value type. In other words, $y_1 = |x|$ and $y_2 = |x-h|+k$. How does the graph of $y_2 = |x-2|-1$ compare to the graph of $y_1 = |x|$? Use page 3.1 of the tns document on your handheld to help determine the transformations.

Answer: To obtain the graph of $y_2 = |x-2|-1$, the graph of $y_1 = |x|$ is shifted right 2 units and down 1 unit.

7. The vertex of the absolute value graph function is where the graph turns back up or goes back down. What is the vertex of $y_2 = |x-2|-1$? Check with page 3.1.

Answer: The vertex is (2,-1)

8. What is the vertex of $y_2 = |x+3|+4$? What is the vertex of $y_2 = |x-h|+k$?

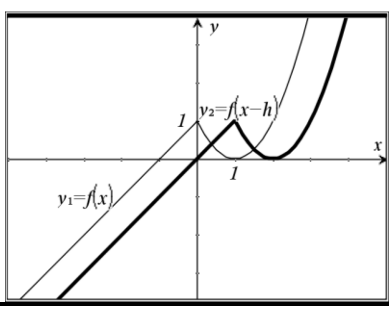
Answer: The vertex is (-3,4) and (h,k)

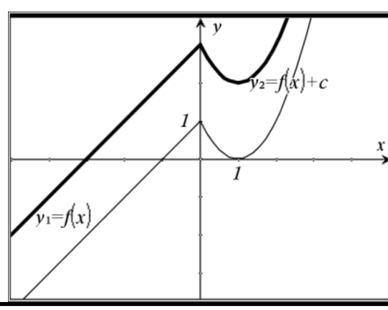
9. Pages 4.1 and 5.1 use a different type of function. For each transformation in a, b and c, describe the changes in the graph of $y_1 = f(x)$ shown. Then sketch the graph of y_2.

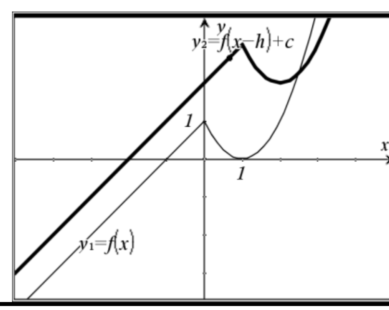
Use pages 4.1 and 5.1 to check answers

Answers:

a. $y_2 = f(x-1)$	b. $y_2 = f(x)+2$	c. $y_2 = f(x-1)+2$
shifts right 1 unit	**shifts up 2 units**	**shifts right 1 unit, up 2 units**







TI-Nspire Navigator Opportunity: *Quick Polls*
See Note 3 at the end of this lesson.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- The transformation $y = f(x) + k$ on a function $y = f(x)$ will result in a vertical shift, while the transformation $y = f(x – h)$ will result in a horizontal shift.
- How to distinguish between horizontal and vertical translation both graphically and symbolically.
- When a table has x-values that repeat but different y-values, then it does not represent a function.
- How to graph a function $y = f(x – h) + k$ given the function $y = f(x)$.
- Why a transformation of $y = (x – h)$ will shift to the right if h is positive and to the left if h is negative.

TI-Nspire Navigator

Note 1

Question 1, *Screen Capture*: Whenever visual patterns from the class are involved, this is an ideal application for the Screen Capture feature of TI-Nspire Navigator.

Note 2

Question 2, *Quick Poll*: Quick Polls are suitable whenever either open or closed questions are asked; used with the slideshow feature, they allow students to learn from the contributions (and mistakes) of others. Use the True False feature to ask students to submit their answers to questions 2a, b, and c. Discuss why and why not.

Note 3

Question 7, *Quick Poll*: Use the Open Response feature. Send students the following prompts, one at a time, and ask them to tell how the graph of the function that is sent to them differs from the original function, $y = f(x)$. Use ‘up’, ‘down’, ‘left’, ‘right’, and how many units in the answers.

1. $y = f(x + 2)$ Answer: left 2
2. $y = f(x) + 2$ Answer: up 2
3. $y = f(x – 3)$ Answer: right 3
4. $y = f(x) – 3$ Answer: down 3
5. $y = f(x + 1) – 4$ Answer: left 1, down 4