## Cipher Solvers

Teacher Notes

## Math Objectives

- Students will discover how to encrypt and decrypt a message.
- Students will learn what a Caesar cipher is and use it to solve a riddle.
- Students will find equations of lines given two points on the line.
- Students will solve systems of linear equations to find vertices of triangles.
- Students will find equations of lines that are perpendicular bisectors of given segments.


## Vocabulary

- Ordered pair
- Vertex of a triangle
- Equation of a line
- Perpendicular bisector


## About the Lesson

- Students need to be able to write equations of lines given two points on the line.
- Students need to be able to solve a system of two linear equations.
- Students will be finding the equation of the perpendicular bisector of a side of a triangle.
- Students are expected to work in groups of size 3 or 4 . The directions will often have students only find one equation instead of three, asking that each student in the group do a different equation.



## TI-Nspire ${ }^{\text {TM }}$ Navigator ${ }^{\text {TM }}$

- Use Class Capture to monitor student's use of the TI-Nspire document.


## Activity Materials

- Compatible TI Technologies: 进 TI-Nspire ${ }^{\text {TM }}$ CX Handhelds,


TI-Nspire ${ }^{\text {TM }}$ Apps for iPad®, $\square_{\text {TI-Nspire }}{ }^{\text {TM }}$ Software

Teacher Note: You have the option of having your students work through this problem using geometry and algebra, or using only algebra. See pages 6 and 7 of this document for the breakdown.

Cipher Solvers
Teacher Notes

1. Open the document: Cipher_Solvers.tns.

Read the opening screen then move to page 1.2,
Day 1, Strange Message.

Read pages 1.3 through 1.8. Make notes as needed.
2. Move to page 1.9. Decode the message in the space provided below:
Answer: Students may need assistance with the shift.
$\begin{array}{llllllllllllll}A & B & C & D & E & F & G & H & I & J & K & L & M\end{array}$
$\begin{array}{lllllllllllll}\mathbf{N} & \mathbf{O} & \mathbf{P} & \mathbf{Q} & R & S & \mathbf{T} & \mathbf{U} & \mathbf{V} & \mathbf{W} & \mathbf{X} & \mathbf{Y} & \mathbf{Z}\end{array}$
The riddle is: The black bird in the book of Poe so the answer is: RAVEN
3. Read pages 1.10 through 1.13.

On page 1.14, type the five-letter word that is the answer to the riddle and select enter.
4. The screen is changed to the screen shown at the right.

Read and follow the instructions displayed.
Once you are ready to continue, select the esc key to close these instructions.
5. Grab the gray template and drag the template to be on top of the grid of numbers.


## 

Using a Caesar shift of 13 letters,
decode the strange message

Gur oynpx oveq va gur obbx bs Cbr

Make a note of the five-letter word that is the answer to this riddle.

### 41.12 1.13 1.14 - *Cipher_Solvers $\nabla$ PAD

Under the grid, Bobby notices another strange phrase,
'According to the clock: $0,90,270 .{ }^{\prime}$
This might tell us how to place the template on the grid,' Bobby said.
Let's try the 0 first.
(make note of the letters and the number that each letter is paired with.)

## Reset


6. The phrase said, "According to the clock: $0,90,270$." Each of the four numbers in the grid have a letter assigned to it. Write the letter and the number assigned to it for the first orientation below.
Answer: (in the first case, rotate $0^{\circ}$.)
A 4 $\qquad$ M 2 H 20
7. With your cursor, select the appropriate button to rotate the template.

Each of the four numbers in the grid have a letter assigned to it. Write the letter and the number assigned to it for the second orientation below.

## Answer: (students should select 'Rotate $90^{\circ}$ ')

$\underline{T 26} \quad \underline{\text { M } 16} \quad \underline{\text { A } 18}$
8. With your cursor, select the appropriate button to rotate the template.

Each of the four numbers in the grid have a letter assigned to it. Write the letter and the number assigned to it for the third orientation below.
Answer: (students should select 'Rotate $180^{\circ}$ ', which is a total of $270^{\circ}$.)
$\underline{\mathrm{A} 6} \quad \underline{\mathrm{H} 9} \quad \underline{\mathrm{M} 10} \quad \underline{1}$
9. Move to page 2.1. Read pages 2.1 through 2.3.

Find at least one word that can be formed with these letters:
$\begin{array}{llll}\boldsymbol{A} & \boldsymbol{T} & \boldsymbol{M} & \boldsymbol{H}\end{array}$
Write your answer(s) below.
Answer:
MATH

Do you have any idea what word can be formed with these letters:
$\begin{array}{llll}\text { A } & \text { T } & \text { M } & H\end{array}$

Find at least one word before proceeding.
10. Move to page 2.4 and read it.
a. Using the instructions on page 2.4 , write the ordered pairs using the numbers from problem 6.
$(2,4)$
$(10,20)$
M: $2 \quad$ A: $4 \quad$ T: 10
H: 20
b. Using the instructions on page 2.4, write the ordered pairs using the numbers from problem 7.
$\qquad$ $(26,3)$
c. Using the instructions on page 2.4, write the ordered pairs using the numbers from problem 8.
$\qquad$
$(10,6)$

## Cipher Solvers

Teacher Notes
11. Move to page 2.5.

Grab and move the points so that each of the points listed in question 10 is plotted. Use the same color for each pair of points. Then, plot the points on the Cipher Solver Graph Paper supplied.
NOTE: have your students do their graphing on both the handheld and on the graph paper - a blank copy and key
 are at the end of the document. Each student needs one graph.
12. Move to page 2.6 and read it.

Determine the equation of the line for each set of points listed in question 10.
(Note: Each person in your group should work on just one equation, but a different equation.)
Show your work below.
Answer: The example shown is for $(10,6)$ and $(1,9)$

Bobby plots the points on the map and says, 'It doesn't look like anything.'

Alison replies, 'Hey, wait... What if we connect the points from each set with a line?'

Create the equations of the three lines and write those equations on your paper.

Go back to page 2.5 and graph
those three equations, then proceed to 2.7 .
$m=\frac{6-9}{10-1}$
$y=m x+b$
$y=-\frac{1}{3} x+\frac{28}{3}$
$m=\frac{-3}{9}$
$6=-\frac{1}{3}(10)+b$
$m=-\frac{1}{3}$
$\frac{18}{3}=-\frac{10}{3}+b$
$\frac{28}{3}=b$
13. Write the three equations determined by the members of your group below.

## Answer:

$y=2 x ; y=-\frac{3}{2} x+42 ; y=-\frac{1}{3} x+\frac{28}{3}$
14. Go back to page 2.5 and graph these three equations (also graph on the graph paper). The equations will not display on the screen, but the graphs will display. Then move to page 2.7. What do these three lines determine?
Answer: A triangle

## Cipher Solvers

Teacher Notes
15. Move to page 2.8
a. Determine the vertices of the triangle created by these lines.

Each person in the group will determine the ordered pair for just one vertex, but a different vertex. Show your work below. Answer: Example below is for $y=2 x$ and $y=-\frac{3}{2} x+42$

$$
\begin{aligned}
& 2 x=-\frac{3}{2} x+42 \\
& 4 x=-3 x+84 \\
& 7 x=84 \\
& x=12 \quad y=24
\end{aligned}
$$

\section*{| 2.6 | 2.7 | 2.8 |
| :--- | :--- | :--- |
| $>$ | Cipher_Solvers $\nabla$ |  | <br> RAD}

The three lines determine a triangle.
On your paper, determine the vertices of the triangle created by these three lines.

Each person in the group will find the ordered pair for a different vertex.

Plot these points on page 2.5.
b. Write the ordered pairs for the three vertices found by the members of your group below.
$\qquad$
$(4,8)$ $(12,24)$ $(28,0)$
c. Plot these points on page 2.5 .
16. Move to page 3.1. Read it and discuss in your group how you might find a point that is equidistant from the three vertices of a triangle.
It is possible that students may not have any idea how to do this. They will get some ideas on pages 3.2 and 3.3.

17. Read pages 3.2 and 3.3. Make notes as necessary.
a. Discuss in your groups how you know that line $k$ is the perpendicular bisector of segment AB. Write your answer below.
Answer: $\vec{k}$ contains E , the midpoint of $\overline{A B}$ and $\vec{k} \perp \overline{A B}$ at point E .
b. This diagram only shows that points C and E are equidistant
 from the segment's endpoints. Grab and move point $C$ along
$\vec{k}$. Does it appear that $A C=B C$ no matter where $C$ is?

## Answer:

Yes. Encourage students to move C to several places.

## IMPORTANT:

If your teacher has instructed you to finish solving this activity with only algebra (no geometry), then move to page 4.1 in the TI-Nspire document and skip to number 21 (Algebra only solution) on the next page.

If you are solving this with geometry and algebra, move to page 3.5 in the TI-Nspire document, then continue with number 18 below.
(Continue here for geometry and algebra solution)
18. Read pages 3.5 and 3.6.

We will first geometrically construct the lines that are perpendicular bisectors of each side of the triangle. Go back to page 2.5 to do this.


Repeat for each of the other two sides.
(You may need to grab the "end" of one of the lines to extend it to the edge of the page.)
What do you notice about the three perpendicular bisectors?

## Answer:

They intersect at a single point (they are concurrent).
20. Confirm your answer to number 19 using algebra.

Determine the equations of the perpendicular bisectors of each of the three sides of the triangle.
(Each person in the group should determine a different one of these). Show your work below.
Answer:
Side: $y=-\frac{3}{2} x+42$
$y=m x+b$
$y=\frac{2}{3} x-\frac{4}{3}$
$m_{\perp}=\frac{2}{3}$
$12=\frac{2}{3}(20)+b$
Midpoint : $\left(\frac{12+28}{2}, \frac{24+0}{2}\right)$
$\frac{36}{3}=\frac{40}{3}+b$
$(20,12)$

$$
-\frac{4}{3}=b
$$

## ***Continue to number 22 on the next page.

## Cipher Solvers

Teacher Notes

## (Continue here for algebra only solution)

21. Each person in the group will pick a different side of the triangle and will find the equation of the line that is perpendicular to that side and that passes through the midpoint of the side. You will need to decide which midpoint to use for each side of the triangle.
Midpoints: $(20,12)$
$(16,4)$
$(8,16)$

Show your work below.
Answer:
$y-12=\frac{2}{3}(x-20)$
$y-16=-\frac{1}{2}(x-8)$
$y-\frac{36}{3}=\frac{2}{3} x-\frac{40}{3}$
$y-16=-\frac{1}{2} x+4$
$y-4=3(x-16)$
$y-4=3 x-48$
$y=\frac{2}{3} x-\frac{4}{3}$
$y=-\frac{1}{2} x+20$
$y=3 x-44$

## (All students continue with this question)

22. Write the three equations found by the members of your group below.

Answer:

$$
y=\frac{2}{3} x-\frac{4}{3} \quad y=-\frac{1}{2} x+20 \quad y=3 x-44
$$

Graph these three lines on page 2.5 (and on the graph paper) to confirm that they are correct. Make any necessary corrections.
Answer: See graph key at end of document.
23. Determine the coordinates where each pair of lines intersects. Use algebra to do so and show your work below. Each person in the group will find the intersection point of two different lines.
Answer:

$$
\begin{array}{rlrl}
\frac{2}{3} x-\frac{4}{3} & =-\frac{1}{2} x+20 & y & =-\frac{1}{2}\left(\frac{128}{7}\right)+20 \\
4 x-8 & =-3 x+120 & y & =-\frac{64}{7}+\frac{140}{7} \\
7 x & =128 & y & =\frac{76}{7} \doteq 10.857 \\
x & =\frac{128}{7} \doteq 18.286 & &
\end{array}
$$

24. Write the coordinates of the points of intersection found by the members of your group below. Express them in two ways: as an exact fraction and rounded to the nearest thousandth.

## Answer:

Eq 1 and Eq $2 \quad\left(\frac{\mathbf{1 2 8}}{\mathbf{7}}, \frac{\mathbf{7 6}}{\mathbf{7}}\right)$
(18.286, 10.857)


Eq 2 and Eq $3 \quad\left(\frac{\mathbf{1 2 8}}{\mathbf{7}}, \frac{\mathbf{7 6}}{\mathbf{7}}\right)$
(18.286, 10.857)

Eq 1 and Eq $3 \quad\left(\frac{128}{7}, \frac{76}{7}\right) \quad(\mathbf{1 8 . 2 8 6}, 10.857)$
25. Move to page 5.1. Follow the instructions and enter the coordinates of the intersection you determined in question 22.
Type the $x$-coordinate, then select tab. Type the $y$-coordinate.
Select enter.
Did you find the treasure?
If so, great!
If not, follow the directions on the screen.


Answer:
A solid gold key
26. Read pages 6.1 and 6.2. Then move to page 6.3.


## Teacher Notes

27. Select $\mathbb{E}$ to Encrypt or select the Encrypt button.

Type an encryption key: any 4- to 12-digit number and select enter . Write it below.
Example: 4321

Type your message using only letters and spaces.
When finished typing, select enter. Write the message below.

## Example: I love math

The encrypted message is displayed, along with the key.
Write it below (it may contain letters and spaces).
N qtaj rfym

Select 'clear' twice, by selecting ctri del twice. This will navigate back to the main menu.

Select $\boldsymbol{\square}$ to decrypt. Follow the instructions on the screen.

Do another encrypted message, if you like. Share within your group.

## 4in Cipher Solvers

## Teacher Notes <br> $\square \square \square$

## Cipher Solver Graph Paper

Students should do ALL graphing on both the calculator and on this graph paper.


Cipher Solver Graph Paper
Students should do ALL graphing on both the calculator and on this graph paper.


## Graph paper key:



