Teachers Teaching with Technology

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Matrices

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MATRICES

Aim

To demonstrate how the TI-83 can be used to solve problems involving matrices and suggest an investigative approach to matrix arithmetic.

Objectives

Mathematical objectives

By the end of this topic you should

- Know what a matrix is composed off.
- Know how to add and subtract matrices.
- Know how to multiply matrices by a scaler.
- Know when it is possible to multiply two matrices and how it is done.
- Know the terms identity matrix , determinant, transpose and inverse matrix.

Calculator objectives

By the end of this topic you should

- Be able to use the MATRIX menu to edit and calculatevarious aspect of matrices.
- Be able to “paste” matrix names into the home screen to allow calculations to be achieved.
- Be able to delete matrices from the TI-83.

MATRICES

A matrix is nothing more than a rectangular array of numbers surrounded by brackets.

For example $$A = \begin{pmatrix} 2 & 3 \\ 6 & 1 \end{pmatrix}$$

A matrix containing *M* rows and *N* columns is said to be an $M \times N$ matix.

Example

Suppose a music shop records the sale of three types of album; album A, Album B and album C and they also record the format on which it is sold either tape or CD.
This information may be recorded as shown.

	A	B	C
Tape	5	2	1
CD	10	7	4

This is an array of numbers which could represent the sales in any given week. Notice that position in the array is important since row 1 makes the sale a tape and row 2 makes the sale a CD.

Clearly standardisation (all branches agreeing that tapes are always row 1 and CDs row 2) of how things are recorded would mean that only the array is required.
Thus sales could be reported as

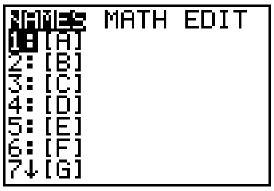
5	2	1
10	7	4

In mathematics it is common to surround such arrays with brackets and give them a name (S for sales say).

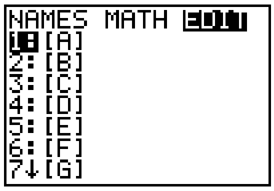
The array above then becomes $$S = \begin{pmatrix} 5 & 2 & 1 \\ 10 & 7 & 4 \end{pmatrix}$$

It is clear then that we may wish to add and subtract matrices (since we may want to add up all the sales of a number of branches) and perform all the usual algebriaic operations.

Entering matricies on the TI-83



Press MATRIX to get to the matrix menu.



Then use the cursor keys to move across to the edit screen to enter a matix then press ENTER

You then enter the matrix size rows first ENTER

Then ENTER twice to move to enter the number of columns.

Then ENTER again to move to putting the elements into thier position in the matrix.

We press ENTER after each element is typed and move across the row and automativcally down the columns.

We have entered the matrix $[A] = \begin{pmatrix} 4 & 3 \\ 1 & 3 \end{pmatrix}$ notice the TI-83 puts square brackets around the matrix name.

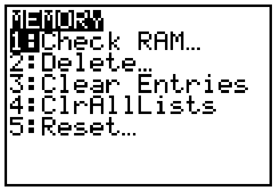
Try entering the following

$$[B] = \begin{pmatrix} 7 & 8 \\ 5 & 2 \end{pmatrix} \qquad [C] = \begin{pmatrix} 1 & 2 & 4 \\ 2 & 1 & 6 \end{pmatrix}$$

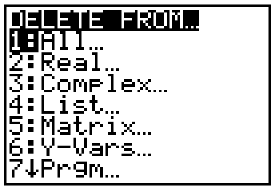
Deleting a Matrix

How do we delete the matricies entered earlier ?

Press 2nd + (MEM !!) followed by 2 since we wish to delete.

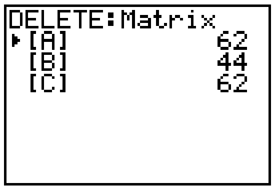


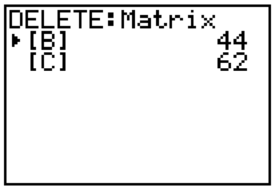
At this screen we choose option 5:Matrix



We now have a list of all the matrices on the TI-83. To delete we simply press ENTER when the matrix we wish to delete is highlighted by the arrow

Pressing ENTER again deletes the next arrowed matrix and so on.





Do this now to remove the previously entered matricies.

Matrix algebra

Example
Suppose two shops return the following sales as matricies.

Shop A $$S_A = \begin{pmatrix} 5 & 2 & 1 \\ 10 & 7 & 4 \end{pmatrix}$$

Shop B $$S_B = \begin{pmatrix} 4 & 3 & 7 \\ 1 & 8 & 6 \end{pmatrix}$$

What would the total sales be?

Clearly we would add each element, so

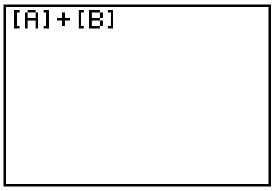
$$S_T = \begin{pmatrix} 5+4 & 2+3 & 1+7 \\ 10+1 & 7+8 & 4+6 \end{pmatrix}$$

$$S_T = \begin{pmatrix} 9 & 5 & 8 \\ 11 & 15 & 10 \end{pmatrix}$$

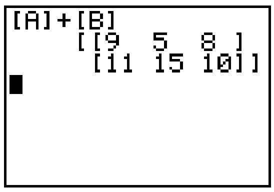
Enter S_A from above in matrix [A] on the calculator and S_B in matrix [B]

To add these on the home screen

Press MATRIX then 1 + MATRIX then 2 to get.



Press ENTER to get



Is this result is true for all matricies?

Enter the matrix $\begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$ in $[C]$ on the calculator.

Now try $[A] + [C]$

What does the TI-83 return as the result?

Write an explaination as to why you think this is

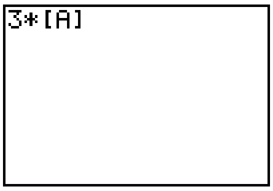
Does your explanation hold true for subtraction?

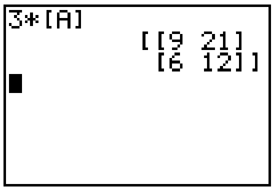
Write down a condition for the addition and subtraction of two matricies and give an example to illustrate what you mean.

Multiplication by a constant

Enter the matrix $\begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix}$ on the calculator in location $[A]$

Now try multiplying this matrix by 3





Look at this result, what can you say about each element of matrix A?

Try this for a few matricies of different sizes and try different multipliers (including negatives)
Write down some of the examples you used.

Write down the general result of multiplying a matrix by a constant.

$$k\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} =$$

Matrix multiplication

Suppose the matrix $[S]$ below represents total sales of some CD's in one week.

$$[S] = [\ 20 \quad 25 \quad 10\ ]$$

Suppose the prices of the three CD's are £5, £6 and £7 respectivly.
We could find the total value of the CD's sold by:

$$£(20 \times 5 + 25 \times 6 + 10 \times 7) = £320$$

The rules for multiplying matricies are quite complicated.
In our example above we can express the cost of CD's in the following form $[C] = \begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix}$

Notice this is in columnar form since we are looking at multiplying a single row by a single column. Enter the two matrices shown and calculate $[S] \times [C]$ on the calculator

$$[S] \times [C] = (20 \times 5 + 25 \times 6 + 10 \times 7)$$

This must mean that

$$\begin{bmatrix} 20 & 25 & 10 \end{bmatrix} \times \begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix} \qquad = (20 \times 5 + 25 \times 6 + 10 \times 7)$$

Now suppose that a second weeks sales are recorded as $[R] = [\ 3 \quad 15 \quad 5\ ]$

We can (using matricies) combine the calculation of weekly sales in matrix multiplication.

Sales could be represented by the matrix $\begin{bmatrix} 20 & 25 & 10 \\ 30 & 15 & 5 \end{bmatrix}$

to calculate the sales for the two weeks we would perform the calculation

$$\begin{bmatrix} 20 & 25 & 10 \\ 30 & 15 & 5 \end{bmatrix} \times \begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix} =$$

Enter the two matricies on the calculator and and write in the answer above.

In order for the answer to be true, what calculation must be carried out in order for it to work?
(fill in the blanks)

$$\begin{bmatrix} 20 & 25 & 10 \\ 30 & 15 & 5 \end{bmatrix} \times \begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix} = \begin{bmatrix} 20\times \quad +25\times \quad +10\times \\ \quad \times 5 + \quad \times 6 + \quad \times 7 + \end{bmatrix} =$$

Given there must be special rules for the multiplication of matricies can we discover them ?

Enter the following matricies and try as many combinations to multiply as you can, enter the results in the table below.

$$[A]=\begin{bmatrix}3 & 1\\ 2 & 0\\ 4 & 1\end{bmatrix} \quad [B]=\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix} \quad [C]=\begin{bmatrix}1 & 5 & 6\\ 2 & 3 & 4\end{bmatrix} \quad [D]=\begin{bmatrix}2 & 3 & 9\end{bmatrix}$$

$$[E]=\begin{bmatrix}3\\ 10\\ 3\\ 8\end{bmatrix} \quad [F]=\begin{bmatrix}3\end{bmatrix} \quad [G]=\begin{bmatrix}2 & 5 & 3\\ 1 & 0 & 9\\ 4 & 5 & 8\end{bmatrix} \quad [H]=\begin{bmatrix}1 & 3 & 10 & 0 & 4\\ 3 & 2 & 5 & 7 & 6\end{bmatrix}$$

Matricies	First Matrix		Second Matrix		Answer	
	Rows	Columns	Rows	Columns	Yes	No
$[A] \times [B]$	3	2	2	2		

Fill out the blanks in the following statement

Before two matrices can be multiplied together:
The first matrix must have the ______________number of ___________ as the second matrix has ____________

Delete all of matricies stored on the TI-83.

Enter the following

$$[A]=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix} \qquad [B]=\begin{bmatrix}5 & 6\\ 7 & 8\end{bmatrix}$$

$$[A]\times[B]=\begin{bmatrix} & \\ & \end{bmatrix} \qquad [B]\times[A]=\begin{bmatrix} & \\ & \end{bmatrix}$$

We can see from the above that $[A]\times[B]\neq\begin{bmatrix} \quad \end{bmatrix}\times\begin{bmatrix} \quad \end{bmatrix}$

Unlike numbers the order of matrix multiplication ***does*** matter.
We say that in matrix algebra, multiplication is ***not commutative.***

The Identity matrix

Enter the following matricies

$$[I]=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} \qquad [A]=\begin{bmatrix}2 & 3\\5 & 9\end{bmatrix}$$

Now perform the follow

$$[A]\times[I]=\begin{bmatrix} & \\ & \end{bmatrix} \qquad [I]\times[A]=\begin{bmatrix} & \\ & \end{bmatrix}$$

Make a statement about your findings

We have found that $[I] \times [A] = [A] \times [I]$

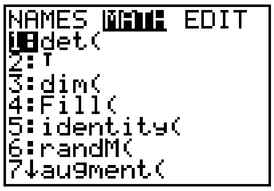
This is very similiar to the way the real number 1 behaves in ordinary algebra. The matrix is a special matrix given a special name, it is usually called the ***unit matrix*** or the ***Identity Matrix*** $[I]$. More generally, if $[A]$ is any $n \times n$ matrix, then the corresponding unit matrix is an matrix, with 1's along the leading diagonal and 0's elswhere.

So the 3×3 unit matrix is $\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$

Other Matrix operations on the TI-83

The TI-83 also performs many of the standard matrix operations. These are to be found in the matrix math menu shown.

To get to these functions press [MATRIX] and the right hand cursor key. It is important to note that some of these functions must be pasted to the home screen first and some second!

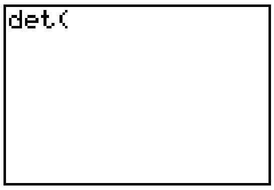


Finding the determinant of a matrix

Enter the following matrix in location $[A]$

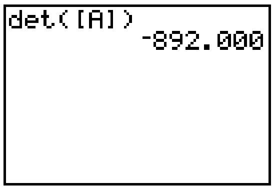
$$[A]=\begin{bmatrix} -10 & 4 & 3 \\ 5 & 8 & 2 \\ -8 & -12 & 6 \end{bmatrix}$$

To find the determinant of this matrix go to the menu above and press 1 to get the screen shown.



Press [MATRIX] followed by 1 to paste $[A]$ into the brackets (remember to close them)as shown.

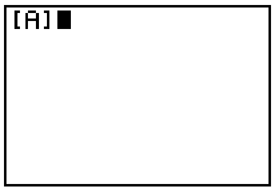
Press enter to see the result



Transpose of a matrix

In this case you paste the matrix to the home screen first.

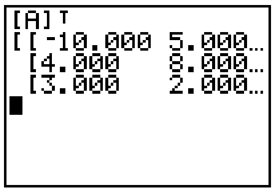
Press [MATRIX] followed by 1 to paste $[A]$ to the home screen



Now access the MATRIX: MATH menu and choose 2.

To see the result press [ENTER].

To view the other values use the cursor keys to move right.



Inverse of a matrix

In the past finding the inverse of a matrix usually involved tedious arithmetic and long calculation where the smallest arithmetic error could cause disaster. Thankfully such thing are behind us and the TI-83 can find quickly the inverse of a matrix. One of the useful applications of inverse matrices is in solving simultaneous equations.

Example

Solve the following:

$$7x + 9y = 3$$
$$5x + 7y = 1$$

Rewrite this as the matrix equation

$$\begin{bmatrix} 7 & 9 \\ 5 & 7 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

The solution is found by premultiplying by the inverse.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 & 9 \\ 5 & 7 \end{bmatrix}^{-1}\begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

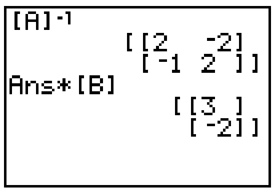
This is the equivalent of dividing in “normal” algebra

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

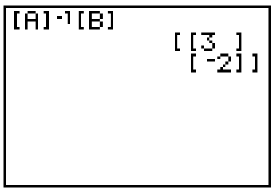
Giving this result

To do this on the TI-83 enter the matrix $\begin{bmatrix} 7 & 9 \\ 5 & 7 \end{bmatrix}$ in location $[A]$ and $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ in $[B]$

Paste matrix $[A]$ to the home screen and press x^{-1} ENTER to get the inverse then multiply by $[B]$ as shown



Alternatively you can do it in one line.



Exercise

1. Given that $[A]=\begin{bmatrix}5 & 2\\ 8 & 4\end{bmatrix}$ and $[B]=\begin{bmatrix}-5 & 3\\ -9 & -3\end{bmatrix}$ evaluate:

a) 3**A** b) 2**B** c) 3**A**+2**B** d) 3**A**-2**B**

2. Given that $[A]=\begin{bmatrix}3 & 1 & 4\\ 5 & 7 & 2\end{bmatrix}$ and $[B]=\begin{bmatrix}2 & -2 & 3\\ -5 & 7 & 5\end{bmatrix}$ find:

a) **AB** b) **BA**

3. Given that $[I]=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ and $[A]=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$ find **AI** and **IA**

4. Two matricies **X** and **Y** are members of the set $\left\{\begin{bmatrix}a & b\\ -b & a\end{bmatrix} : a, b \in \Re\right\}$

Prove that the product **XY** is also a member of the set.