

6 Two distributions

6.1 The binomial distribution

Suppose we throw a dice 20 times and the random variable X counts the number of times six spots appear. X has a binomial distribution with parameters $n = 20$, the number of trials, and $p = \frac{1}{6}$, the probability of success, six spots. The probability to have after 20 throws:

- exactly 4 times six spots, is: $P(X = 4) = \binom{20}{4} \left(\frac{1}{6}\right)^4 \left(\frac{5}{6}\right)^{16} = 0.202$,
- exactly 8 times "six or three": $\binom{20}{8} \left(\frac{1}{3}\right)^8 \left(\frac{2}{3}\right)^{12} = 0.148$,
- maximum 4 times a six: $P(X \leq 4) = \sum_{x=0}^4 \binom{20}{x} \left(\frac{1}{6}\right)^x \left(\frac{5}{6}\right)^{20-x} = 0.769$.

The **TI-83 Plus** can replace the distribution table or the calculation above by the following commands:

2nd[DISTR] 0:binompdf(and **2nd[DISTR] A:binomcdf(**

```
binomPdf(20,1/6,
4)
.2022035812
binomPdf(20,1/3,
8)
.1479796456
```

```
sum(seq(binomPdf(
20,1/6,X),X,0,4
)
.768749219
binomcdf(20,1/6,
4)
.768749219
```

```
binomPdf(2,1/2)
(.25 .5 .25)
binomcdf(2,1/2)
(.25 .75 1)
cumSum(binomPdf(
2,1/2))
(.25 .75 1)
```

6.2 The normal distribution

Suppose that the weight X of a group of students has a normal distribution with mean $\mu = 82$ kg and standard deviation $\sigma = 2$ kg – $X \sim N(82,2)$.

We will check the "68-95-99,7 rule" with the **normalcdf** command. This rule tells us that the surface under the graph of a normal distribution function between:

- $\mu \pm \sigma$ is equal to $0,68269 \approx 68\%$
- $\mu \pm 2\sigma$ is equal to $0,95450 \approx 95\%$
- $\mu \pm 3\sigma$ is equal to $0,99730 \approx 99,7\%$

```
0516 DRAW
1:normalcdf(
2:normalcdf(
3:invNorm(
4:tpdf(
5:tcdf(
6:X^2pdf(
7:X^2cdf(
```

```
normalcdf(80,84,
82,2)
.6826894809
normalcdf(78,86,
82,2)
.954499876
.954499876
```

```
.6826894809
normalcdf(78,86,
82,2)
.954499876
normalcdf(76,88,
82,2)
.9973000656
```

We will determine successively the percentage of students with a weight less than 79 kg and the weight below which 90% of the students' weights are situated. This weight is called the 90th percentile of the distribution.

```
normalcdf(-10^99,
79,82,2)
.0668072287
normalcdf(20,79,
82,2)
.0668072287
```

```
0516 DRAW
1:normalcdf(
2:normalcdf(
3:invNorm(
4:tpdf(
5:tcdf(
6:X^2pdf(
7:X^2cdf(
```

```
invNorm(.90,82,2)
84.56310313
normalcdf(-10^99,
84.56,82,2)
.9997273665
```

With the **normalcdf** command you can plot normal distribution functions.

Define **Y1** as **normalpdf(X,82,2)** and plot **Y1** with the window setting below.

It's also possible to shade a region under the graph of a normal distribution function.

2nd[DISTR]<DRAW> 1:ShadeNorm(80,84,82,2) gives you a graphical representation of the fact that the surface under the graph between $\mu \pm \sigma$ is equal to 68%.

```
WINDOW
Xmin=77
Xmax=87
Xscl=1
Ymin=-.07
Ymax=.23
Yscl=1
Xres=1
```

