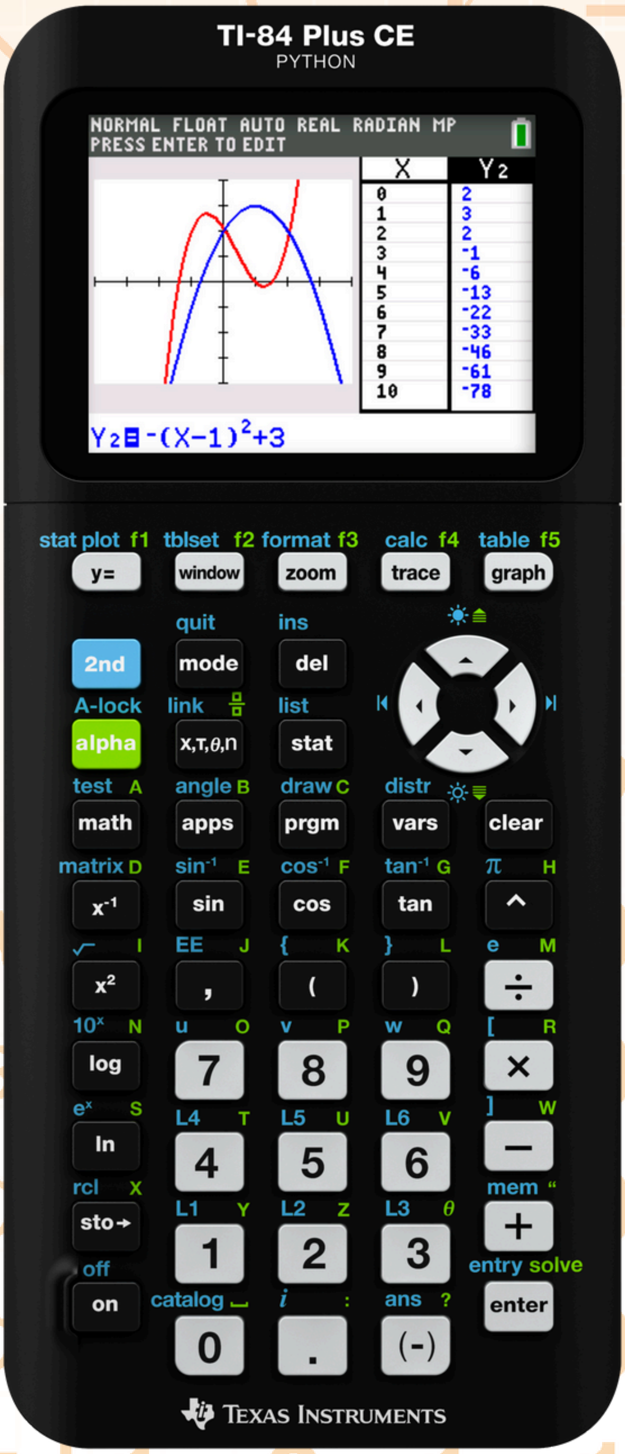
QCE- QCAA Mathematical Methods

2024 Examination 2 Solutions

For more **QCE resources**, visit:

http://www.education.ti.com/aus/qld



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TEXAS INSTRUMENTS

LUI | | | | | | | | | | School code | | | |

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Book   of   books used

External assessment 2024

Question and response book

Mathematical Methods

Paper 2 — Technology-active

Time allowed

- Perusal time — 5 minutes
- Working time — 90 minutes

General instructions

- Answer all questions in this question and response book.
- QCAA-approved calculator **permitted.**
- QCAA formula book provided.
- Planning paper will not be marked.

Section 1 (10 marks)

- 10 multiple choice questions

Section 2 (45 marks)

- 9 short response questions

Queensland Government | QCAA Queensland Curriculum & Assessment Authority

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Section 1

Instructions

- This section has 10 questions and is worth 10 marks.
- Use a 2B pencil to fill in the A, B, C or D answer bubble completely.
- Choose the best answer for Questions 1–10.
- If you change your mind or make a mistake, use an eraser to remove your response and fill in the new answer bubble completely.

	A	B	C	D
Example:	●	○	○	○

	A	B	C	D
1.	○	○	●	○
2.	●	○	○	○
3.	○	○	●	○
4.	○	●	○	○
5.	○	○	○	●
6.	●	○	○	○
7.	○	●	○	○
8.	○	○	○	●
9.	○	○	●	○
10.	○	●	○	○

Ensure you have filled an answer bubble for each question.

Do not write outside this box.

Section 2

Instructions

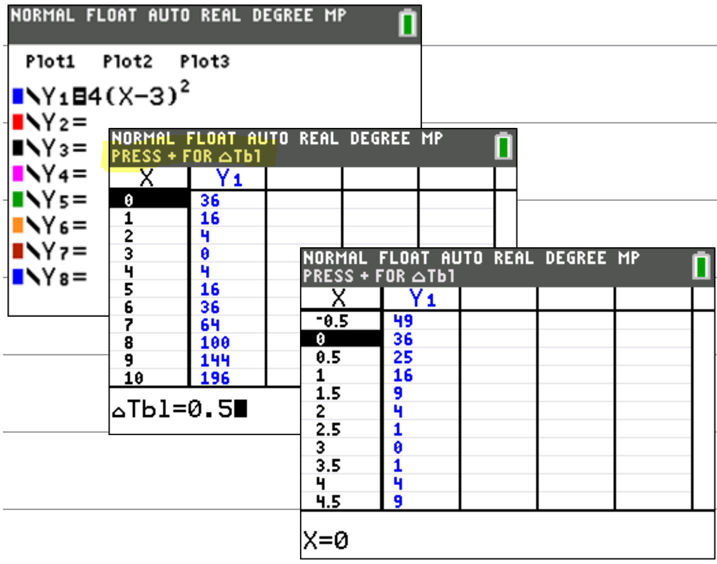
- Write using black or blue pen.
- Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.
- If you need more space for a response, use the additional pages at the back of this book.
  - On the additional pages, write the question number you are responding to.
  - Cancel any incorrect response by ruling a single diagonal line through your work.
  - Write the page number of your alternative/additional response, i.e. See page …
  - If you do not do this, your original response will be marked.
- This section has nine questions and is worth 45 marks.

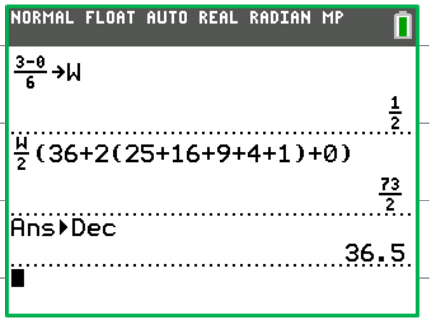
QUESTION 11 (4 marks)

State the trapezoidal rule and use it with six strips to determine an approximate value of the definite integral for the curve of $f(x) = 4(x-3)^2$ from $x = 0$ to $x = 3$. Show all substitutions made into the rule.

trapezoidal rule	$\int_a^b f(x)\,dx \approx \frac{w}{2}\left[f(x_0)+2\left(f(x_1)+f(x_2)+f(x_3)+\ldots f(x_{n-1})\right)+f(x_n)\right]$ where $w = \frac{b-a}{n}$

[1 mark]



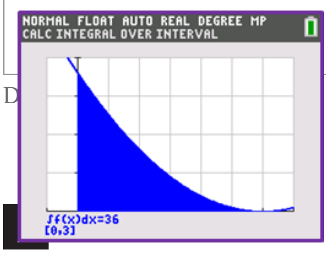


[1 mark]
[1 mark]

0.5/2(36+2(25+16+9+4+1)+0)

= 36.5 units squared [1 mark]

Note: It's good to calculate the exact integral, so that you know what your approximation should be close to.



QUESTION 12 (5 marks)

The magnitude of an earthquake can be modelled by the logarithmic equation $M_A = \log_{10}\left(\frac{I_A}{I_0}\right)$, where

M_A is the magnitude at a location A, I_A is the intensity of the earthquake at location A and I_0 is a constant.

An earthquake at location P had a magnitude of 5.2.

A different earthquake at location Q had a magnitude of 3.5.

a) Determine an equation involving logarithms that expresses the difference in magnitudes between the earthquakes at locations P and Q. *[1 mark]*

$M_P = \log_{10}\left(\frac{I_P}{I_0}\right)$, $M_Q = \log_{10}\left(\frac{I_Q}{I_0}\right)$

$$M_P - M_Q = \log_{10}\left(\frac{I_P}{I_0}\right) - \log_{10}\left(\frac{I_Q}{I_0}\right)$$ [1 mark]

b) How many times more intense was the earthquake at location P than the earthquake at location Q? *[4 marks]*

$$M_P - M_Q = \log_{10}\left(\frac{I_P/I_0}{I_Q/I_0}\right)$$

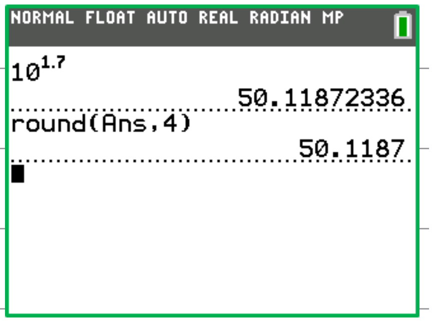
$$M_P - M_Q = \log_{10}\left(\frac{I_P}{I_Q}\right)$$ [1 mark]

$M_P = 5.2$ $\quad M_Q = 3.5$

$$5.2 - 3.5 = \log_{10}\left(\frac{I_P}{I_Q}\right)$$ [1 mark]

$$1.7 = \log_{10}\left(\frac{I_P}{I_Q}\right)$$

$$10^{1.7} = \frac{I_P}{I_Q}$$ [1 mark]



Therefore, the intensity at P is 50.1187 times that of the intensity at Q. [1 mark]

Do not write outside this box.

QUESTION 13 (8 marks)

The number of termites in a particular nest can be modelled by $N(t) = \dfrac{A}{2 + e^{-t}}$, where A is a constant and t represents time (months) since the nest first became a visible mound above ground level.

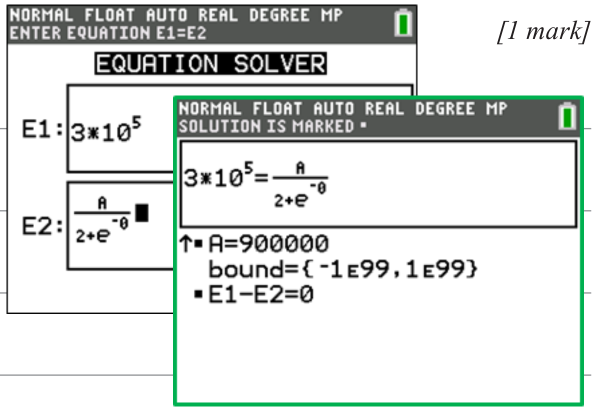
It is estimated that when the mound first became visible, the population was 3×10^5 termites.

a) Determine the value of A. *[1 mark]*

$A = 900000$

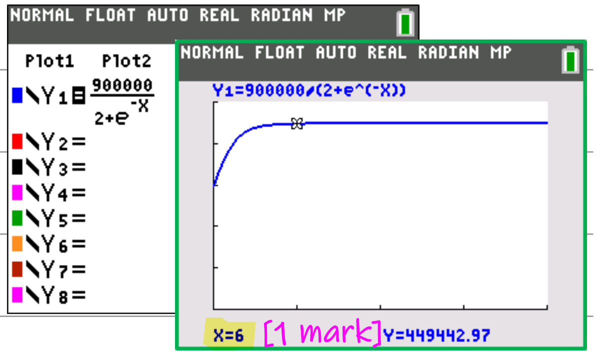
or

$A = 9 \times 10^5$ [1 mark]



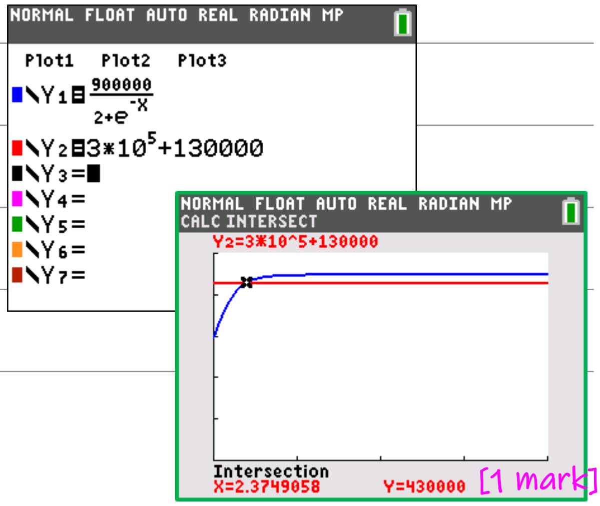
b) Determine the number of termites in the nest half a year after the mound became visible. *[2 marks]*

449443 termites [1 mark]



c) Determine the time in months after the mound became visible for the initial population to increase by 130 000 termites. Express the time as a decimal. *[2 marks]*

$t = 2.3749$ months [1 mark]



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d) Develop a formula for the rate of change in the number of termites at any time after the mound became visible. Express your formula as a fraction. *[2 marks]*

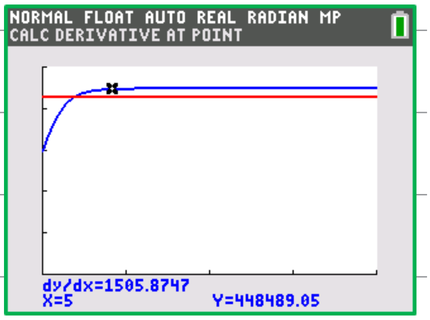
$$N(t) = \frac{A}{2+e^{-t}} = A(2+e^{-t})^{-1}$$

$$N'(t) = -A(2+e^{-t})^{-2} \times (-e^{-t}) = \frac{Ae^{-t}}{(2+e^{-t})^2} = \frac{A}{e^t(2+e^{-t})^2}$$

[1 mark]

$$N'(t) = \frac{9\times10^5}{e^t(2+e^{-t})^2}$$ [1 mark]

e) Determine the rate of change in the number of termites five months after the mound became visible. *[1 mark]*



1506 termites / months [1 mark]

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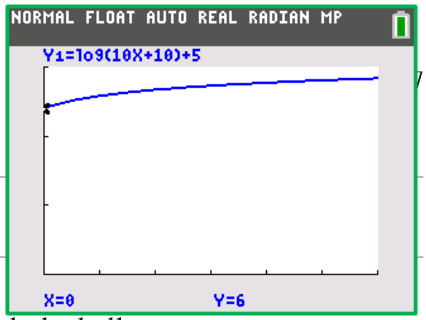
QUESTION 14 (6 marks)

A football coach offered a 12-day intensive training clinic. During the clinic, the height that each player could kick a football was monitored.

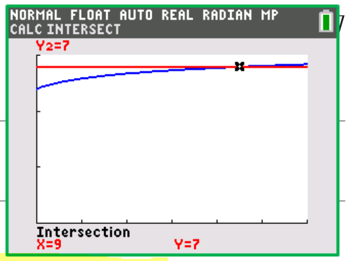
One player's kick heights could be modelled by $H(t) = \log_{10}(10t + 10) + 5$, $0 \le t \le 12$, where $H(t)$ is vertical height (m) and t is the time (days) spent in training.



a) Determine the initial height that the player could kick the ball.

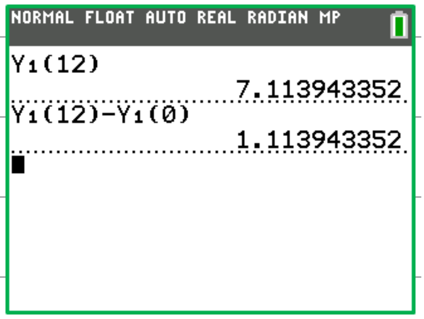
$H(0) = 6$ m [1 mark]

b) Determine the training time needed for the player to be able to kick the ball to a height of 7 m.



$t = 9$ days [1 mark]

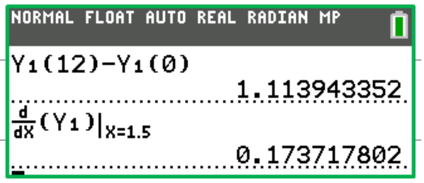
c) Determine the overall improvement in kick height achieved by completing the clinic. *[2 marks]*



[1 mark]

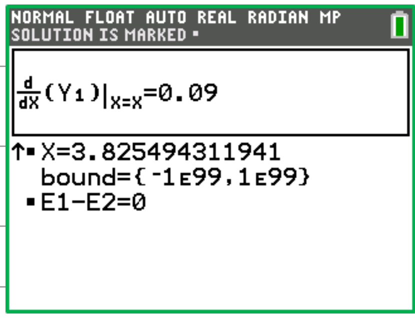
The kick height has increased by 1.1139 metres during the course.
[1 mark]

d) Determine the rate of change in kick height when $t = 1.5$ days. *[1 mark]*



$H'(1.5) = 0.17372$ m/day [1 mark]

e) Determine the training time (as a decimal) when the rate of change in kick height is 0.09 m/day. *[1 mark]*



$t = 3.82549$ days [1 mark]

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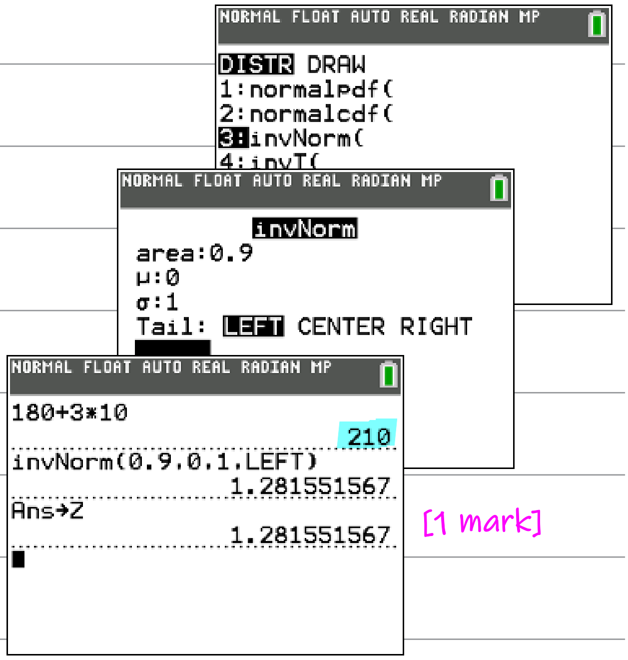
QUESTION 15 (4 marks)

The term *extremely tall* is used to describe any person whose height is three standard deviations or more above the mean height of the population.

A person who just qualifies as extremely tall in a country where heights are normally distributed with a mean height of 180 cm and a standard deviation of 10 cm travels to another country. The person discovers they are taller than exactly 90% of the destination country's population.

Assuming that the standard deviation of both countries is the same, determine the minimum height required to be considered extremely tall in the destination country.

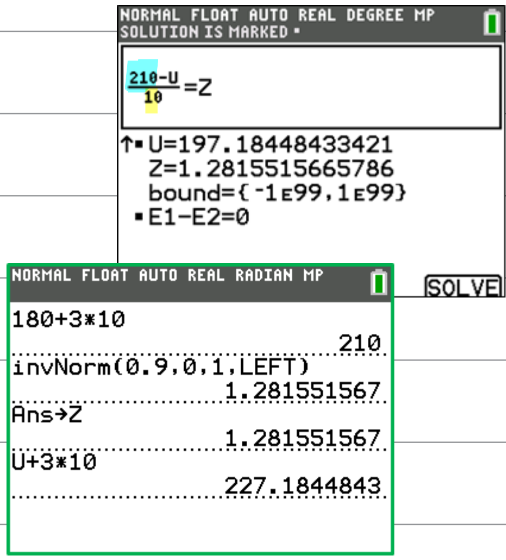


In the original country, the cut-off for extremely tall is:

210 cm

[1 mark]

[1 mark]



[1 mark]

The cut-off for extremely tall in the destination country is:

227.18 cm

[1 mark]

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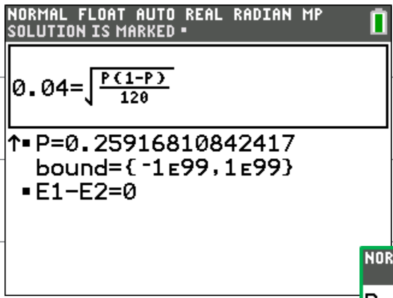
QUESTION 16 (4 marks)

At council meetings in a particular town, new proposals are only discussed if more than 80% of the community are in favour of the proposal.

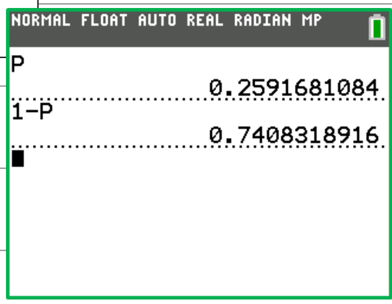
To discover community opinion on a new bus route proposal, the council conducted several surveys, each with a sample size of 120. The distribution of the sample proportions from the surveys had a standard deviation of 0.04.

Make a justified decision as to whether the new bus route proposal would be discussed at a council meeting.

sample proportion	mean	p
	standard deviation	$\sqrt{\frac{p(1-p)}{n}}$



[1 mark]



[1 mark]

[1 mark]

The council only discusses proposals if MORE THAN 80% are in favour, i.e. $p>0.8$

Our estimate from the reported sampling sd is $p=0.7408$, which is LESS THAN 0.8.

Therefore, based on the information given, the surveys do NOT indicate more than 80% support, so the proposal WOULD NOT be discussed. [1 mark]

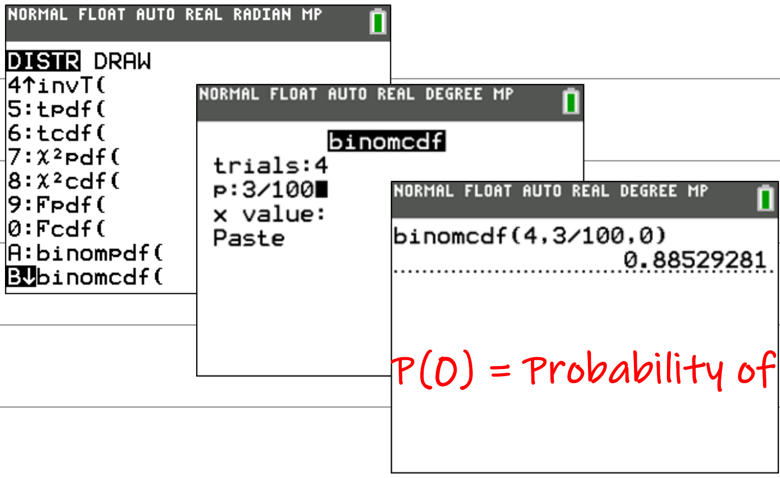
Do not write outside this box.

QUESTION 17 (3 marks)

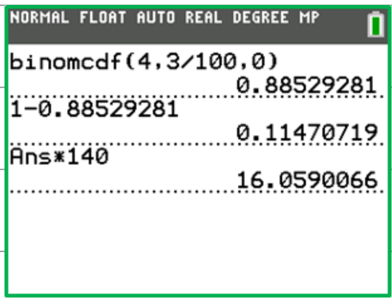
At a particular orchard, 3% of fruit is bruised during picking. After picking, the fruit is packed into boxes, each containing four pieces of fruit.

A grocery shop orders 140 boxes of fruit to sell to their customers.

Determine the expected number of boxes that will contain bruised fruit.



P(0) = Probability of NO bruised fruit = 0.8853



[1 mark]
[1 mark]

16 boxes can be expected to contain bruised fruit [1 mark]

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QUESTION 18 (5 marks)

An object experiencing straight-line motion along a path has an acceleration (m s^{-2}) defined by the function $a(t) = 3\sin(2t)$ where t is time (s) since the object begins moving, $t \geq 0$.

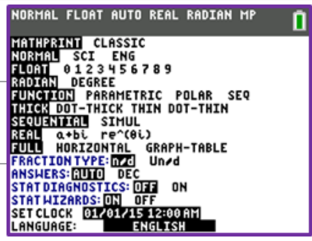
When $t = 0$, both displacement and velocity are zero.

On the path is a motion sensor that is able to detect motion up to 2 metres away.

The object passes directly by the motion sensor when $t = 3$.

Determine the average velocity of the object while it moves through the range of the sensor.

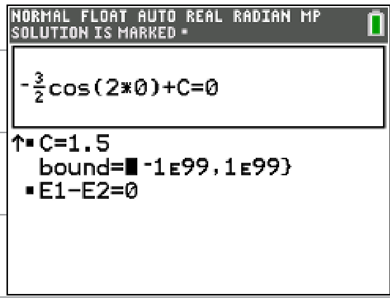
Note: make sure you are in RADIAN mode



$$v(t) = \int a(t)\,dt$$

$$= \int 3\sin(2t)\,dt$$

$$= -\frac{3}{2}\cos(2t) + c$$



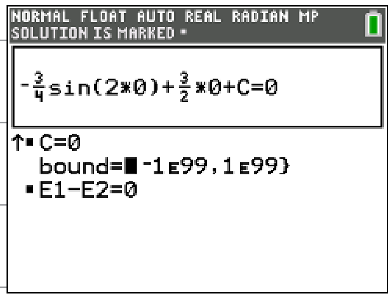
$$\therefore v(t) = -\frac{3}{2}\cos(2t) + \frac{3}{2}$$

[1 mark]

$$d(t) = \int v(t)\,dt$$

$$= \int -\frac{3}{2}cos(2t) + \frac{3}{2}\,dt$$

$$= -\frac{3}{4}sin(2t) + \frac{3}{2}t + c$$



$$\therefore d(t) = -\frac{3}{4}\sin(2t) + \frac{3}{2}t$$

[1 mark]

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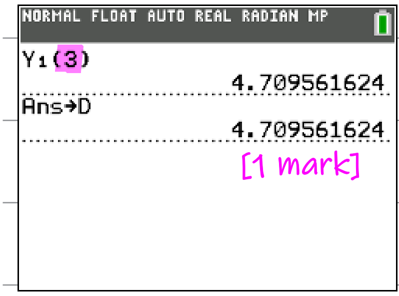

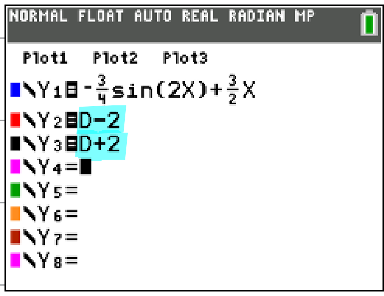

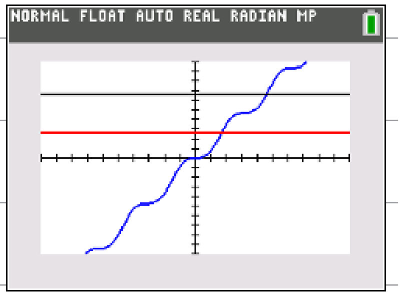

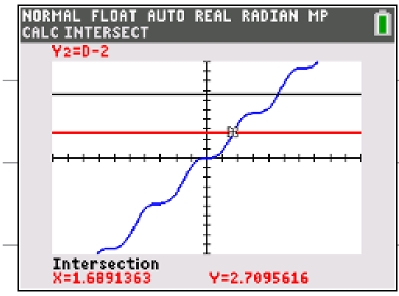

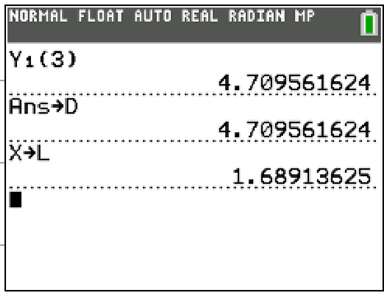

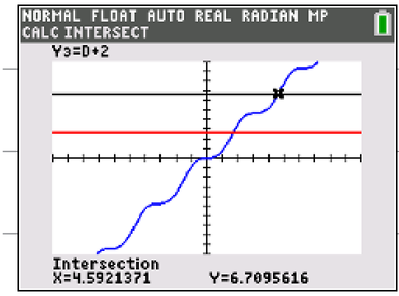

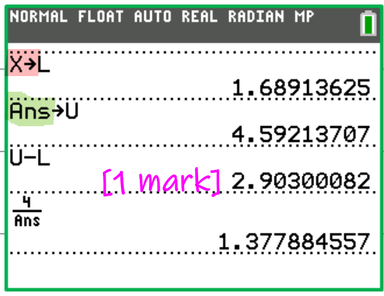

Note: You can recall (& therefore save) a value from the Graph screen either:
using the assigned variable (e.g. X)
or
if it was the last value (ans)

Average velocity within sensor range = 1.378 m/s [1 mark]

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QUESTION 19 (6 marks)

The normal distribution probability density function is

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

, with the parameters mean, μ, and standard deviation, σ.

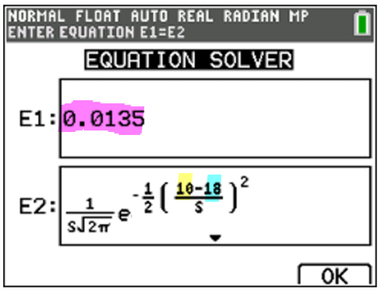
The speeds of electric scooter (e-scooter) riders on a particular section of a bike path are approximately normally distributed with a mean of 18 km/h. It is known that $p(10) = 0.0135$.

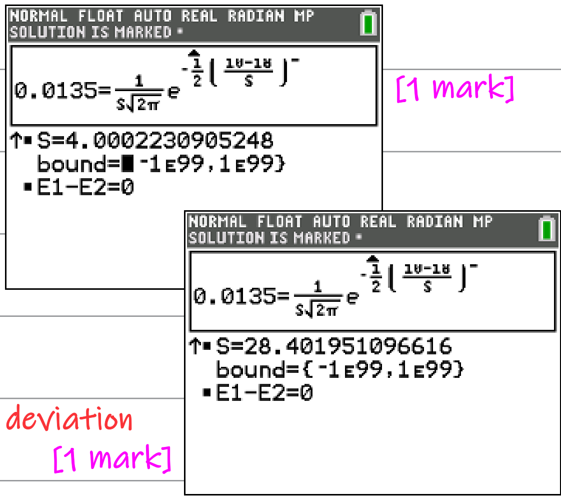
The speed limit for e-scooters on this section of bike path is 23 km/h.

A speed camera is set up and records the speeds of 75 e-scooter riders. Every rider travelling faster than the speed limit is given a $143 fine. Before setting up the speed camera, the following suggestion was made.

The total of the fines expected to be issued will be more than $1500.

Evaluate the reasonableness of this suggestion.

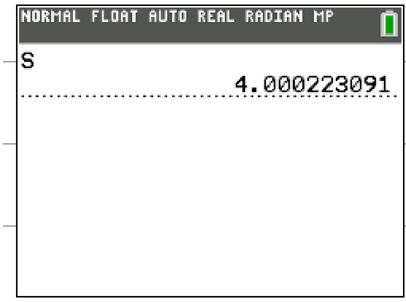


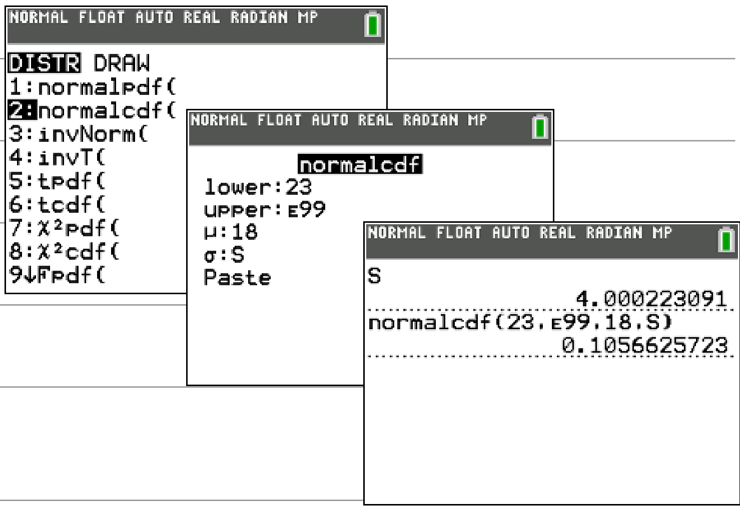


[1 mark]

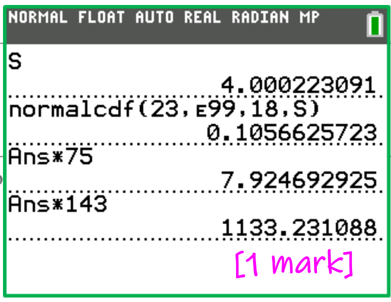
Two solutions are obtained for the std deviation
σ = 4.0002 and 28.4020 [1 mark]

Note: Reject 28.4020 as it is not a possible standard deviation because for example three standard deviations less than the mean would produce a negative speed or three above would result in an impossible speed on an e-scooter (>100 km/h).





[1 mark]



[1 mark]

The expected total fines is about $1133, which is less than the suggested $1500, so the suggestion is NOT reasonable. [1 mark]

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ADDITIONAL PAGE FOR STUDENT RESPONSES

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[1 mark]