

Specialist Mathematics Exam Preparation MC Questions

Poll Question

The subset of the complex plane defined by the complex equation $\left| \frac{z-2}{z+2} \right| = 1$ is

- A. a circle
- B. an ellipse
- C. a ray
- D. a straight line
- E. a hyperbola

Question: 1.

The asymptotes of the hyperbola $\frac{(x-2)^2}{9} - \frac{(y+3)^2}{25} = 1$ have equations

- A. $y = \frac{5}{3}x + \frac{4}{3}$ and $y = -\frac{5}{3}x - \frac{16}{3}$
- B. $y = \frac{5}{3}x - \frac{4}{3}$ and $y = \frac{5}{3}x - \frac{16}{3}$
- C. $y = \frac{3}{5}x + \frac{4}{3}$ and $y = -\frac{3}{5}x - \frac{16}{3}$
- D. $y = \frac{5}{3}x - \frac{19}{3}$ and $y = -\frac{5}{3}x + \frac{1}{3}$
- E. $y = \frac{3}{5}x - \frac{4}{3}$ and $y = -\frac{5}{3}x + \frac{4}{3}$

Question: 2.

Which of the following is an even function?

- A. $f(x) = \operatorname{cosec}(x)$
- B. $f(x) = \arcsin(|x|)$
- C. $f(x) = \arctan(x) + 1$
- D. $f(x) = \arccos(x)$
- E. $f(x) = \sec\left(x - \frac{\pi}{4}\right)$

Question: 3.

Consider the circle $|z + 3 - 2i| = 2$. Which of the following lines intersects the circle exactly twice?

- A. $\text{Im}(z) = 0$
- B. $\text{Re}(z) = 1$
- C. $|z + 3 - 2i| = |z - 5|$
- D. $|z + 3 - 2i| = |z + 8i|$
- E. $|z + 3 - 2i| = |z + 1 + i|$

Question: 4.

The sum and product of the roots of the equation $z^5 + z^4 + z^3 + z^2 + z + 1 = 0$, $z \in C$ are respectively:

- A. $-1, -1$
- B. $-1, 0$
- C. $0, -1$
- D. $1, -1$
- E. $-1, 1$

Question: 5.

If $z = (1 - i)^n$ and $|z| = 32$ then

- A. $n = 8$
- B. $n = 10$
- C. $n = 5$
- D. $n = 4$
- E. $n = 2$

Question: 6.

The complex number $z = k \left(\cos \frac{\pi}{m} + i \sin \frac{\pi}{m} \right)$ is a root of the equation $z^3 = w$. Given $w = 4 - 4\sqrt{3}i$ then

- A. $k = 1, m = 3$
- B. $k = 2, m = 3$
- C. $k = 2, m = -9$
- D. $k = 3, m = 9$
- E. $k = \frac{1}{2}, m = -9$

Question: 7.

The solution of differential equation $\frac{dy}{dx} = e^{2x}(1 + y^2)$ given that $x = 0$ when $y = 1$ is

- A. $y = \tan\left(\frac{e^{2x}}{2} - \frac{\pi}{4} + \frac{1}{2}\right)$
- B. $y = \tan\left(\frac{e^{2x}}{2} + \frac{\pi}{4} + \frac{1}{2}\right)$
- C. $y = \arctan\left(\frac{e^{2x}}{2} + \frac{\pi}{4}\right)$
- D. $y = \tan\left(\frac{e^{2x}}{2} + \frac{\pi}{4} - \frac{1}{2}\right)$
- E. $y = \arctan\left(\frac{e^{2x}}{2} + \frac{\pi}{4} - \frac{1}{2}\right)$

Question: 8.

The velocity, v , of the particle P, at time t is given by $v(t) = e^{3t} - 2e^t$. The distance covered by P between $t = 0$ and $t = \log_e 3$ is closest to

- A. 4.7
- B. 5.1
- C. 5.2
- D. 12.7
- E. 0.8

Question: 9.

A curve is defined by the equation $4x^2 + 9y^2 = 36$. The section of the curve in the first quadrant is rotated through 360° about the y -axis to form a solid of revolution with volume equal to

- A. 4π
- B. 8π
- C. 12π
- D. 16π
- E. 9π

Question: 10.

The vectors $\underline{a} = -\underline{i} + 2\underline{j} + 2\underline{k}$, $\underline{b} = \underline{i} - 3\underline{j} + \underline{k}$ and $\underline{c} = \lambda\underline{i} - 5\underline{j} - 2\underline{k}$ are **linearly dependent** when the value of λ is

- A. $-\frac{53}{12}$
- B. $\frac{17}{8}$
- C. $-\frac{5}{8}$
- D. $\frac{8}{17}$
- E. $\frac{5}{8}$

Question: 11.

A particle is moving along a curve defined by the following parametric equations

$$x(t) = \sec(t)$$

$$y(t) = \sin(t)$$

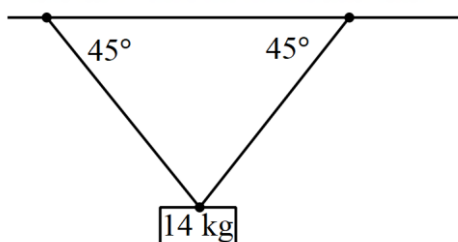
where $0 \leq t \leq \pi$.

The equation of the tangent to the curve at $t = \frac{\pi}{6}$ is

- A. $y = \frac{3\sqrt{3}}{4}x - \frac{1}{2}$
- B. $y = \frac{3\sqrt{3}}{4}x - 1$
- C. $y = \frac{3\sqrt{3}}{4}x + 1$
- D. $y = \frac{3\sqrt{3}}{4}x$
- E. $y = \frac{3\sqrt{3}}{2}x - 1$

Question: 12.

A 14 kg mass is suspended in equilibrium from a horizontal ceiling by two identical light strings. Each string makes an angle of 45° with the ceiling as shown in the diagram.



The magnitude, in newtons, of the tension in each string is equal to

- A. $14\sqrt{2}$
- B. $7\sqrt{2}$
- C. $14\sqrt{2}g$
- D. $\frac{7g}{\sqrt{2}}$
- E. $7\sqrt{2}g$

Question: 13.

The length of arc of the graph of $f : [0, 4] \rightarrow R$, $f(x) = \arctan(x) + 1$, correct to 3 decimal places is:

- A. 4.345
- B. 4.350
- C. 18.880
- D. 4.620
- E. 4.068

Question: 14.

Euler's method, with a step size of 0.2, is used to approximate the solution of the differential equation

$\frac{dy}{dx} = x - y^2$, with $y = 0$ when $x = 1$. The estimated value of y , to five decimal places, when $x = 2$ is

- A. 1.00233
- B. 1.09090
- C. 1.09091
- D. 1.10033
- E. 0.01033

Question: 15.

The position vector $\underline{r}(t)$ of a mass of 5 kg after t seconds, where $t \geq 0$, is given by

$$\underline{r}(t) = \sin(2t)\underline{i} + \cos(t)\underline{j} + \frac{5}{3}t^3\underline{k}.$$

The force, in newtons, acting on the mass when $t = \pi$ seconds is

- A. $5\underline{j} + 50\pi\underline{k}$
- B. $\underline{j} + 10\pi\underline{k}$
- C. $2\underline{i} + 5\pi\underline{j}$
- D. $25\pi\underline{k}$
- E. $2\underline{j} + 5\pi\underline{k}$

Question: 16.

Domain and range of $h(x) = \frac{3}{\sqrt{\arcsin(2x)}}$ are, respectively

- A. $\left[-\frac{1}{2}, \frac{1}{2}\right]$ and $\left[\frac{3\sqrt{2}}{\sqrt{\pi}}, \infty\right)$
- B. $\left(0, \frac{1}{2}\right]$ and $\left[\frac{3\sqrt{2}}{\sqrt{\pi}}, \infty\right)$
- C. $\left[0, \frac{1}{2}\right]$ and $\left(\frac{3\sqrt{2}}{\sqrt{\pi}}, \infty\right)$
- D. $\left[0, \frac{1}{2}\right)$ and $\left(\frac{3\sqrt{2}}{\sqrt{\pi}}, \infty\right)$
- E. $\left(0, \frac{1}{2}\right)$ and $\left(\frac{3\sqrt{2}}{\sqrt{\pi}}, \infty\right)$

Question: 17.

If α is acute and $\cos(2\alpha) = \frac{3}{4}$, then $\operatorname{cosec}(\alpha)$ is

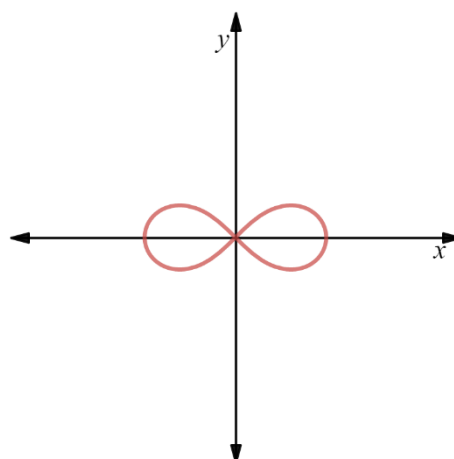
- A. $\frac{1}{2\sqrt{2}}$
- B. $\frac{\sqrt{2}}{\sqrt{7}}$
- C. $\frac{5}{4}$
- D. $2\sqrt{2}$
- E. $\sqrt{2}$

Question: 18.

The graph shows the relation $(x^2 + y^2)^2 = x^2 - y^2$.

Point P lies in the first quadrant and the tangent to the graph at P is horizontal. The coordinates of P are

- A. $\left(\frac{\sqrt{6}}{2}, \frac{\sqrt{2}}{2}\right)$
- B. $\left(\frac{\sqrt{3}}{4}, \frac{\sqrt{2}}{4}\right)$
- C. $\left(\frac{\sqrt{3}}{2}, \frac{\sqrt{2}}{2}\right)$
- D. $\left(\frac{\sqrt{6}}{4}, \frac{\sqrt{2}}{4}\right)$
- E. $(1, 0)$



Question: 19.

Which of the following is true for the graph of $y = \frac{x^2 + 2x}{x^2 - 1}$

- A. no points of inflection and two asymptotes
- B. three asymptotes and one point of inflection
- C. two asymptotes and one point of inflection
- D. three asymptotes and no points of inflection
- E. two asymptotes and no stationary points

Question: 20.

A particle moves in a straight line such that its velocity, $v \text{ ms}^{-1}$, at time t seconds is given by

$$v(t) = \begin{cases} 6t - t^2, & \text{for } 0 \leq t \leq 5 \\ \frac{1}{2}(15 - t), & \text{for } t > 5 \end{cases}$$

The particle returns to its initial position at $t = T$.

The value of T , to three decimal places, is

- A. 31.234
- B. 30.275
- C. 14.550
- D. 29.550
- E. 30.272

Question: 21.

A particle of mass 3 kg is traveling along a path so that its position vector, r , in metres, at time, t , in seconds, is

$$\underline{r}(t) = 2t^3 \underline{i} - 3t^2 \underline{j} + t \underline{k}.$$

The magnitude, to the nearest integer, of momentum, in kg ms^{-1} , of the particle at $t = 3$ is

- A. 171
- B. 445
- C. 148
- D. 454
- E. 154

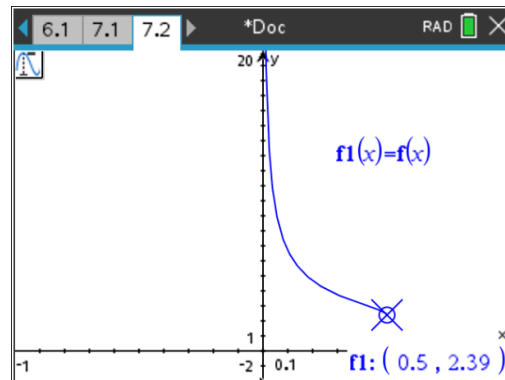
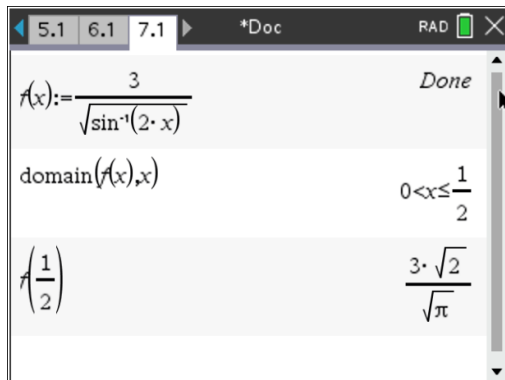
Question: 22

Evaluate the following $i + i^2 + i^3 + i^4 + \dots + i^{199} + i^{200} + i^{201}$

- A. 0
- B. -1
- C. i
- D. $-i$
- E. 1

Answers

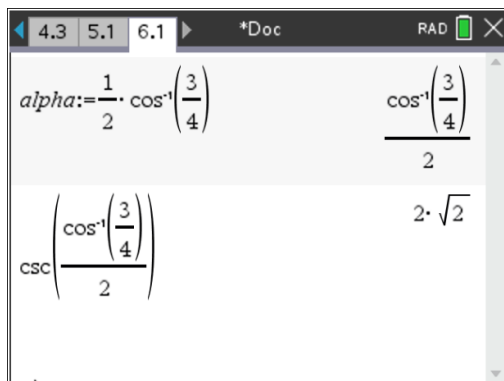
Question 16 Option B



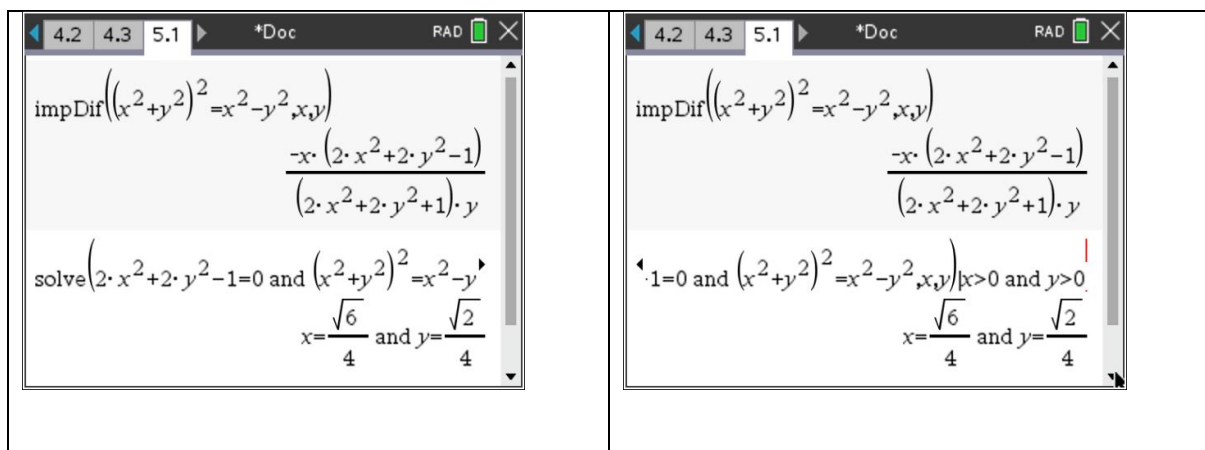
Domain: $\left(0, \frac{1}{2}\right]$

Range: $\left[\frac{3\sqrt{2}}{\sqrt{\pi}}, \infty\right)$

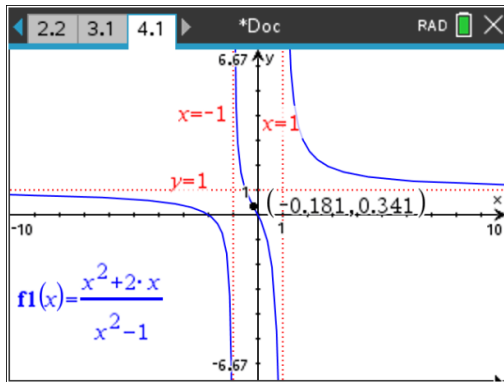
Question 17 Option D



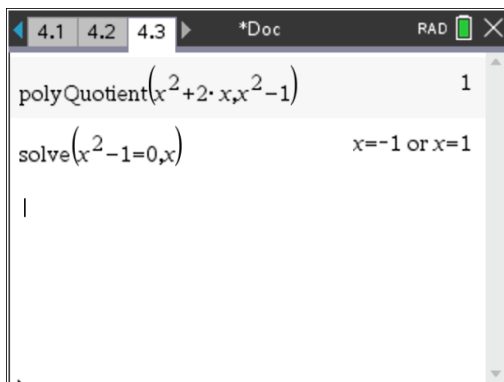
Question 18 Option D



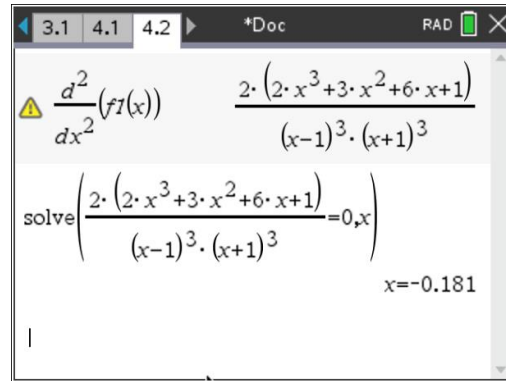
Question 19 Option B



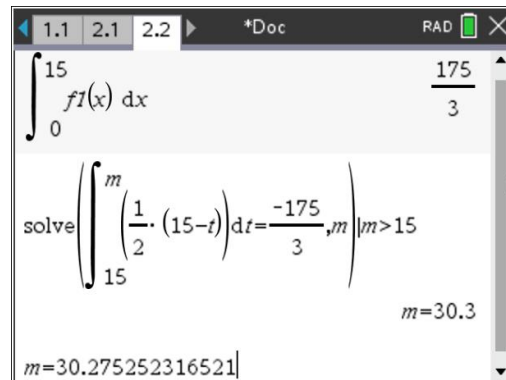
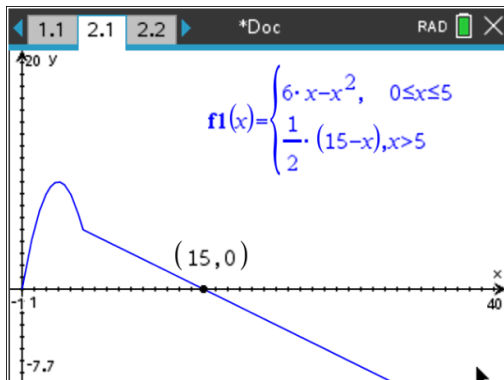
Asymptotes:



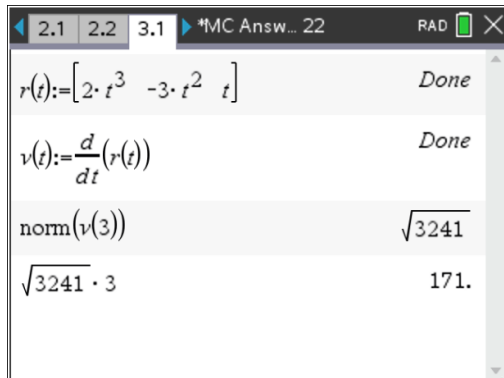
Point of inflection:



Question 20 Option B



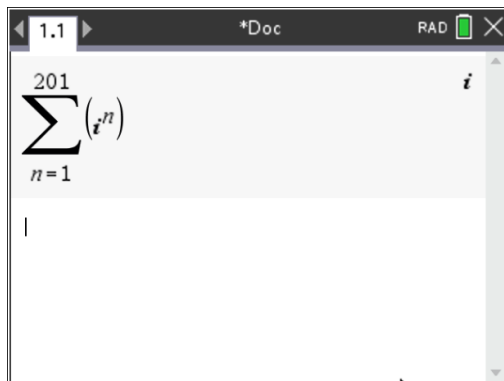
Question 21 Option A



A TI-84 Plus calculator screenshot showing the following steps and results:

- Input: $r(t) = [2 \cdot t^3 \quad -3 \cdot t^2 \quad t]$ Done
- Input: $v(t) = \frac{d}{dt}(r(t))$ Done
- Input: $\text{norm}(v(3))$ $\sqrt{3241}$
- Input: $\sqrt{3241} \cdot 3$ 171.

Question 22 Option C



A TI-84 Plus calculator screenshot showing the input of a summation formula:

- Input: $\sum_{n=1}^{201} i^n$