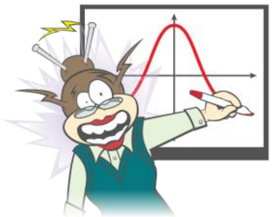
# Mechanics (Dynamics)



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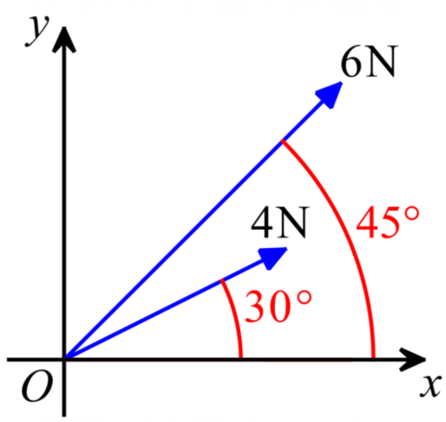
Each of the questions included here can be solved using TI-Nspire CX CAS technology.
Take the acceleration due to gravity to have magnitude $g\ \text{m/s}^2$, where $g = 9.8$.
Ensure that the Angle mode is set to **Degrees**.

## Question 1

Three forces of magnitude $4\ \text{N}$, $6\ \text{N}$ and $\sqrt{10}\ \text{N}$ are acting on a particle at $O$. The $4\ \text{N}$ force acts at an angle of $30^\circ$ to the $x$-axis and the $6\ \text{N}$ force acts at an angle of $45^\circ$ to the $x$-axis, both in the horizontal plane, as shown in the diagram. Given that $\underset{\sim}{i}$, $\underset{\sim}{j}$ and $\underset{\sim}{k}$ are unit vectors in the direction of the $x$-, $y$- and $z$-axis respectively, and the third force acting is $\sqrt{3}\underset{\sim}{i} + \sqrt{2}\underset{\sim}{j} + \sqrt{5}\underset{\sim}{k}$, the magnitude of the net force is closest to




**A.** $9.91\ \text{N}$
**B.** $9.92\ \text{N}$
**C.** $10\ \text{N}$
**D.** $12.35\ \text{N}$
**E.** $12.36\ \text{N}$

## Question 2

A surfboarder of mass $70\ \text{kg}$ is being towed on the end of a rope by a jet ski.

The rope maintains an angle of $5^\circ$ above the horizontal throughout the first stage of the manoeuvre. The tension in the rope is $T\ \text{N}$ and there is a total resistance (air & water) force of $30\ \text{N}$ acting on the surfboarder.

a) Write down an equation of motion for the surfboarder, given that the acceleration is $a\ \text{m/s}^2$, $a > 0$.
b) If $a = 3.3$, find the value of $T$, correct to the nearest integer.
c) After some time, the second stage of the the manoeuvre begins.
The tension in the rope changes to $300\ \text{N}$, the acceleration of the surfboarder increases to $3.85\ \text{m/s}^2$ and the angle between the rope and the horizontal decreases to $\theta^\circ$. Find $\theta$, correct to one decimal place.

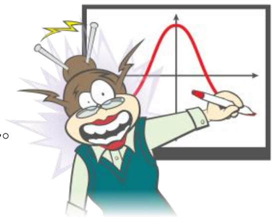
Response:

## Question 3

A $5\ \text{kg}$ box is on the floor of a lift that is accelerating upwards at $3\ \text{m/s}^2$. The reaction force of the floor on the box is

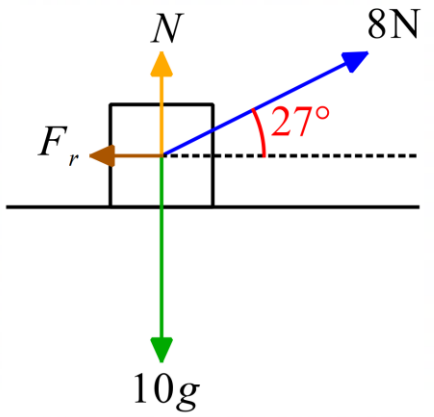
**A.** $49\ \text{N}$
**B.** $34\ \text{N}$
**C.** $64\ \text{N}$
**D.** $15\ \text{N}$
**E.** $0\ \text{N}$

## Question 4



A $10\ \text{kg}$ box is pulled along a horizontal surface by an $8\ \text{N}$ force, acting at an angle of $27^\circ$ to the horizontal, as shown. As the box moves, a resistive force impedes its motion, with a magnitude $F_r = 0.1v$, where $v$ m/s is the speed of the box at time $t$ s.

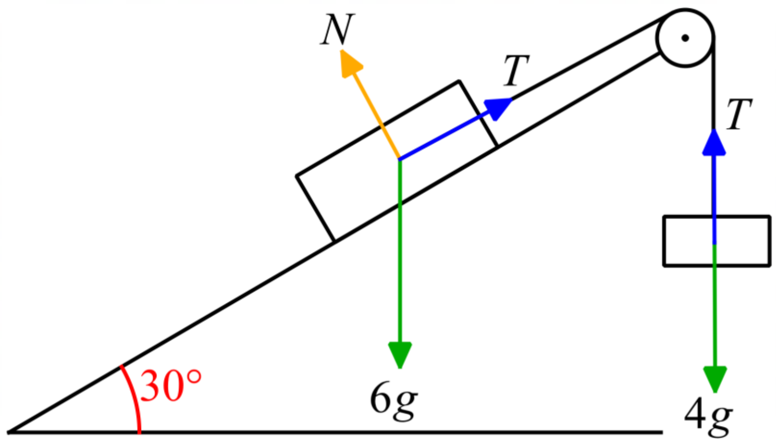
Write an equation of motion for the box and hence write a definite integral to evaluate the time taken for the box to reach a speed of $9\ \text{m/s}$ from rest. Evaluate this definite integral, stating the answer correct to two decimal places.



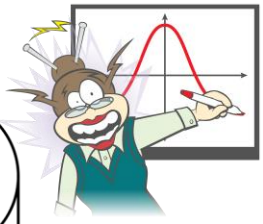

Response:

## Question 5

A box of mass $6\ \text{kg}$ on a smooth plane inclined to the horizontal at an angle of $30^\circ$ is connected by a light inelastic string over a smooth pulley at the top of the plane to a box of $4\ \text{kg}$ which is hanging vertically. At the instant the $4\ \text{kg}$ box is moving downwards at $1\ \text{m/s}$, it is $7.41\ \text{m}$ above the floor. Calcuate how long it takes to reach the floor.



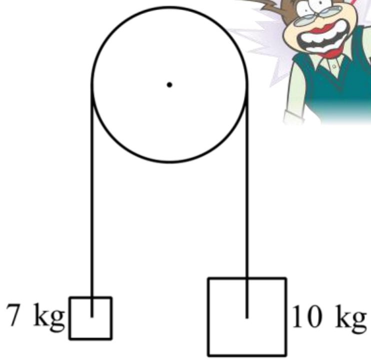

Response:



**Question 6**

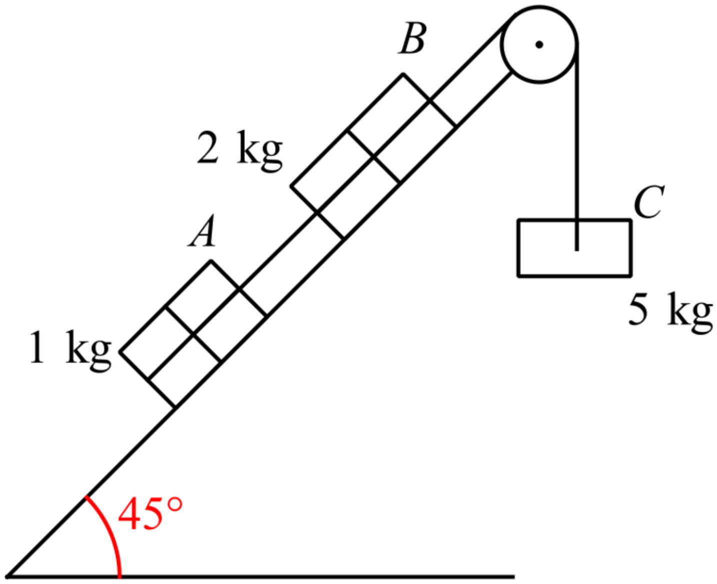
The diagram shows two objects of mass $7\ \mathrm{kg}$ and $10\ \mathrm{kg}$ connected by a light inelastic string passing over a smooth pulley. The objects are initially at rest. The distance that the $10\ \mathrm{kg}$ object moves downwards in two seconds after being released from rest is closest to

**A.** $3.00\ \mathrm{m}$
**B.** $3.53\ \mathrm{m}$
**C.** $3.52\ \mathrm{m}$
**D.** $3.46\ \mathrm{m}$
**E.** $3.45\ \mathrm{m}$




**Question 7**

Two parcels $A$ and $B$ are connected by a light inelastic string and are placed on a smooth inclined plane inclined at $45^\circ$ to the horizontal. Parcel $B$ is connected by a light inelastic string over a smooth pulley at the top of the plane to a box $C$ which is hanging vertically, as shown in the diagram below. Find the acceleration of the system in $\mathrm{m/s^2}$.

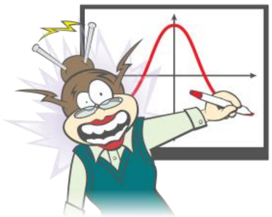



Response:

**Question 8**

An object of mass $8\ \mathrm{kg}$ at rest on a smooth horizontal surface is acted on by a horizontal force that decreases uniformly with the distance travelled. This force is $20\ \mathrm{N}$ at the start ( $x = 0$ ) and $10\ \mathrm{N}$ after travelling a distance of $20\ \mathrm{m}$. Calculate the exact speed of the object at the end of this $20\ \mathrm{m}$ movement.

Response:

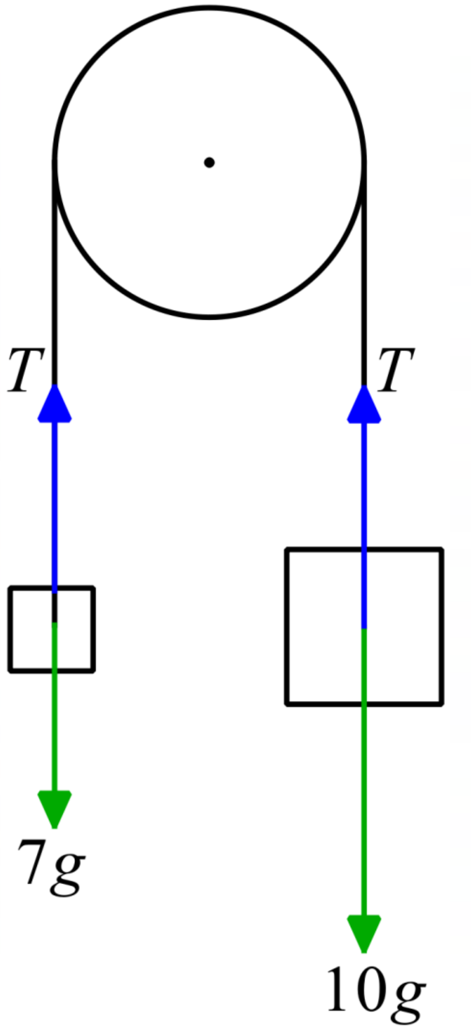


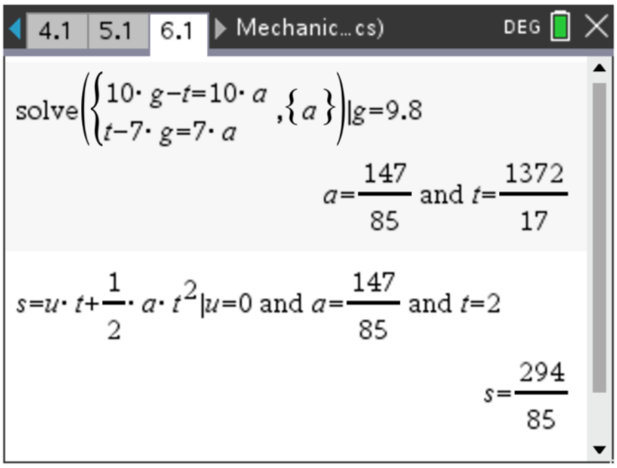
## Answers*

### Question 6

Distance $= \dfrac{294}{85}$ m $\approx 3.46$ m (**D.**)

Label the diagram with the weight and tension forces (taking care with arrow sizes, especially $T$ ).




For the $10$ kg object:

$10g - T = 10a \cdots\cdots \boxed{1}$

For the $7$ kg object:

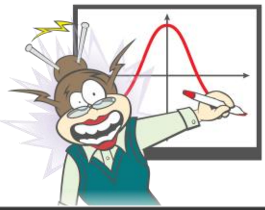
$T - 7g = 7a \cdots\cdots\cdots \boxed{2}$

Solving $\boxed{1}$ and $\boxed{2}$ using CAS gives $a = \dfrac{147}{85}$ and

$T = \dfrac{1372}{17}$.

Using $s = ut + \dfrac{1}{2}at^2$ with $u = 0$ and $t = 2$ gives $s = \dfrac{294}{85}$.

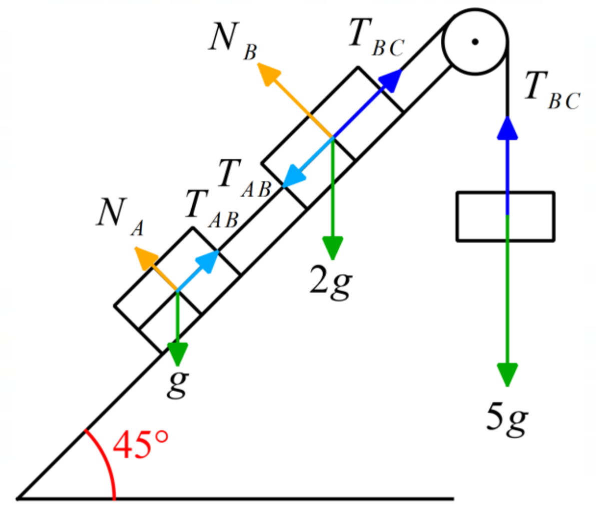
* When using CAS as a calculation and/or algebraic manipulation tool, it is important to set out working clearly.

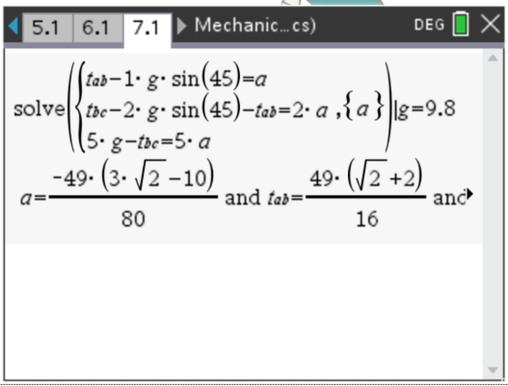


## Question 7

$a = \frac{49}{80}\left(10 - 3\sqrt{2}\right)\ \text{m/s}^2$

Label the diagram with the weight, normal reaction and tension forces.




For parcel $A$:

$T_{AB} - 1g\sin\left(45^\circ\right) = 1a \cdots\cdots\cdots\cdots\cdots \boxed{1}$

For parcel $B$:

$T_{BC} - 2g\sin\left(45^\circ\right) - T_{AB} = 2a \cdots\cdots \boxed{2}$

For parcel $C$:

$5g - T_{BC} = 5a \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \boxed{3}$

Solving $\boxed{1}$, $\boxed{2}$ and $\boxed{3}$ using CAS gives $a = \frac{49}{80}\left(10 - 3\sqrt{2}\right)$,

$T_{AB} = \frac{49}{16}\left(2 + \sqrt{2}\right)$ and $T_{BC} = \frac{147}{16}\left(2 + \sqrt{2}\right)$.

Alternatively, given that there is no friction involved (on either the inclined plane or the pulley), the acceleration may also be obtained using the net force:

$$\Sigma F = 5g - 2g\sin\left(45^\circ\right) - 1g\sin\left(45^\circ\right)$$

$$= 49 - \frac{147\sqrt{2}}{10}$$

Therefore the acceleration, using Newton's 2nd law, is:

$$a = \frac{\Sigma F}{m}$$

$$= \frac{1}{8}\left(49 - \frac{147\sqrt{2}}{10}\right)$$

$$= \frac{49}{80}\left(10 - 3\sqrt{2}\right)$$

## Question 8

$\text{Speed} = 5\sqrt{3}\ \text{m/s}$

The force is linear, with negative gradient of $\frac{10-20}{20} = -\frac{1}{2}$. Finding the function, using $F - 20 = -\frac{1}{2}(x - 0)$, gives $F = 20 - \frac{1}{2}x$. This can be divided by the mass of $8$ to get the acceleration $a = \frac{40 - x}{16}$.

Using $a = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$, the change in $\frac{1}{2}v^2$ from $x = 0$ to $x = 20$ is found by calculating $\int_0^{20} \frac{40 - x}{16}\, dx$. Given that the object starts at rest, its increase and thus its final value of $\frac{1}{2}v^2$ is $\frac{75}{2}$.

Therefore the final speed, that is, $|v|$, is $5\sqrt{3}\ \text{m/s}$.