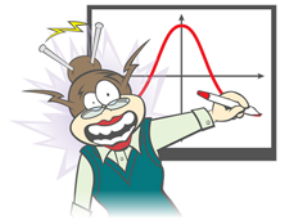


Mathematical Methods - The Normal Distribution Revision Questions



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Each of the questions included here can be solved using either the TI-Nspire CX or CX CAS.

Question 1

A random variable X is normally distributed with a mean of 80 and variance 20. Find, to three decimal places, $\Pr(68 \leq X < 86)$.

Response:

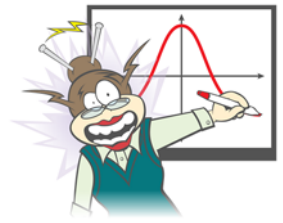
Question 2

The age, in months, of children starting three-year old kindergarten is normally distributed with a mean of 41 and a standard deviation of 3.

The percentage of children who start three-year old kindergarten who are older than 38 months is closest to:

- A. 63%
- B. 68%
- C. 84%
- D. 95%
- E. 99.7%

Response:



Question 3

A random variable X is normally distributed with a mean of 35 and variance 25. Given that $\Pr(X < a) = 0.82$, find, to one decimal place, the value of a .

Response:

Question 4

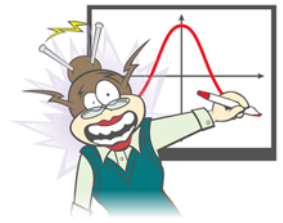
Y is a random variable with a normal distribution. The mean of Y is 32 and the variance is 10. The values of c and d are such that $\Pr(c < Y < d) = 0.95$ where this is the middle 95% of values. Find, to two decimal places, the values of c and d .

Response:

Question 5

If $X \sim N(\mu, 0.4)$ and $\Pr(X < 20) = 0.85$, find, to two decimal places, the mean of the normally distributed variable X .

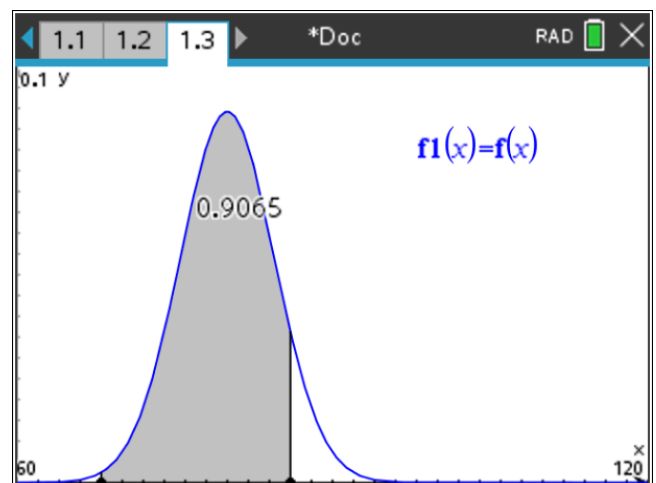
Response:



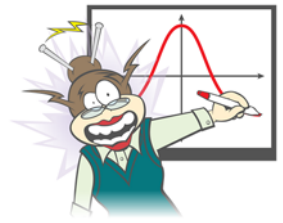
Answers

Question 1

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$
$$\mu = 80, \sigma = 2\sqrt{5}$$

A screenshot of the TI-84 Plus calculator's 'Normal Cdf' dialog box. The 'Lower Bound' is set to 68, the 'Upper Bound' is 86, the mean μ is 80, and the standard deviation σ is $\sqrt{20}$. The 'OK' and 'Cancel' buttons are at the bottom.A screenshot of the TI-84 Plus calculator screen showing the result of the normal CDF function: `normCdf(68,86,80,sqrt(20))` resulting in `0.906498465348`.A screenshot of the TI-84 Plus calculator screen showing the function definition: $f(x) := \frac{1}{sd \cdot \sqrt{2 \cdot \pi}} \cdot e^{-\frac{(x-\mu)^2}{2 \cdot sd^2}}$. Below the function, the parameters are defined: `mu:=80` and `sd:=sqrt(20)`. A green arrow points to the word 'Done'.

Solution: 0.906



Question 2

Solution: C

```
1.1 1.2 *Doc RAD X  
normCdf(38,∞,41,3) 0.841344740437
```

Question 3

```
1.2 1.3 1.4 *Doc RAD X  
Mean mu:=35 ▶ 35  
Variance var:=25 ▶ 25  
Standard Deviation sd:=√var ▶ 5  

$$f(x) := \frac{1}{sd \cdot \sqrt{2 \cdot \pi}} \cdot e^{-\frac{(x-\mu)^2}{2 \cdot sd^2}} \quad \blacktriangleright \text{Done}$$

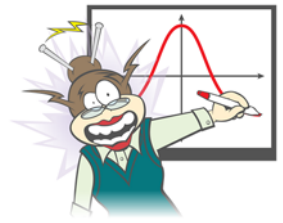
```

```
1.3 1.4 1.5 *Doc RAD X  
invNorm(0.82,35,5) 39.5768254101
```

```
1.2 1.3 1.4 *Doc RAD X  

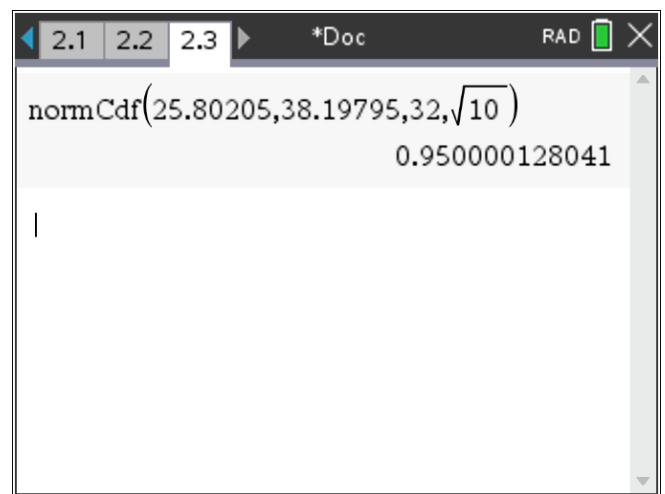
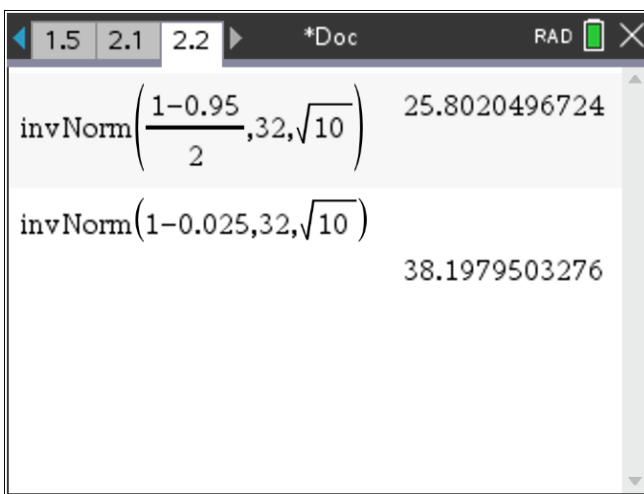
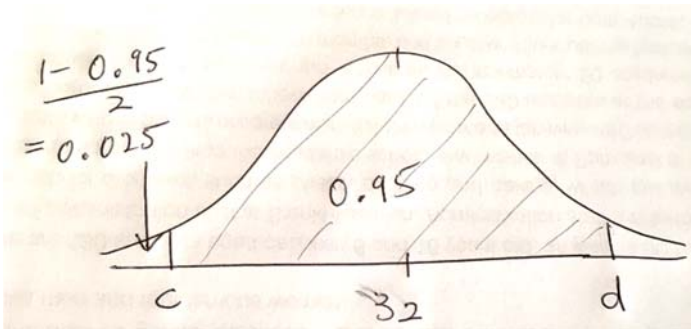
$$f(x) := \frac{1}{sd \cdot \sqrt{2 \cdot \pi}} \cdot e^{-\frac{(x-\mu)^2}{2 \cdot sd^2}} \quad \blacktriangleright \text{Done}$$
  
solve  $\left( \int_{-\infty}^a f(x) dx = 0.82, a \right)$   
▶ a=39.5768254392 ⚠
```

Solution: $a = 39.6$

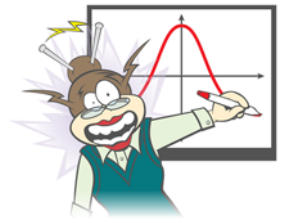


Question 4

The middle 95% of the distribution, therefore $\Pr(Y < c) = 0.025$ and $\Pr(Y < d) = 0.975$



Solution: $c = 25.80$ and $d = 38.20$



Question 5

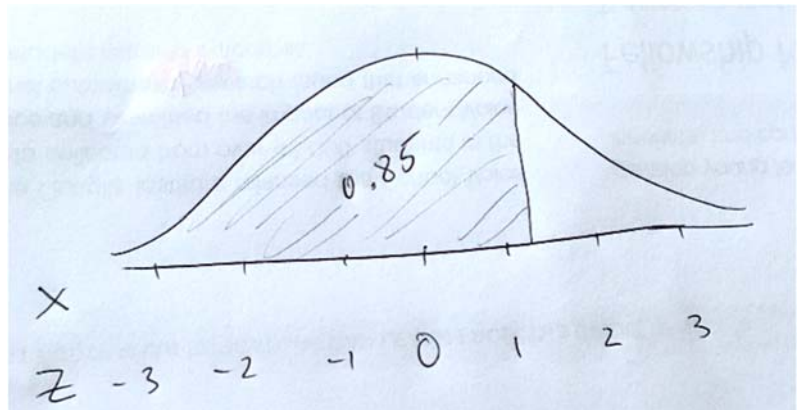
$$\Pr(X < 20) = 0.85$$

$$\Pr(Z < 1.03643337977) = 0.85$$

$$Z = \frac{X - \mu}{\sigma}$$

$$1.03643337977 = \frac{20 - \mu}{\sqrt{0.4}}$$

$$\mu = 19.34$$



```
2.2 2.3 2.4 *Doc RAD X
invNorm(0.85,0,1) 1.03643337977
solve(1.0364333797684 = (20 - mu) / sqrt(0.4), mu)
mu = 19.3445019754
```

Solution: $\mu=19.34$