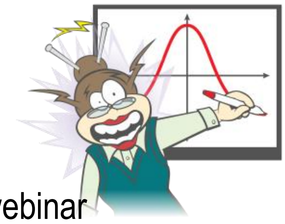


Mathematical Methods with TI-Nspire™ CX CAS

# Applications of Integral Calculus

**Revision Worksheet with solutions** – may be completed after viewing the webinar

**Author:** Frank Moya

Each of the questions included here can be solved using either the TI-Nspire CX or CX CAS.

## Finding a function from a known rate of change given a boundary condition

### Question 1

Water is pumped out of a swimming pool at a constant rate of 800 litres/hour. That is, the rate of change in the volume of water is given by $V'(t) = -800$ litres/hour.

If the volume of water remaining in the pool is 12 000 litres at time $t = 5$ hours, find an expression for $V(t)$, the of water in the pool at time $t$ hours.

Response:

## Calculation of the area of a region under a curve

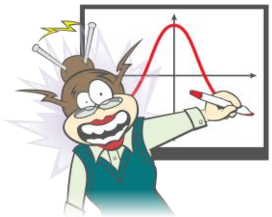
### Question 2

Consider the function $f : [-2, \infty) \to R,\ f(x) = 4 - x^2$.

a. Find the area of the region bounded by the graph of $f$, the $x$-axis and the line $x = 3$, over the interval $x \in [-2, 3]$.

b. Evaluate $\int_{-2}^{3} (4 - x^2)\, dx$. Explain why this answer is not the same as the answer to part a. above.

Response:



## Question 3

Consider the piecewise (hybrid) function $f(x) = \begin{cases} \sqrt{x} & 0 \le x \le 4 \\ 6 - x & 4 < x \le 6 \end{cases}$.

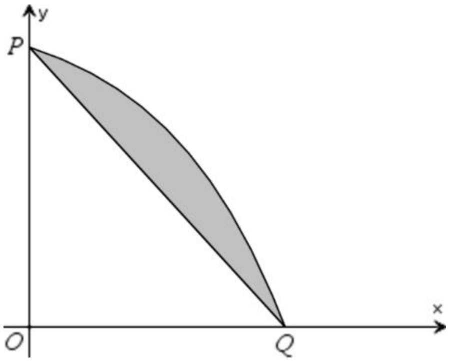
Find the area of the region enclosed by the graph of $f$ and the $x$-axis.

Response:

## Calculation of the areas between curves

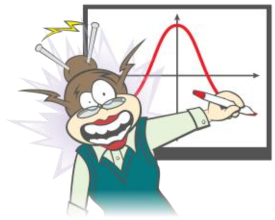
## Question 4

The graph of the function with rule $f(x) = 8 - 2^x$ intersects the axes at the points $P$ and $Q$, as shown below. Also shown on the graph is the line segment joining $P$ and $Q$.




Find the area of the shaded region.

Response:



## Question 5

Consider the function $g : R \rightarrow R,\ f(x) = 9 - x^2$

a. Find the equation of the tangent to the graph of $g$ at the point where $x = 1$.
b. Find the area of the region bounded by the graph of $g$, the tangent and the line $x = 3$.
c. Find the area of the region bounded by the graph of $g$, the tangent and the $x$-axis.

Response:

## Distance travelled in a straight line
## Question 6

The speed, $v$ m/s of a body moving in a straight line is modelled by the function $v(t) = 2t + 1, t \geq 0$.

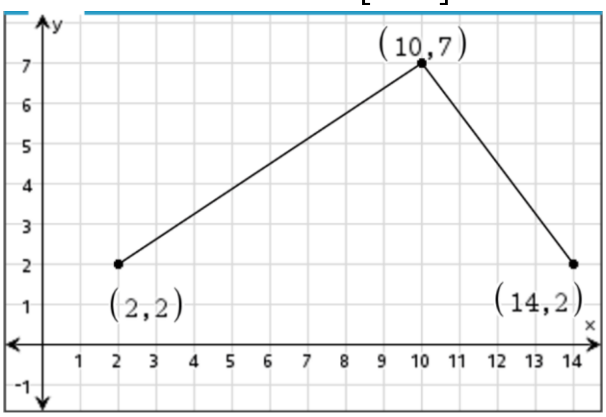
The distance travelled by the body is given by the area under the graph of $v$.

a. Find the distance that the body travels in the first 20 seconds
b. Given that the distance travelled by the body over the interval $t \in [12, k]$ is 656 m, find the value of $k$.

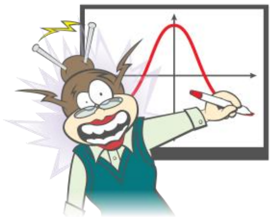
Response:

## Average value of a function
## Question 7

The graph of a function $f : [2,14] \rightarrow R$ is shown below.




Find average value of $f$ over the interval $[2,14]$.



## Question 8

The amount, $c$ milligrams, of a medication in the bloodstream, $t$ minutes after it is administered, is modelled by the function $c(t) = kte^{-\frac{t}{10}}, t \geq 0$, where $k > 1$.

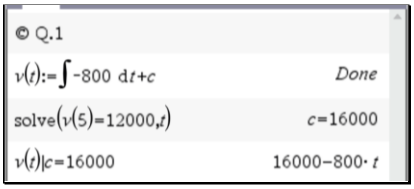
The average amount of the medication in the bloodstream over the interval $t \in [3, 16k]$ is found to be 5.89 milligrams. Find the value of $k$, correct to two decimal places.

Response:

# Answers

## Question 1

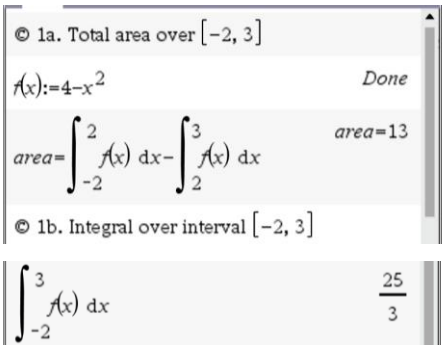
Answer: $V(t) = 16000 - 800t$



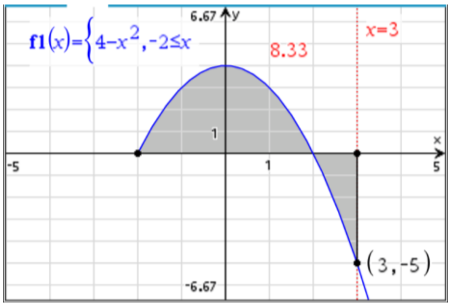

## Question 2

a. Area = 13

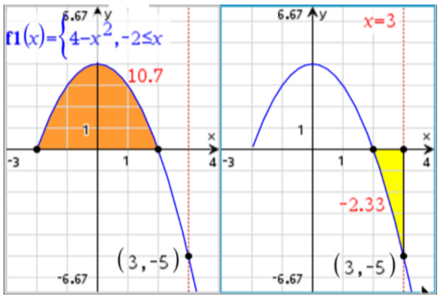
b. b. Integral = $\frac{25}{3} = 8\frac{1}{3}$ . Part a. $10\frac{2}{3} - \left(-2\frac{1}{3}\right) = 13$, whereas Part b. $10\frac{2}{3} - 2\frac{1}{3} = 8\frac{1}{3}$.



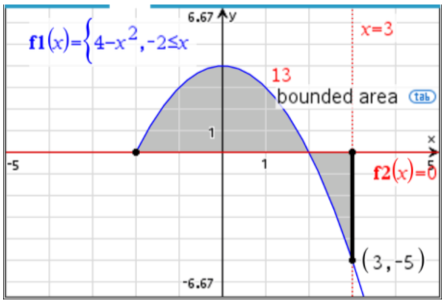

**Integral** – calculated graphically

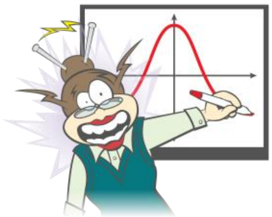



**Integral** – calculated graphically



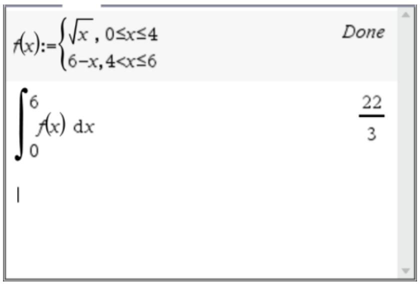

**Bounded area** calculated graphically between $y = f(x)$ and $y = 0$ over interval $[-2, 3]$.

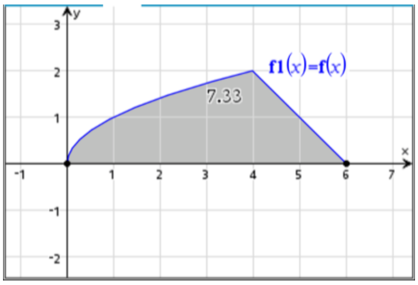


## Question 3

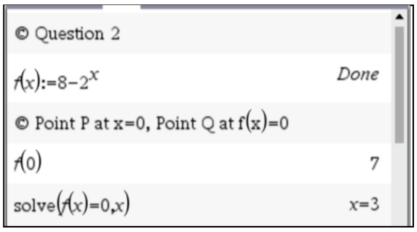
Answer: $\frac{22}{3} = 7\frac{1}{3}$

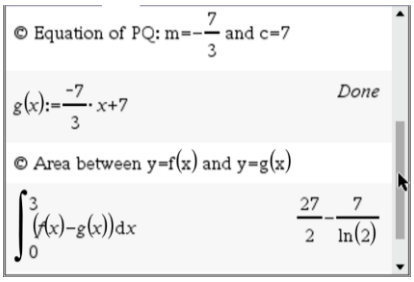


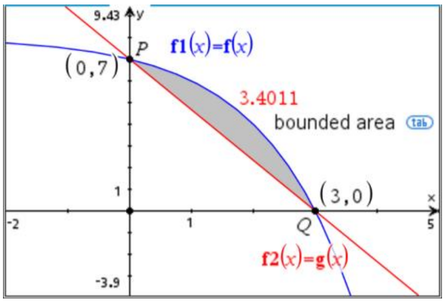




## Question 4

Answer: $\frac{27}{2} - \frac{7}{\log_e(2)}$

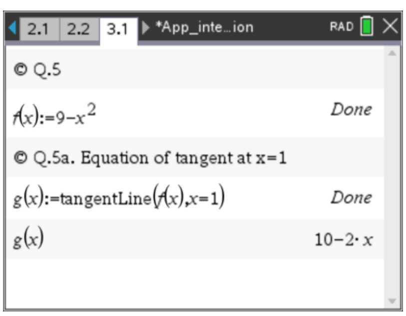


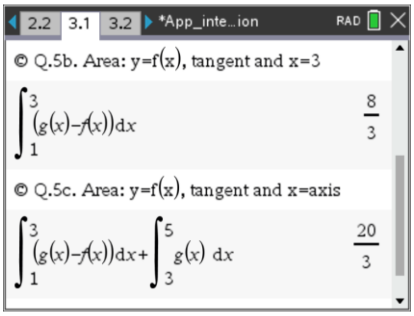



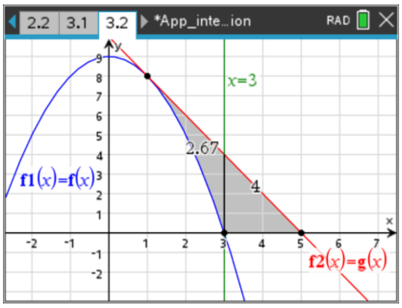



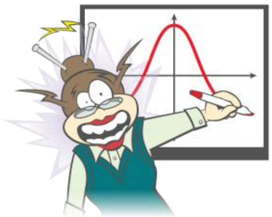

## Question 5

a. $y = 10 - 2x$ b. $\frac{8}{3}$ c. $\frac{20}{3}$



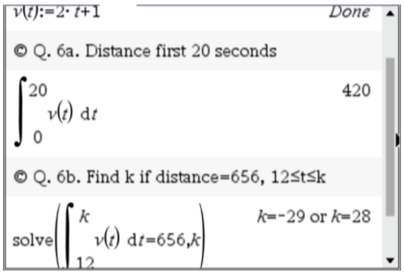



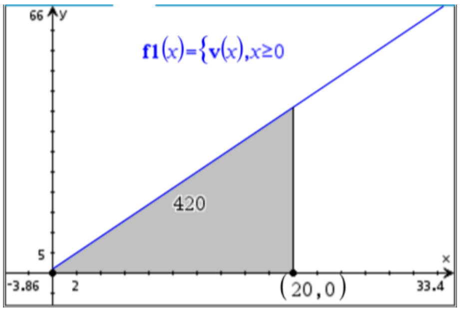



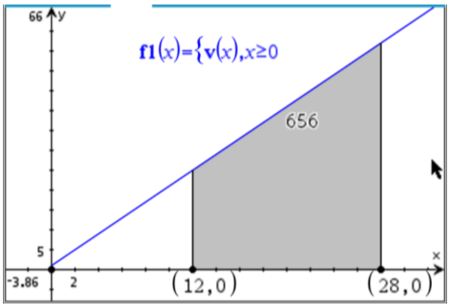


## Question 6

a. 420 metres   b. $k = 28$




## Question 7

$Av = 2 + \left(\frac{7-2}{2}\right) = \frac{9}{2}$. (The function increases and decreases at a constant rate, so the average value is half-way between $y = 2$ and $y = 7$. Calculus is not required to determine the average value in this case.)

## Question 8

$k = 2.50$ (correct to two decimal places).