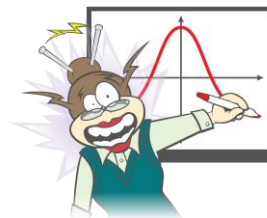


Mathematical Methods – Differential Calculus

Revision Question Sheet



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Each of the questions included here can be solved using the TI-Nspire CX CAS.

Question 1

Consider the function $f(x) = \frac{2x+5}{x+1}$.

- Find the equation of the gradient function.
- Find the value of the gradient function when $x = -2$.
- Find the coordinates of the point(s) on the function $y = f(x)$, where the derivative is -1 .

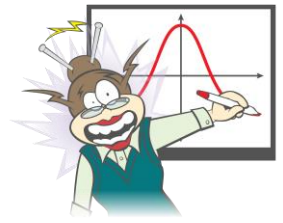
Response:

Question 2

Consider the function $h(x) = x^3 - ax^2 - 4$.

If $a \in \mathbb{Z}$, find the value of a , given that a tangent line to the function $y = h(x)$ is $y = -5x - 2$.

Response:



Question 3

Consider the functions,

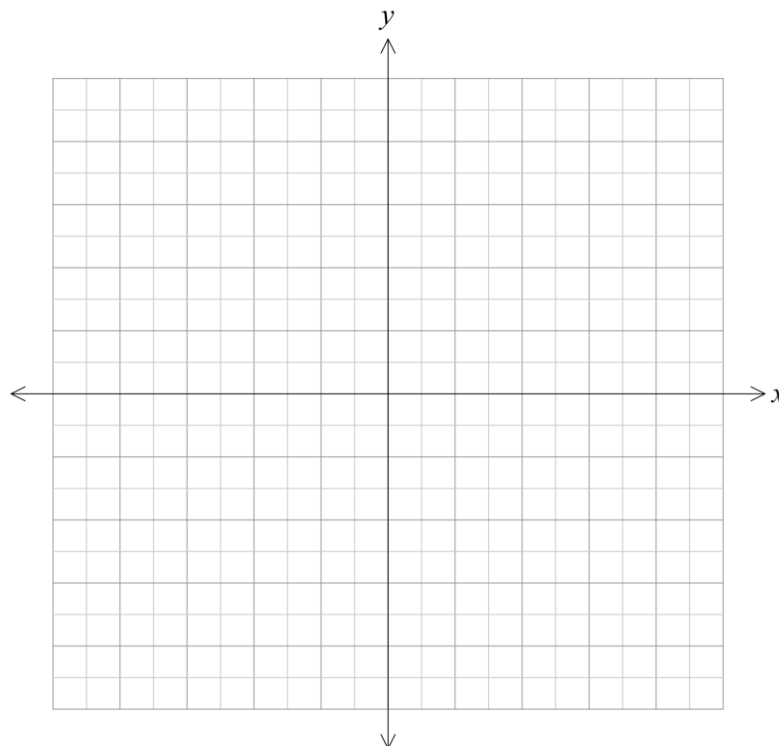
$$f : [-3, \infty) \rightarrow \mathcal{R}, \text{ where } f(x) = e^{2x} + 1 \text{ and}$$

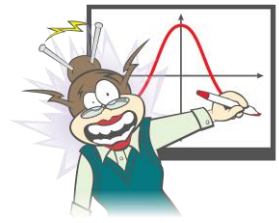
$$g : (-\infty, 1] \rightarrow \mathcal{R}, \text{ where } g(x) = 2x - 1.$$

If $h(x) = f(x) + g(x)$,

- state the maximal domain of $y = h(x)$.
- state the maximal domain of the derivative of $y = h(x)$.
- sketch the graph of the derivative of $y = h(x)$, showing the coordinates of all axial intercepts and endpoints correct to 2 decimal places.

Response:



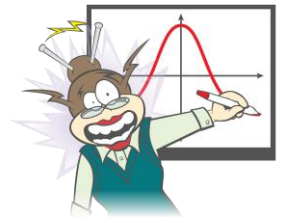


Question 4

Consider the function $f(x) = 2 \cos\left(3x - \frac{\pi}{4}\right) + 3$ for $-\pi < x \leq 2\pi$.

- How many stationary points does $y = f(x)$ have?
- Find the sum of the x-values of the stationary points of $y = f(x)$.
- Find the product of the x-values of the stationary points of $y = f(x)$.

Response:



Answers

Question 1

1.1 1.2 1.3 Doc RAD Done

$$f(x) := \frac{2 \cdot x + 5}{x + 1}$$

© (a)

$$\frac{d}{dx}(f(x)) = \frac{-3}{(x+1)^2}$$

© $f'(x) = \frac{-3}{(x+1)^2}$

1.1 1.2 1.3 Doc RAD Done

© (b)

© Define $f'(x)$ as used in part (c)

$$df(x) := \frac{-3}{(x+1)^2}$$

$df(-2) = -3$

1.1 1.2 1.3 Doc RAD

© (c) Method 1

$$\text{solve}(df(x) = -1, x) \quad x = -\sqrt{3} - 1 \text{ or } x = \sqrt{3} - 1$$

$$f(-\sqrt{3} - 1) = 2 - \sqrt{3}$$

$$f(\sqrt{3} - 1) = \sqrt{3} + 2$$

© Points $(-\sqrt{3} - 1, 2 - \sqrt{3})$ or $(\sqrt{3} - 1, \sqrt{3} + 2)$

1.2 1.3 1.4 Doc RAD

© (c) Method 2

$$\Delta \text{zeros}(df(x) + 1, x) \quad \{-\sqrt{3} + 1, \sqrt{3} - 1\}$$

$$f\{-\sqrt{3} + 1, \sqrt{3} - 1\} \quad \{2 - \sqrt{3}, \sqrt{3} + 2\}$$

© Points $(-\sqrt{3} - 1, 2 - \sqrt{3})$ or $(\sqrt{3} - 1, \sqrt{3} + 2)$

Question 2

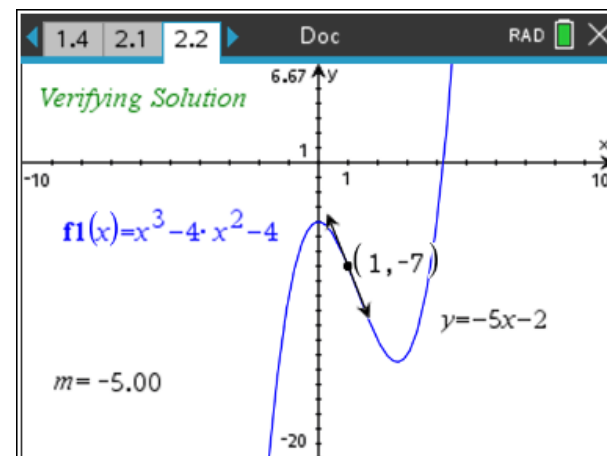
1.3 1.4 2.1 Doc RAD Done

Define $h(x) = x^3 - a \cdot x^2 - 4$

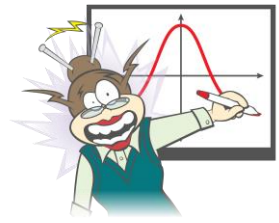
$$\text{solve} \left(\begin{cases} x^3 - a \cdot x^2 - 4 = -5 \cdot x - 2 \\ \frac{d}{dx}(h(x)) = -5 \end{cases}, \{a, x\} \right)$$

$$\left(\frac{7 \cdot \sqrt{17} + 7}{16} \right) \text{ or } x = 1 \text{ and } a = 4 \text{ or } x = \frac{\sqrt{17} - 1}{2} \text{ a}$$

© $a = 4$



Scroll across the Calculator Page to find the required solution. Remember, $a \in \mathbb{Z}$.



Question 3

2.1 2.2 3.1 Doc RAD

a)

$f(x) := e^{2 \cdot x} + 1 | x \geq -3$ ▶ Done

$g(x) := 2 \cdot x - 1 | x \leq 1$ ▶ Done

$h(x) := f(x) + g(x)$ ▶ Done

$\text{domain}(h(x), x)$ ▶ $-3 \leq x \leq 1$

2.2 3.1 3.2 Doc RAD

b)

$dh(x) := \frac{d}{dx}(h(x))$ ▶ Done

$\text{domain}(dh(x), x)$

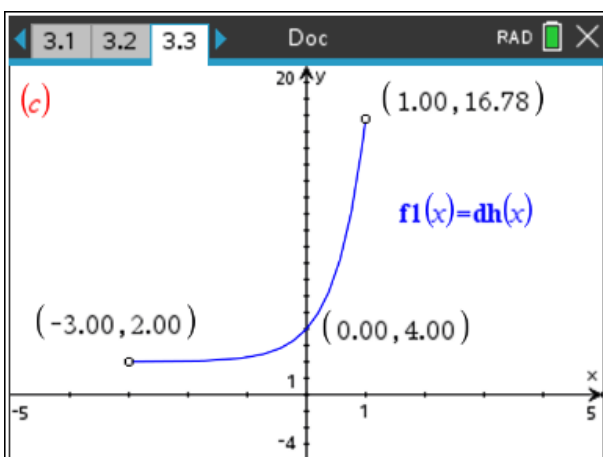
▶ $\text{domain}(\left(\left\{2 \cdot e^{2 \cdot x}, x > -3\right\} + \left\{2, x < 1\right\}, x\right)$

or $\frac{d}{dx}(h(x))$ ▶ $\left\{2 \cdot e^{2 \cdot x}, x > -3\right\} + \left\{2, x < 1\right\}$

Combining rules. Domain: $-3 < x < 1$

The endpoints of the function $y = h(x)$ are defined.

The endpoints of the derivative of a restricted function are not defined, by definition. CAS is used to verify this.

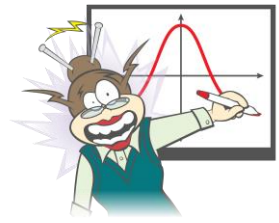


3.2 3.3 3.4 Doc RAD

Finding the coordinates of the undefined endpoints on the Graphs Page on Page 3.3.

* menu>5:Trace>1:Graph Trace. To find the left endpoint on $y=f1(x)$ type in -2.999999 and press enter for coordinates to appear.

A solid dot always appears. To make it an open circle hover over the point and $\text{ctrl}>\text{menu}>3:\text{Attributes}$.



Question 4

```

3.3 3.4 4.1 Doc RAD
f(x):=2*cos(3*x-pi/4)+3|-pi<x<=2*pi Done
a)
d/dx(f(x)) -> {6*cos(3*x+pi/4), -pi<x<2*pi}
df(x):={6*cos(3*x+pi/4), -pi<x<2*pi} Done

```

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3.3 3.4 4.1 Doc RAD
solve(df(x)=0,x)
-> x=-11*pi/12 or x=-7*pi/12 or x=-pi/4 or
x=pi/12 or x=5*pi/12 or x=3*pi/4 or x=13*pi/12 or
x=17*pi/12 or x=7*pi/4

```

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3.3 3.4 4.1 Doc RAD
Too many to count, try zeros().
zeros(df(x),x)
-> {-11*pi/12, -7*pi/12, -pi/4, pi/12, 5*pi/12, 3*pi/4, 13*pi/12, 17*pi/12, 7*pi/4}
Takes too long to count, use count(). Found
in the catalog.
count(zeros(df(x),x)) -> 9
**There are 9 stationary points.

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3.4 4.1 4.2 Doc RAD
b) sum() is found in the catalog.
sum(zeros(df(x),x)) -> 15*pi/4
c) product() is found in the catalog.
product(zeros(df(x),x)) -> -595595*pi^9/63700992

```