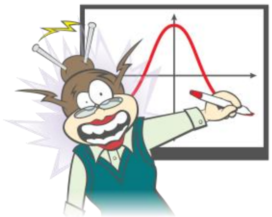
Mathematical Methods – Differential Calculus Revision Question Sheet



Author: Chris Ireson

Each of the questions included here can be solved using the TI-Nspire CX CAS.

Question 1

Consider the function $f(x)=\dfrac{2x+5}{x+1}$.

a) Find the equation of the gradient function.
b) Find the value of the gradient function when $x=-2$.
c) Find the coordinates of the point(s) on the function $y=f(x)$, where the derivative is -1.

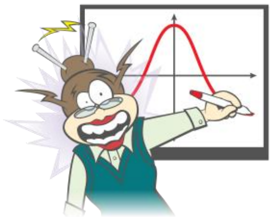
Response:

Question 2

Consider the function $h(x)=x^3-ax^2-4$.

If $a\in Z$, find the value of a, given that a tangent line to the function $y=h(x)$ is $y=-5x-2$.

Response:



Question 3

Consider the functions,

$$f:[-3,\infty)\to R \text{, where } f(x)=e^{2x}+1 \text{ and}$$
$$g:(-\infty,1]\to R \text{, where } g(x)=2x-1.$$

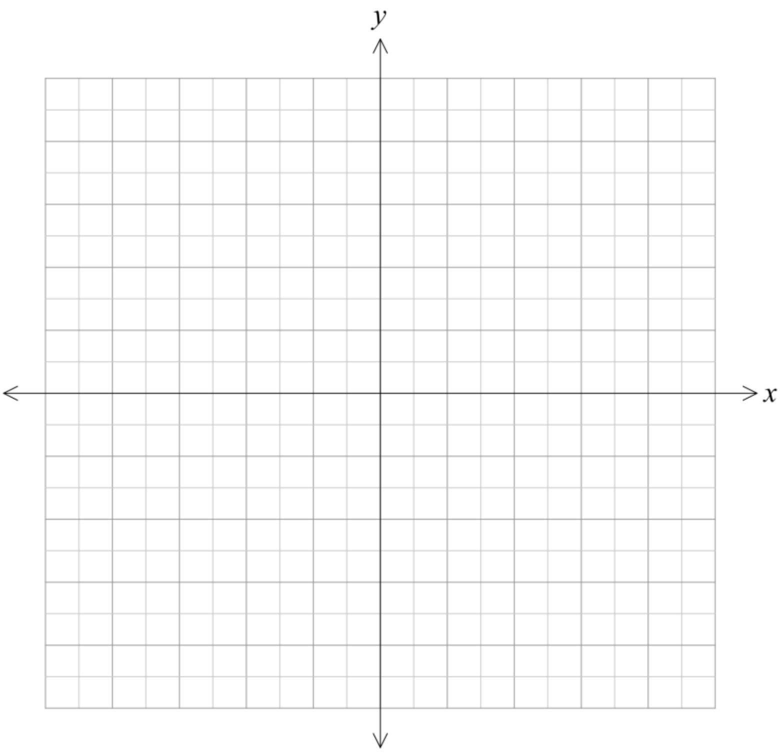
If $h(x)=f(x)+g(x)$,

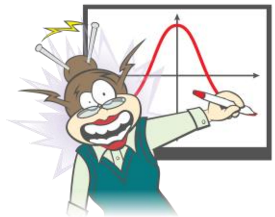
a) state the maximal domain of $y=h(x)$.

b) state the maximal domain of the derivative of $y=h(x)$.

c) sketch the graph of the derivative of $y=h(x)$, showing the coordinates of all axial intercepts and endpoints correct to 2 decimal places.

Response:





Question 4

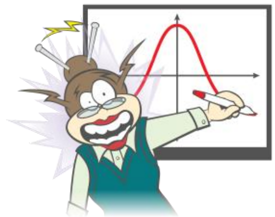
Consider the function $f(x) = 2\cos\left(3x - \frac{\pi}{4}\right) + 3$ for $-\pi < x \leq 2\pi$.

a) How many stationary points does $y = f(x)$ have?

b) Find the sum of the x-values of the stationary points of $y = f(x)$.

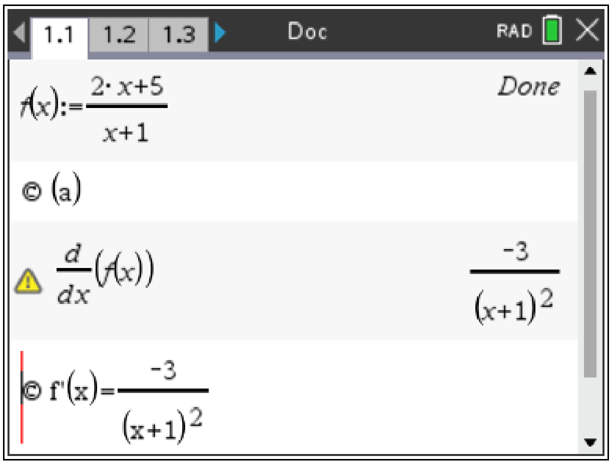
c) Find the product of the x-values of the stationary points of $y = f(x)$.

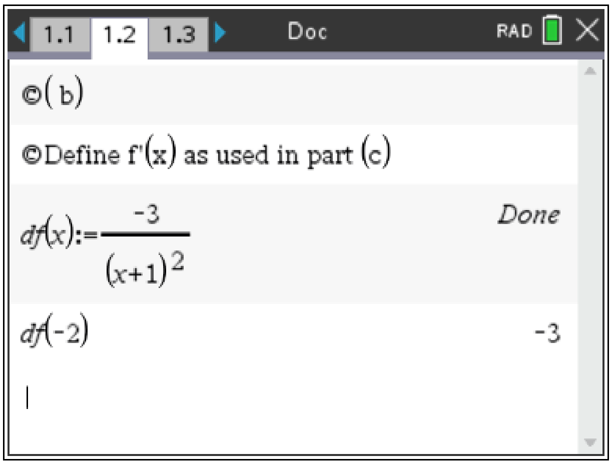
Response:

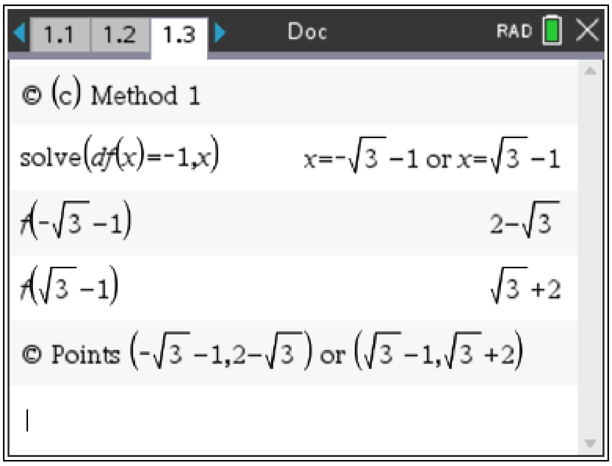


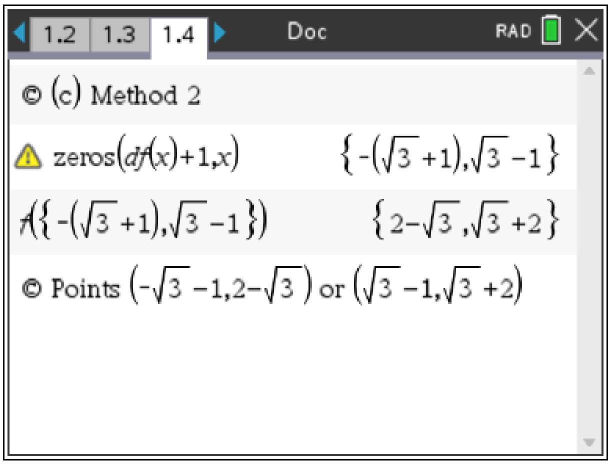
Answers

Question 1

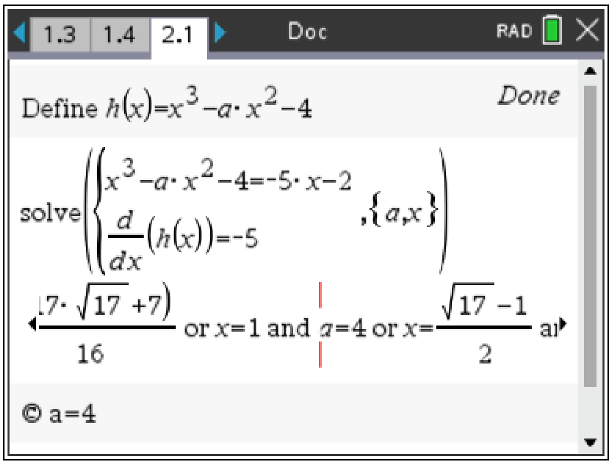


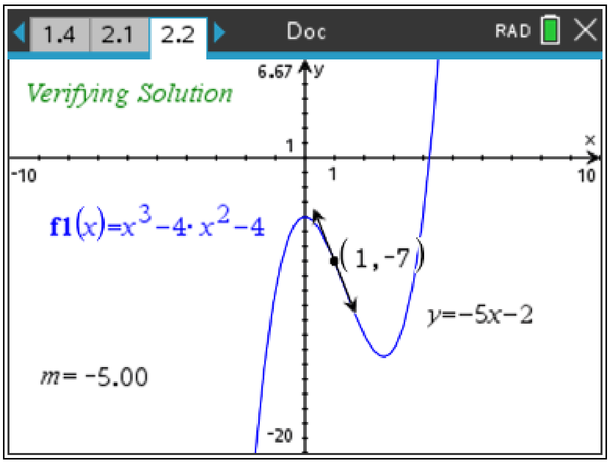




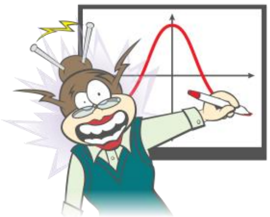


Question 2

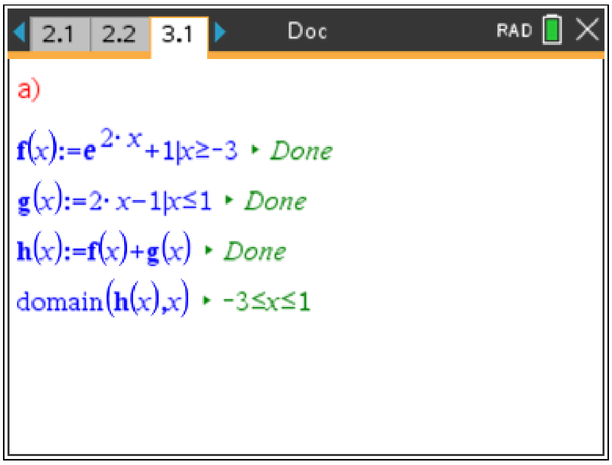


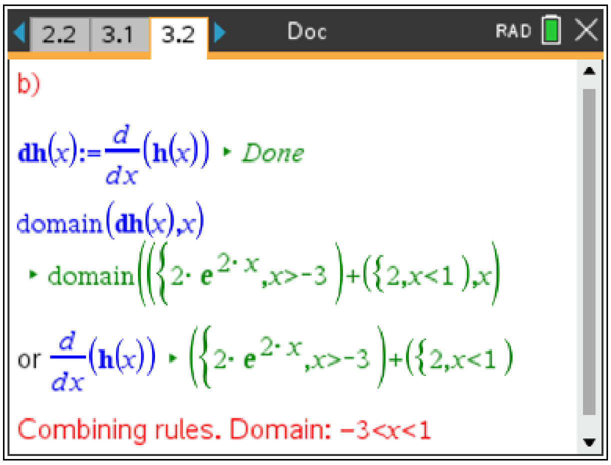


Scroll across the Calculator Page to find the required solution. Remember, $a \in Z$.



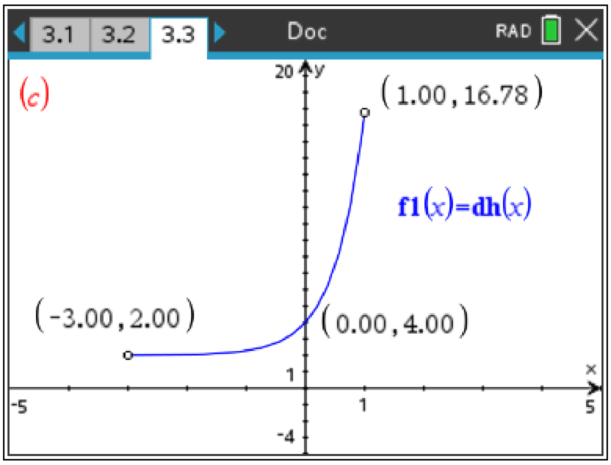
Question 3

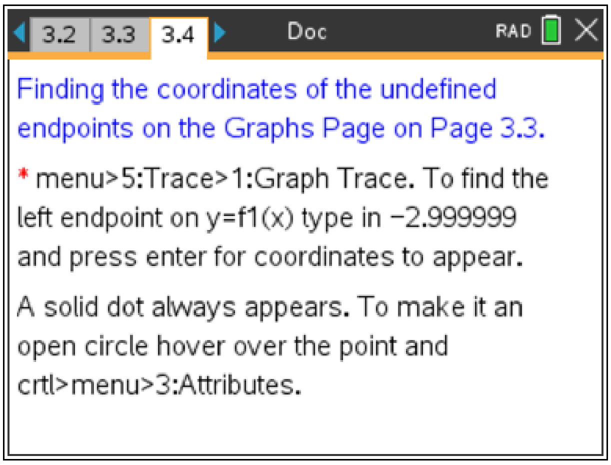


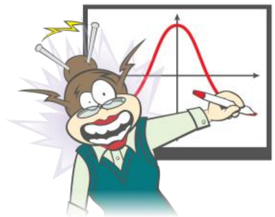


The endpoints of the function $y=h(x)$ are defined.

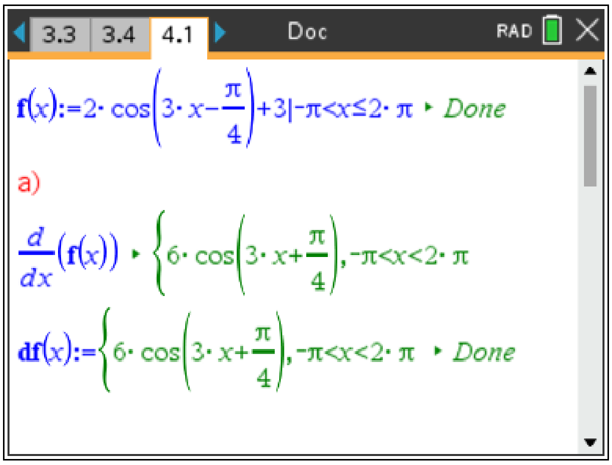
The endpoints of the derivative of a restricted function are not defined, by definition. CAS is used to verify this.

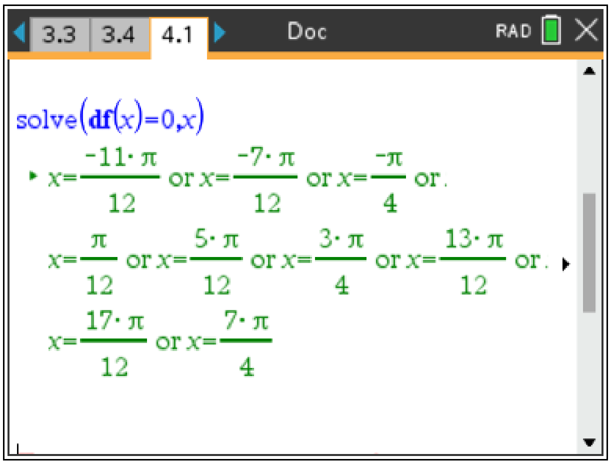


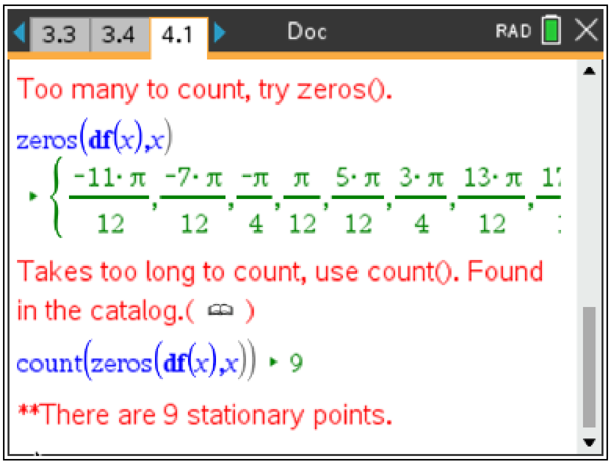


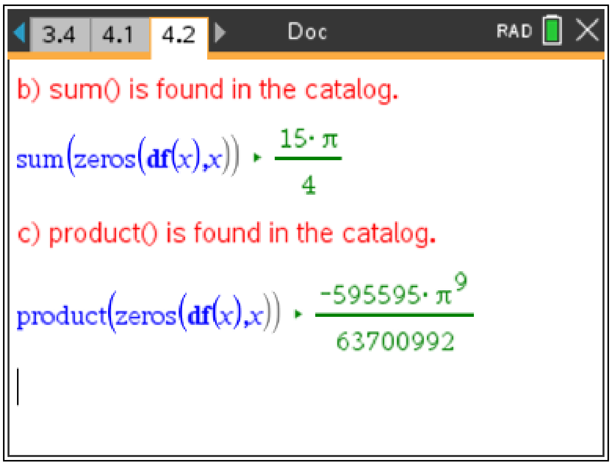


Question 4


3.3 3.4 4.1 Doc RAD
$f(x):=2\cdot\cos\left(3\cdot x-\frac{\pi}{4}\right)+3|-\pi<x\leq 2\cdot\pi$ ▸ Done
a)
$\frac{d}{dx}(f(x))$ ▸ $\left\{6\cdot\cos\left(3\cdot x+\frac{\pi}{4}\right),-\pi<x<2\cdot\pi\right.$
$df(x):=\left\{6\cdot\cos\left(3\cdot x+\frac{\pi}{4}\right),-\pi<x<2\cdot\pi\right.$ ▸ Done


3.3 3.4 4.1 Doc RAD
$\text{solve}(df(x)=0,x)$
▸ $x=\frac{-11\cdot\pi}{12}$ or $x=\frac{-7\cdot\pi}{12}$ or $x=\frac{-\pi}{4}$ or
$x=\frac{\pi}{12}$ or $x=\frac{5\cdot\pi}{12}$ or $x=\frac{3\cdot\pi}{4}$ or $x=\frac{13\cdot\pi}{12}$ or
$x=\frac{17\cdot\pi}{12}$ or $x=\frac{7\cdot\pi}{4}$


3.3 3.4 4.1 Doc RAD
Too many to count, try zeros().
$\text{zeros}(df(x),x)$
▸ $\left\{\frac{-11\cdot\pi}{12},\frac{-7\cdot\pi}{12},\frac{-\pi}{4},\frac{\pi}{12},\frac{5\cdot\pi}{12},\frac{3\cdot\pi}{4},\frac{13\cdot\pi}{12},\right.$ 1!
Takes too long to count, use count(). Found in the catalog.( )
$\text{count}(\text{zeros}(df(x),x))$ ▸ 9
**There are 9 stationary points.


3.4 4.1 4.2 Doc RAD
b) sum() is found in the catalog.
$\text{sum}(\text{zeros}(df(x),x))$ ▸ $\frac{15\cdot\pi}{4}$
c) product() is found in the catalog.
$\text{product}(\text{zeros}(df(x),x))$ ▸ $\frac{-595595\cdot\pi^{9}}{63700992}$