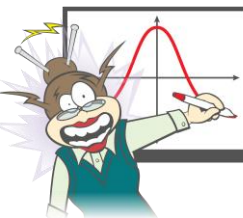


# Creating Notes Pages for Further Mathematics Modules



**Author:** Craig Browne

Each of the questions included here can be solved using either the TI-Nspire CX or CX CAS.

## Question 1a: Matrices

On any given day, 80% of the customers who visit Jenny's Ice-cream Shop and buy soft-serve ice cream will also buy it again on the return visit. However, 20% of these customers will buy a thick-shake next time. 90% of customers who buy a thick-shake will buy a thick-shake next time, but 10% of them will buy soft serve ice cream next time. This information is represented in a transition matrix as shown

$$T = \begin{bmatrix} 0.8 & 0.1 \\ 0.2 & 0.9 \end{bmatrix}$$

If 50 customers buy a thick-shake initially how many of these will buy a thick-shake on the next visit?

Response:

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## Question 1b

The following matrix shows the initial number of people who bought soft serve ice-creams and thick-shakes, where 50 customers initially bought soft serve ice-creams and 40 initially bought thick-shakes.

$$S_0 = \begin{bmatrix} 50 \\ 40 \end{bmatrix}$$

Use recursion to find the number of customers who bought soft-serve ice-creams and thick-shakes on the third return visit,  $S_3$ .

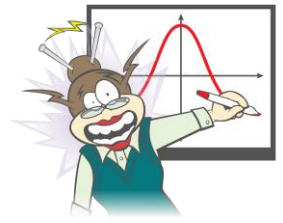
Response:

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### Question 1c

Calculate the number of thick-shakes bought by customers on their 10<sup>th</sup> return visit.

Response:

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### Question 1d

Over the long term, how many soft-serve ice creams and thick-shakes would she expect to sell would Jenny expect to sell to returning visitors.

Response:

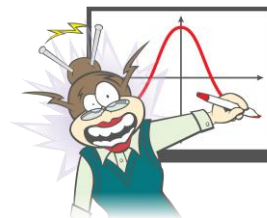
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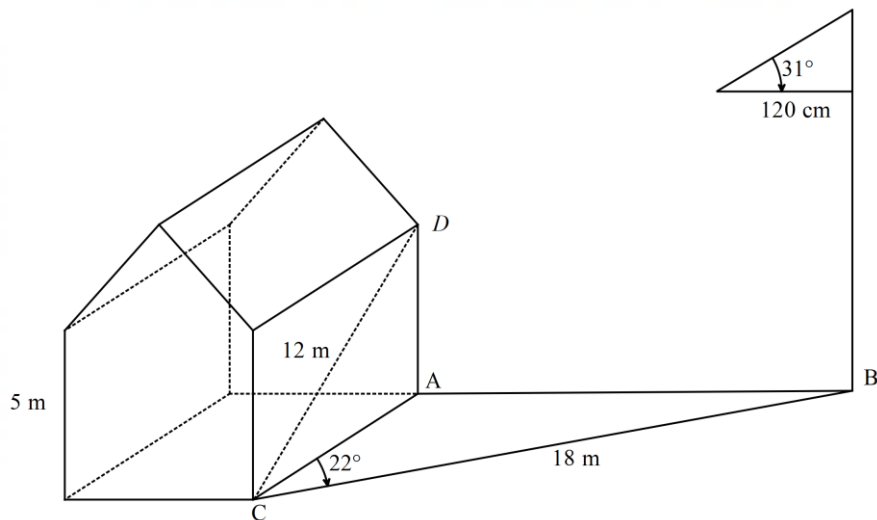
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## Question 2a: Geometry and Trigonometry



A garden shed is located in the back yard. There is a flagpole that has a triangular flag that forms a right-angled triangle. The height of the flagpole is 13 m. There is a path from the shed to the flagpole that is marked by the line AB. The distance from along BC is 18 m. [Note: Do not assume that Angle BAC is a right angle]



State the length of AD.

Response:

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## Question 2b

Calculate height of the flag, expressing your answer to the nearest cm.

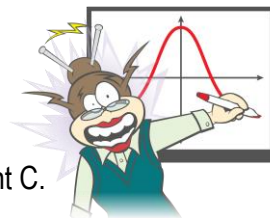
Response:

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### Question 2c

Calculate the angle of elevation to the top of the flagpole from the corner of the shed at point C. Write your answer to the nearest degree.

Response:

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### Question 2d

Calculate the length of the shed to 2 decimal places.

Response:

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### Question 2e

Calculate the length of the path from the shed to the flagpole shown by the line AB to 2 decimal places.

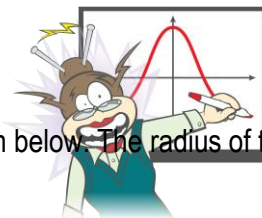
Response:

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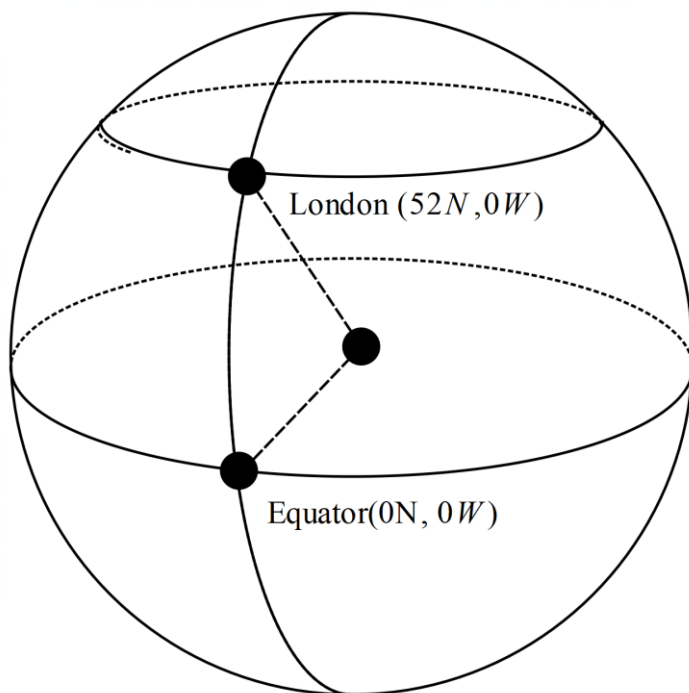
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### Question 3a: Geometry and Trigonometry

London is Located at a Latitude of  $52^\circ\text{N}$  and a Longitude of  $0^\circ$ . This is shown on the diagram below. The radius of the earth is 6400 km



Use this information to find:

The distance to the Equator to the nearest km.

Response:

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### Question 3b

The distance to the North Pole to the nearest km.

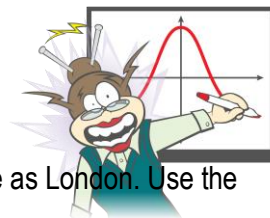
Response:

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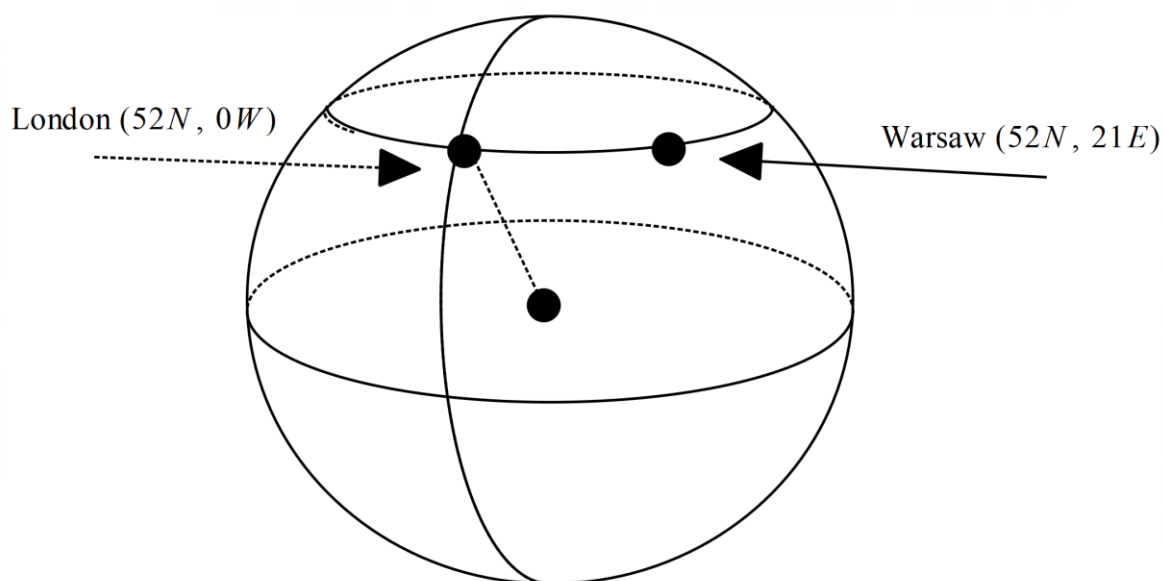
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### Question 3c

The Distance between London and Warsaw if Warsaw is approximate on the same Latitude as London. Use the diagram below as an aid.



Response:

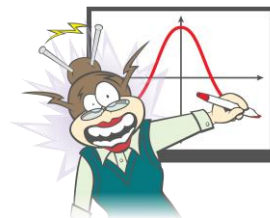
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Now your Try!!



### Question 1a: Matrices

On any given day 70% of customers who visit Peter's pie shop and buy a pie, will buy a pie again on the following visit. 30% of these customers will buy a sausage roll next time. 15% of customers who buy sausage roll will buy a pie next time but 85% of them will buy sausage roll next time.

This information is represented in a transition matrix as shown

$$T = \begin{bmatrix} 0.7 & 0.15 \\ 0.3 & 0.85 \end{bmatrix}$$

If 70 customers buy a pie initially how many of these will buy a pie on the next visit?

Response:

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### Question 1b

The following matrix shows the initial number of people who bought pies and sausage rolls, where 70 customers initially bought pies and 60 initially bought sausage rolls.

$$S_0 = \begin{bmatrix} 70 \\ 60 \end{bmatrix}$$

Use recursion to find the number of customers who bought pies and sausage rolls on the third return visit,  $S_3$ .

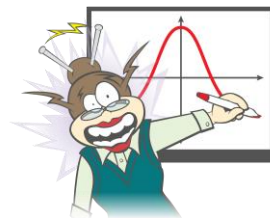
Response:

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### Question 1c

Calculate the number of sausage rolls bought by customers on their 10<sup>th</sup> return visit.

Response:

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### Question 1d

Over the long term, how many pies and sausage rolls would she expect to sell would Peter expect to sell to returning visitors.

Response:

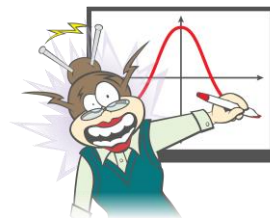
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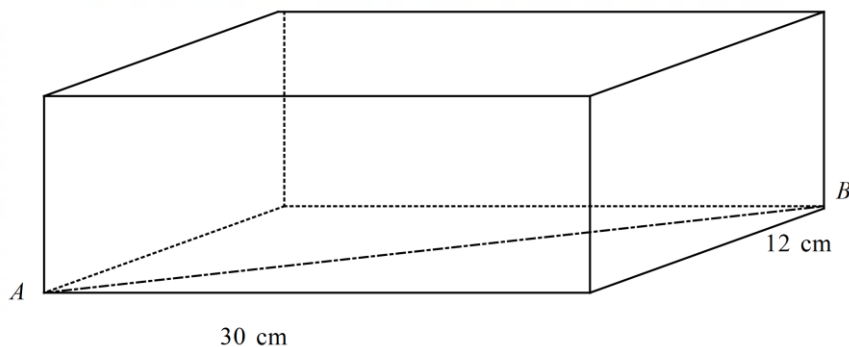
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### Question 2: Geometry and Trigonometry

A shoe box that is 30 cm long and 12 cm wide. The diagram is shown below.



Calculate the length of the diagonal line AB. Express your answer to three significant figures.

Response:

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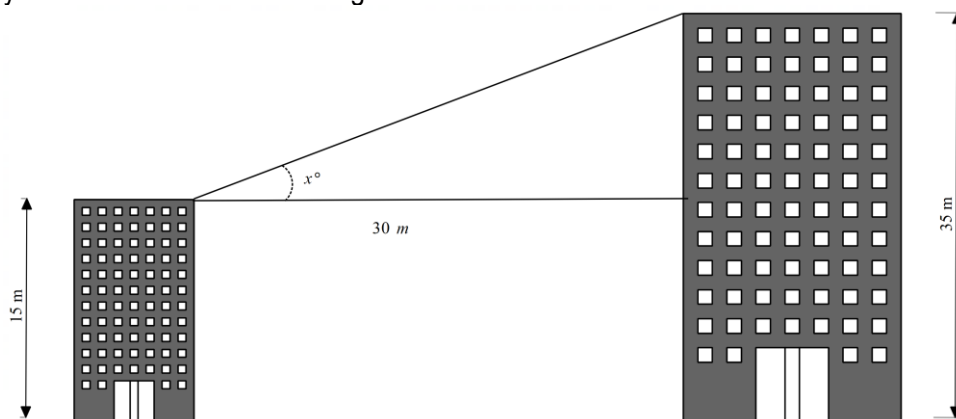
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### Question 3: Geometry and Trigonometry

Calculate the angle of elevation from the corner of the smaller building to the corner of the taller building. Express your answer to the nearest degree.



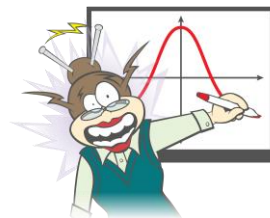
Response:

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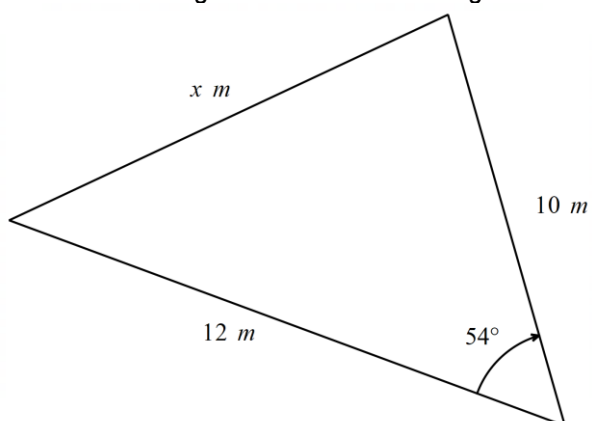
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#### Question 4: Geometry and Trigonometry

Calculate the length marked  $x$  on the diagram below, to two decimal places.



Response:

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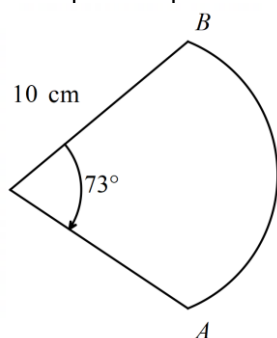
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#### Question 5: Geometry and Trigonometry

A piece of pie is shown below.



Calculate the arc length from A to B to two decimal places.

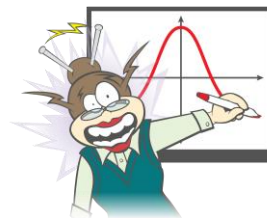
Response:

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## Answers

### Question 1a: Matrices

20% of 50 = 10

### Question 1b

Soft serve = 37, Thick shakes = 53

Take note of the screen shot from the calculator below when the information has been entered:

```

1.1 2.1 3.1 matrices RAD
transition matrices
Transition matrix:
t:= $\begin{bmatrix} 0.8 & 0.1 \\ 0.2 & 0.9 \end{bmatrix}$  *  $\begin{bmatrix} 0.8 & 0.1 \\ 0.2 & 0.9 \end{bmatrix}$ 
Initial state matrix
s0:= $\begin{bmatrix} 50 \\ 40 \end{bmatrix}$  *  $\begin{bmatrix} 50. \\ 40. \end{bmatrix}$ 
-----
[44.]

```

```

1.1 2.1 3.1 matrices RAD
s1:=t*s0 *  $\begin{bmatrix} 44. \\ 46. \end{bmatrix}$ 
s2:=t*s1 *  $\begin{bmatrix} 39.8 \\ 50.2 \end{bmatrix}$ 
s3:=t*s2 *  $\begin{bmatrix} 36.86 \\ 53.14 \end{bmatrix}$ 
s4:=t*s3 *  $\begin{bmatrix} 34.802 \\ 55.198 \end{bmatrix}$ 

```

### Question 1c

59 thick shakes sold after 10 visits

Type 10 in the maths box for n and press enter

Scroll down the page or press **ctrl 3** to move **down** the page one **screen** at a time. Note to move **up** a **screen** at a time press **ctrl 9**. To move to the **top** of the **screen** press **ctrl 7**. To move to the **bottom** of the **page** press **ctrl 1**.

```

1.1 2.1 3.1 *matrices RAD
s3:=t*s2 *  $\begin{bmatrix} 36.86 \\ 53.14 \end{bmatrix}$ 
s4:=t*s3 *  $\begin{bmatrix} 34.802 \\ 55.198 \end{bmatrix}$ 
-----
Enter the value for n: n:=10 * 10.
sn=t^n*s0 *  $\begin{bmatrix} 30.564950498 \\ 59.435049502 \end{bmatrix}$ 
make n a large number to find the steady state

```

### Question 1d

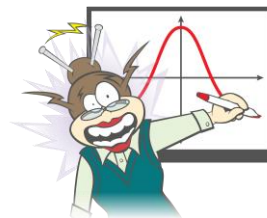
30 soft serve and 60 thick shakes.

Type 100 in the maths box for n and press enter.

```

1.1 2.1 3.1 matrices RAD
s3:=t*s2 *  $\begin{bmatrix} 36.86 \\ 53.14 \end{bmatrix}$ 
s4:=t*s3 *  $\begin{bmatrix} 34.802 \\ 55.198 \end{bmatrix}$ 
-----
Enter the value for n: n:=100 * 100.
sn=t^n*s0 *  $\begin{bmatrix} 30. \\ 60. \end{bmatrix}$ 
make n a large number to find the steady state

```



## Question 2a: Geometry and Trigonometry

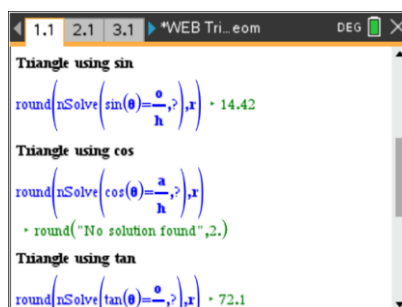
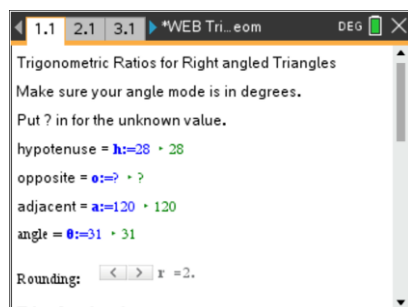
5 m

## Question 2b

72 cm

Note: The question focuses on using the tan ratio so you only worry about entering information for “o”, “a” and “ $\theta$ ”. You type a “?” for the unknown value.

Then move down the page and read the answer under “Triangle using tan”.

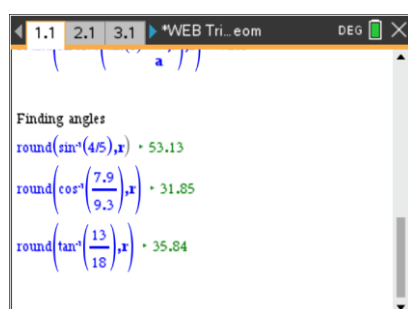


## Question 2c

36°

Note: The question focuses on using the tan ratio so you only worry about entering information for “o”, “a” and “ $\theta$ ”. You type a “?” for the unknown value.

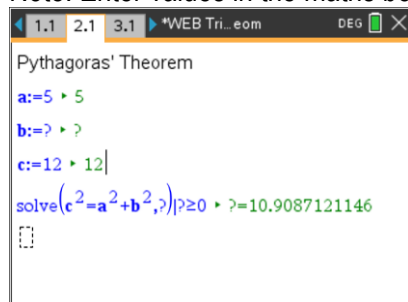
Then move down the page and read the answer under “Triangle using tan”.

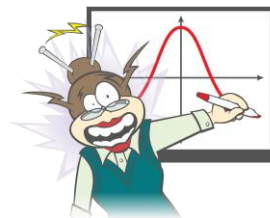


## Question 2d

10.91 m

Note: Enter values in the maths boxes. Enter “?” for the unknown value of “b”.





## Question 2e

8.88 m

Note: Enter values in the maths boxes. Enter “?” for the unknown value of “c”.

```

2.1 3.1 4.1 *WEB Tri...eom DEG
Cos Rule for Non-Rightangled Triangle
side a: sa:=10.91 → 10.91 side b: sb:=18 → 18
side c: sc:=? → ? angle C: ac:=22 → 22
< > r =3.
round(nSolve(sc^2=sa^2+sb^2-2*sa*sb*cos(ac),?),r)
→ 8.881

```

## Question 3a: Geometry and Trigonometry

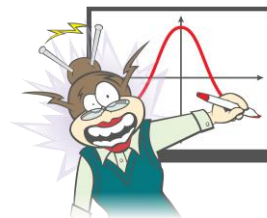
5808 km

Note: Enter values in the maths boxes. Enter “?” for the unknown value of “l”.

```

2.1 3.1 4.1 *WEB Tri...eom DEG
Arc Length
radius: r:=6400 → 6400.
θ:=52 → 52.
l:=? → ?
solve(l=π*r*θ/180,?) → ?=5808.45575064

```



### Question 3b

4245 km

Note: Enter values in the maths boxes. Enter “?” for the unknown value of “I”.

```

2.1 3.1 4.1 *WEB Tri...eom DEG
Arc Length
radius: r:=6400 ▶ 6400.
θ:=90-52 ▶ 38.
l:=? ▶ ?
solve(1=frac(π·r·θ,180),?) ▶ ?=4244.64074085

```

### Question 3c

a. 1444 km

First a triangle is created using the information so the radius of the minor circle can be found.

Then the cos ratio is used to calculate the radius.

```

1.1 2.1 3.1 *WEB Tri...eom DEG
Trigonometric Ratios for Right angled Triangles
Make sure your angle mode is in degrees.
Put ? in for the unknown value.
hypotenuse = h:=6400 ▶ 6400.
opposite = o:=? ▶ ?
adjacent = a:=? ▶ ?
angle = θ:=52 ▶ 52.
Rounding: < > x = 2.

```

```

1.1 2.1 3.1 *WEB Tri...eom DEG
round(nSolve(sin(θ)=frac(o,h),?),x) ▶ 5043.27
Triangle using cos
round(nSolve(cos(θ)=frac(a,h),?),x) ▶ 3940.23
Triangle using tan
round(nSolve(tan(θ)=frac(o,a),?),x)
▶ round("No solution found",2.)⚠

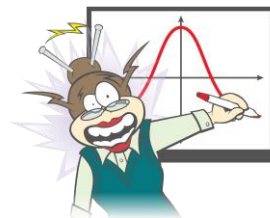
```

Then we can use the arc length formula to calculate the distance that is required.

```

2.1 3.1 4.1 *WEB Tri...eom DEG
Arc Length
radius: r:=3940 ▶ 3940.
θ:=21 ▶ 21.
l:=? ▶ ?
solve(1=frac(π·r·θ,180),?) ▶ ?=1444.0854231

```



Now your try Answers!!

### Question 1a

70% of 70 = 49 pies.

### Question 1b

48 pies and 82 sausage rolls

```

2.1 3.1 4.1 *matrices RAD
Transition matrices
Transition matrix:
t:= $\begin{bmatrix} 0.7 & 0.15 \\ 0.3 & 0.85 \end{bmatrix}$   $\rightarrow$   $\begin{bmatrix} 0.7 & 0.15 \\ 0.3 & 0.85 \end{bmatrix}$ 
Initial state matrix
s0:= $\begin{bmatrix} 70 \\ 60 \end{bmatrix}$   $\rightarrow$   $\begin{bmatrix} 70. \\ 60. \end{bmatrix}$ 
-----
[ca]

```

```

2.1 3.1 4.1 *matrices RAD
s1:=t·s0  $\rightarrow$   $\begin{bmatrix} 58. \\ 72. \end{bmatrix}$ 
s2:=t·s1  $\rightarrow$   $\begin{bmatrix} 51.4 \\ 78.6 \end{bmatrix}$ 
s3:=t·s2  $\rightarrow$   $\begin{bmatrix} 47.77 \\ 82.23 \end{bmatrix}$ 
s4:=t·s3  $\rightarrow$   $\begin{bmatrix} 45.7735 \\ 84.2265 \end{bmatrix}$ 

```

### Question 1c

43 pies and 87 sausage rolls.

```

2.1 3.1 4.1 *matrices RAD
s3:=t·s2  $\rightarrow$   $\begin{bmatrix} 47.77 \\ 82.23 \end{bmatrix}$ 
s4:=t·s3  $\rightarrow$   $\begin{bmatrix} 45.7735 \\ 84.2265 \end{bmatrix}$ 
-----
Enter the value for n: n:=10  $\rightarrow$  10.
sn=t^n·s0  $\rightarrow$   $\begin{bmatrix} 43.4008787099 \\ 86.5991212901 \end{bmatrix}$ 
make n a large number to find the steady state

```

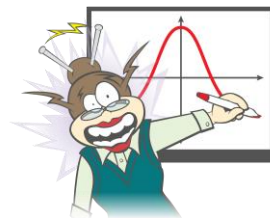
### Question 1d

43 pies and 87 sausage rolls.

```

2.1 3.1 4.1 *matrices RAD
s3:=t·s2  $\rightarrow$   $\begin{bmatrix} 47.77 \\ 82.23 \end{bmatrix}$ 
s4:=t·s3  $\rightarrow$   $\begin{bmatrix} 45.7735 \\ 84.2265 \end{bmatrix}$ 
-----
Enter the value for n: n:=100  $\rightarrow$  100.
sn=t^n·s0  $\rightarrow$   $\begin{bmatrix} 43.3333333333 \\ 86.6666666667 \end{bmatrix}$ 
make n a large number to find the steady state

```



## Question 2

32.3 cm

```

1.1 2.1 3.1 *WEB Tri...eom DEG
Pythagoras' Theorem
a:=30 → 30.
b:=12 → 12.
c:=? → ?
solve(c^2=a^2+b^2,?)>0 → ?=32.3109888428

```

## Question 3

34°

```

1.1 2.1 3.1 *WEB Tri...eom DEG
Trigonometric Ratios for Right angled Triangles
Make sure your angle mode is in degrees.
Put ? in for the unknown value.
hypotenuse = h:=6400 → 6400.
opposite = o:=20 → 20.
adjacent = a:=30 → 30.
angle = θ:=? → ?
Rounding: < > r = 2.

```

```

1.1 2.1 3.1 *WEB Tri...eom DEG
Triangle using sin
round(nSolve(sin(θ)=o/h,?),r) → 0.18
Triangle using cos
round(nSolve(cos(θ)=a/h,?),r) → 89.73
Triangle using tan
round(nSolve(tan(θ)=o/a,?),r) → 33.69

```

## Question 4

10.15 m

```

1.1 2.1 3.1 *WEB Tri...eom DEG
Cos Rule for Non-Rightangled Triangle
side a: sa:=12 → 12. side b: sb:=10 → 10.
side c: sc:=? → ? angle C:ac:=54 → 54.
< > r = 3.
round(nSolve(sc^2=sa^2+sb^2-2*sa*sb*cos(ac),?),r)
→ 10.146

```

## Question 5

12.74 cm

```

2.1 3.1 4.1 *WEB Tri...eom DEG
Arc Length
radius: r:=10 → 10.
θ:=73 → 73.
l:=? → ?
solve(l=π*r*θ/180,?) → ?=12.7409035396

```