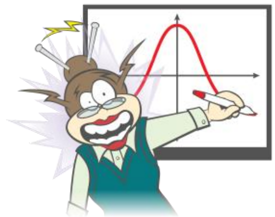
Webinar problems: Matrices



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Each of the questions included here can be solved using either the TI-Nspire CX or CX CAS.

Problem 1

The table below shows the data from one week's worth of sales at a clothing store:

Item	T-shirt	Pants	Jacket	Shoes
Cost	$18	$45	$60	$98
Number	27	15	23	9

a) Construct a matrix equation that, when solved, displays the total number of items sold, and the total revenue earnt from the sales.

b) Solve the matrix equation in part a.

Response:

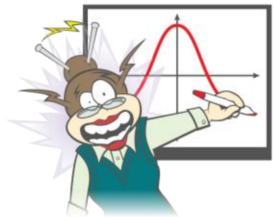
Problem 2

Consider the following pair of simultaneous equations:

$$5x - 2y - 16 = 0$$
$$4 + 6y + 7x = 0$$

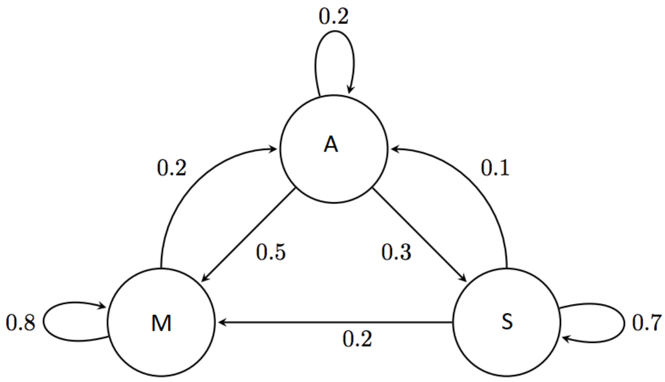
a) Set up a matrix equation to show how you would begin to solve for x and y.
b) Solve the equations

Response:



Problem 3

In year 2000, a group of 720 students were asked to choose how they would prefer to spend their spare time. Their choices were Art, Music or Sport. 380 students chose art, 250 chose sport, and 90 students chose music. The transition diagram below shows how these preferences are expected to change from year to year:



How many students will choose music in 2003?

Response:

Problem 4

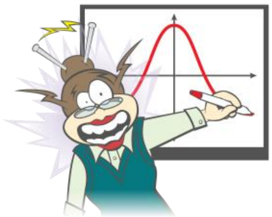
The students from Problem 3 were tracked over many years, even decades.

a) Set up an equation to show how you would find how many students chose each activity in the n^{th} year after 2000.

b) In the long term, numbers per activity type are expected to stabilise and reach a steady state. How many of the initial students would be expected to be in each category in this steady state? Justify your response.

Response:

Extra Practice problems: Matrices



Extra Practice Problem 1

Don's Pizza received two big orders from customers throwing parties to celebrate the end of lockdowns. The first customer bought 7 regular pizzas and 1 gourmet pizza and paid \$74. The second customer ordered 5 regular pizzas and 1 gourmet pizza and paid a total of \$58.

a) Using a matrix equation, show how you would represent this information.
b) What is the price of each pizza?

Response:

Extra Practice Problem 2

The following transition table is data taken from Netflix statistics and it describes the proportion of people who change from one movie genre to another each time they watch their next movie.

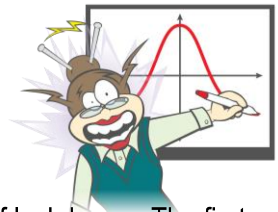
		FROM			
		Action	Comedy	Romance	Thriller
TO	Action	0.6	0.1	0.1	0.3
	Comedy	0.2	0.05	0.35	0.2
	Romance	0.05	0.85	0.3	0
	Thriller	0.15	0	0.25	0.5

This is initially tracking 100 people whose initial movie choices are as below:

Action	18
Comedy	27
Romance	30
Thriller	25

a) Approximately how many people will choose Romance as their next movie?
b) Set up a matrix equation to show how you would find how many people chose each genre in their n^{th} movie choice.
c) In the long term, numbers per genre of movie are expected to stabilise and reach a steady state. How many of the initial people would be expected to be in each genre in this steady state? Justify your response.

Response:



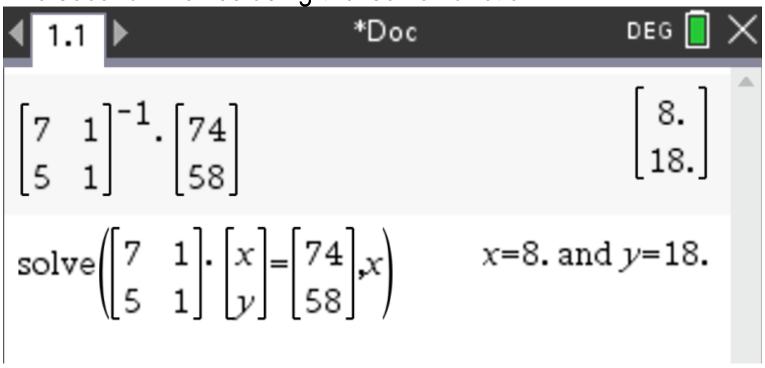
Answers

Extra Practice Problem 1

Don's Pizza received two big orders from customers throwing parties to celebrate the end of lockdowns. The first customer bought 7 regular pizzas and 1 gourmet pizza and paid \$74. The second customer ordered 5 regular pizzas and 1 gourmet pizza and paid a total of \$58.

a) Let x represent the cost of a regular pizza, and let y represent the cost of a gourmet pizza:
$\begin{bmatrix} 7 & 1 \\ 5 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 74 \\ 58 \end{bmatrix}$

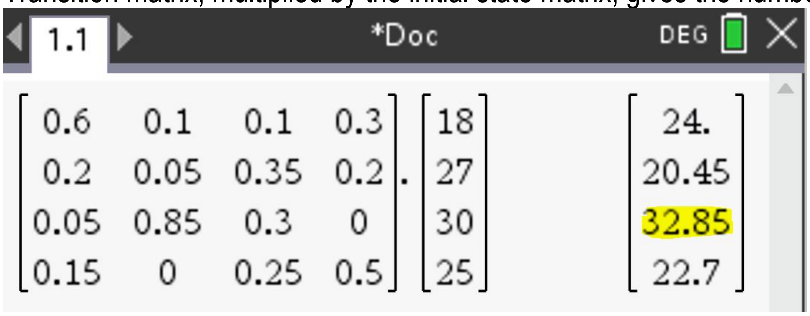
b) What is the price of each pizza?
Below are two methods for solving the equation from part a. The first involves taking the inverse of matrix $\begin{bmatrix} 7 & 1 \\ 5 & 1 \end{bmatrix}$ and multiplying it on the other side.
The second involves using the 'solve' function.



Answer: A regular pizza costs \$8 and a gourmet pizza costs \$18.

Extra Practice Problem 2

a) Approximately many people will choose Romance as their next movie?
$S_1 = T \times S_0$
Transition matrix, multiplied by the initial state matrix, gives the numbers for the next state.

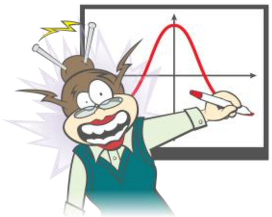


In this case, Romance gets approximately 33 viewers in the next choosing.

Set up a matrix equation to show how you would find how many people chose each genre in their n^{th} movie choice.
$S_n = T^n \times S_0$

$$S_n = \begin{bmatrix} 0.6 & 0.1 & 0.1 & 0.3 \\ 0.2 & 0.05 & 0.35 & 0.2 \\ 0.05 & 0.85 & 0.3 & 0 \\ 0.15 & 0 & 0.25 & 0.5 \end{bmatrix}^n \times \begin{bmatrix} 18 \\ 27 \\ 30 \\ 25 \end{bmatrix}$$

b)



c) In the long term, numbers per genre of movie are expected to stabilise and reach a steady state. How many of the initial people would be expected to be in each genre in this steady state? Justify your response.

In the long term you would expect stabilizing numbers of approximately:

Action	29
Comedy	21
Romance	28
Thriller	22

As you can see in the examples below, having chosen two numbers for *n* (50 and 500) we observe no significant change in the result. Therefore, indicating a steady state has been reached.

1.1 *Doc DEG

$$\begin{bmatrix} 0.6 & 0.1 & 0.1 & 0.3 \\ 0.2 & 0.05 & 0.35 & 0.2 \\ 0.05 & 0.85 & 0.3 & 0 \\ 0.15 & 0 & 0.25 & 0.5 \end{bmatrix}^{50} \cdot \begin{bmatrix} 18 \\ 27 \\ 30 \\ 25 \end{bmatrix}$$

$$\begin{bmatrix} 28.9892252437 \\ 20.9851205747 \\ 27.5525910724 \\ 22.4730631093 \end{bmatrix}$$

1.1 *Doc DEG

$$\begin{bmatrix} 0.6 & 0.1 & 0.1 & 0.3 \\ 0.2 & 0.05 & 0.35 & 0.2 \\ 0.05 & 0.85 & 0.3 & 0 \\ 0.15 & 0 & 0.25 & 0.5 \end{bmatrix}^{500} \cdot \begin{bmatrix} 18 \\ 27 \\ 30 \\ 25 \end{bmatrix}$$

$$\begin{bmatrix} 28.9892252437 \\ 20.9851205747 \\ 27.5525910724 \\ 22.4730631093 \end{bmatrix}$$