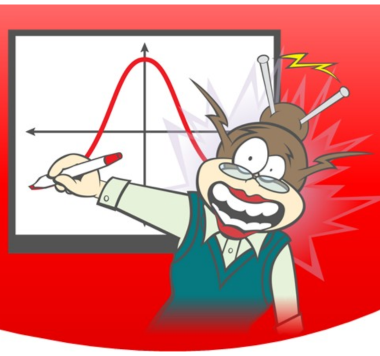
STUDENT REVISION SERIES



Calculus 2

Question: 1.

Find the gradient at $x = 2$ on the curve $x^3 + y^3 = 9$.

Question: 2.

Find the equation of the tangent to the curve $3x^2 - 2x^2y + 4y^2 = 1$ at the point $(3, 1)$

Question: 3.

Find the coordinates of the points on the curve $x^2 - 3xy + 3y^2 = 9$ at which the tangent is parallel to the x-axis

Question: 4.

For the curve defined by the parametric equations
$x = t - t^2$
$y = t - t^3$
Find $\frac{dy}{dx}$ at $t = 2$

Question: 5.

For the curve defined by the parametric equations
$x = t - \sin(t)$
$y = 1 - \cos(t)$
Find $\frac{dy}{dx}$ at $y = \frac{1}{2}$

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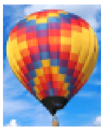
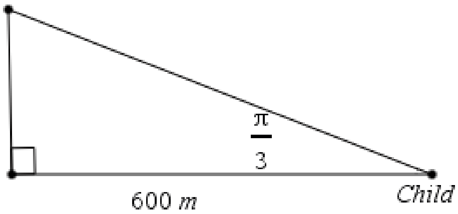
Question: 6.
For the curve defined by the parametric equations
$x = t^3 - 2t + 2$
$y = e^{2t}$ for $0 \le t \le \frac{3}{2}$
Find the coordinates of the point where $\frac{dy}{dx}$ is undefined.

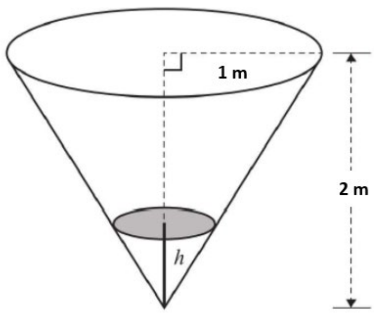
Question: 7.
An ascending hot air balloon rises vertically. As it rises the balloon is observed by a child at ground level 600 meters away. When the angle of elevation of the balloon is $\frac{\pi}{3}$ radians from the horizontal direction, and is increasing at a rate of 0.01 radians per second. Find the speed of the balloon. Give your answer as an exact value.





Question: 8.
A pancake is in the shape of a circle cooking in a frying pan. Its surface area is increasing at a rate of $0.001\ m^2/s$. Let r meters be the radius of the pancake at time t seconds. Find $\frac{dr}{dt}$ in terms of r.

Question: 9
A conical tank of radius of 1 meter and height of 2 meters is filling with water at a constant rate of $1.5\ m^3/min$. At what rate is the water rising when the depth is 0.75 meters?



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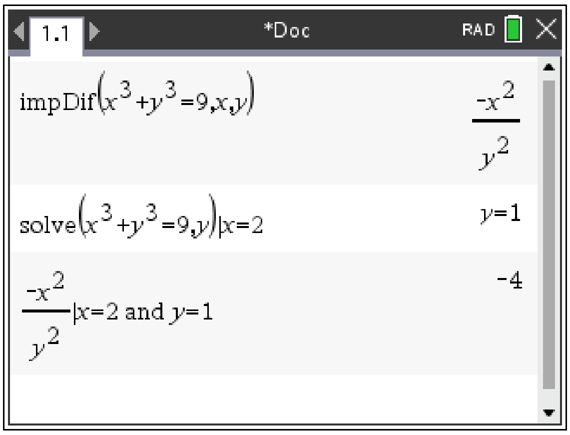
Answers

Question 1 Gradient is -4



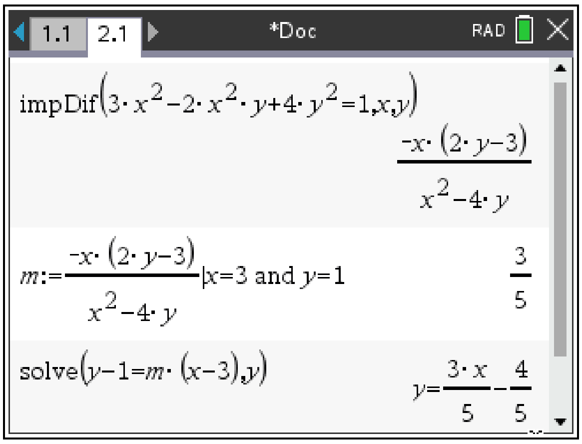
To perform implicit differentiation on CAS use the command ‘impdif’, which can be accessed from the menu:

- Menu
- Calculus
- Implicit Differentiation

To find the y value we need to substitute in $x = 2$ and solve for y. See 2nd line.

Once we have our x and y values, we can substitute them in using the ‘given/such that’ symbol |.

Question 2 $y = \frac{3x}{5} - \frac{4}{5}$



To find the equation of a straight line we can use the formula $y - y_1 = m(x - x_1)$, where m is the gradient, and (x_1, y_1) is a point on the graph.

By finding and defining the gradient, we can substitute in the gradient and given point into the above formula and solve for y to make y the subject. See 3rd line.

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Question 3 $\left(-3\sqrt{3},-2\sqrt{3}\right)$ & $\left(3\sqrt{3},2\sqrt{3}\right)$

$$\text{impDif}\left(x^2-3\cdot x\cdot y+3\cdot y^2=9,x,y\right) \qquad \frac{2\cdot x-3\cdot y}{3\cdot(x-2\cdot y)}$$

$$\text{solve}\left(\begin{cases}\frac{2\cdot x-3\cdot y}{3\cdot(x-2\cdot y)}=0\\ x^2-3\cdot x\cdot y+3\cdot y^2=9\end{cases},\{x,y\}\right)$$

$$x=-3\cdot\sqrt{3}\text{ and }y=-2\cdot\sqrt{3}\text{ or }x=3\cdot\sqrt{3}\text{ and }y=2\cdot\sqrt{3}$$

Question 4 $\frac{dy}{dx}$ at $t=2$ is $\frac{11}{3}$

2.1 3.1 4.1 *Doc RAD

$$x:=t-t^2 \qquad t-t^2$$

$$y:=t-t^3 \qquad t-t^3$$

$$\frac{d}{dt}(y)\cdot\frac{1}{\frac{d}{dt}(x)} \qquad \frac{3\cdot t^2-1}{2\cdot t-1}$$

$$\frac{3\cdot t^2-1}{2\cdot t-1}|t=2 \qquad \frac{11}{3}$$

Question 5 $\frac{dy}{dx}$ at $y=\frac{1}{2}$ is $\sqrt{3}$

$$x:=t-\sin(t) \qquad t-\sin(t)$$

$$y:=1-\cos(t) \qquad 1-\cos(t)$$

$$\frac{dy}{dx}=\frac{d}{dt}(y)\cdot\frac{1}{\frac{d}{dt}(x)} \qquad \frac{dy}{dx}=\frac{1}{\tan\left(\frac{t}{2}\right)}$$

$$\text{solve}\left(y=\frac{1}{2},t\right)|0<t<\frac{\pi}{2} \qquad t=\frac{\pi}{3}$$

$$\frac{dy}{dx}=\frac{1}{\tan\left(\frac{t}{2}\right)}|t=\frac{\pi}{3} \qquad \frac{dy}{dx}=\sqrt{3}$$

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Question 6 $\left(2-\frac{4\sqrt{6}}{9}, e^{\frac{2\sqrt{6}}{9}}\right)$

$x:=t^3-2\cdot t+2$	$t^3-2\cdot t+2$
$y:=e^{2\cdot t}$	$e^{2\cdot t}$
$\frac{dy}{dx}=\frac{dy}{dt}\cdot\frac{dt}{dx}$	true
$\frac{dy}{dx}=\frac{d}{dt}(y)\cdot\frac{1}{\frac{d}{dt}(x)}$	$\frac{dy}{dx}=\frac{2\cdot e^{2\cdot t}}{3\cdot t^2-2}$
$\text{solve}(3\cdot t^2-2=0,t)\vert 0\le t\le\frac{3}{2}$	$t=\frac{\sqrt{6}}{3}$
$x\vert t=\frac{\sqrt{6}}{3}$	$2-\frac{4\cdot\sqrt{6}}{9}$
$y\vert t=\frac{\sqrt{6}}{3}$	$e^{\frac{2\cdot\sqrt{6}}{3}}$

To determine where the derivative is undefined, we require the denominator in the derivative to be zero.

Note: When solving do not forget any domain restrictions.

Question 7 $\frac{dx}{dt}$ at $\theta=\frac{\pi}{3}$ is $24\ m/s$

$\frac{dx}{dt}=\frac{dx}{d\theta}\cdot\frac{d\theta}{dt}$	true
$x:=600\cdot\tan(\theta)$	$600\cdot\tan(\theta)$
$\frac{dx}{dt}=\frac{d}{d\theta}(x)\cdot 0.01$	$\frac{dx}{dt}=\frac{6.}{(\cos(\theta))^2}$
$\frac{dx}{dt}=\frac{6.}{(\cos(\theta))^2}\vert\theta=\frac{\pi}{3}$	$\frac{dx}{dt}=24.$

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Question 8 $\frac{dr}{dt} = \frac{1}{2000\pi r}\ m/s$

$\frac{dr}{dt}=\frac{dr}{da}\cdot\frac{da}{dt}$	true
$a:=\pi\cdot r^2$	$\pi\cdot r^2$
$\frac{d}{dr}(a)$	$2\cdot\pi\cdot r$
$\frac{dr}{dt}=\frac{1}{\frac{d}{dr}(a)}\cdot\frac{1}{1000}$	$\frac{dr}{dt}=\frac{1}{2000\cdot\pi\cdot r}$

Note: It is a good idea to convert decimals into fractions when entering them into CAS. If you're not sure how to convert the decimal, use the 'exact' command.

exact(0.001)	$\frac{1}{1000}$

Question 9 $\frac{dh}{dt}$ at $h = 0.75$ is $3.40\ m/s$

$\frac{dh}{dt}=\frac{dh}{dv}\cdot\frac{dv}{dt}$	true
$\text{solve}\left(\frac{h}{r}=\frac{2}{1},r\right)$	$r=\frac{h}{2}$
$v:=\frac{1}{3}\cdot\pi\cdot r^2\cdot h\mid r=\frac{h}{2}$	$\frac{h^3\cdot\pi}{12}$
$\frac{dh}{dt}=\frac{1}{\frac{d}{dh}(v)}\cdot\frac{3}{2}$	$\frac{dh}{dt}=\frac{6}{h^2\cdot\pi}$
$\frac{dh}{dt}=\frac{6}{h^2\cdot\pi}\mid h=0.75$	$\frac{dh}{dt}=3.39531$

Note: Since the volume has two variables, r and h, we need to find a relationship between r and h. Similarity is a common strategy for geometric problems.

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