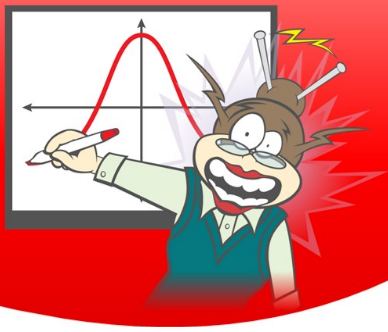

Calculus 1

Question: 1.

Given that $f(x) = \arcsin\left(\frac{x}{2}\right)$, find $f''\left(\frac{3}{2}\right)$.

Question: 2.

Let $y = \arctan(2x)$. Find the value of a given that $\frac{d^2y}{dx^2} = ax\left(\frac{dy}{dx}\right)^2$, for $x \in \mathbb{R}$.

Question: 3.

Let $f(x) = -mx^3 + 3x^2 - 1$, where $m \in \mathbb{R}^+$. The gradient of f will always be strictly decreasing for:

A. $x \geq \frac{1}{m}$ B. $x \leq \frac{1}{m}$ C. $x \geq \frac{2}{m}$ D. $x \leq \frac{2}{m}$ E. $x \geq 0$

Question: 4.

The graph of the function f, where $f(x) = x^4 - 2x^3 + x$ is concave up for:

A. $x < 0$ and $x > 1$

B. $0 < x < 1$

C. $\frac{1-\sqrt{3}}{2} < x < 1$ and $x > \frac{1+\sqrt{3}}{2}$

D. $x < \frac{1-\sqrt{3}}{2}$ and $1 < x < \frac{1+\sqrt{3}}{2}$

E. $x < \frac{1-\sqrt{3}}{2}$ and $x > \frac{1+\sqrt{3}}{2}$

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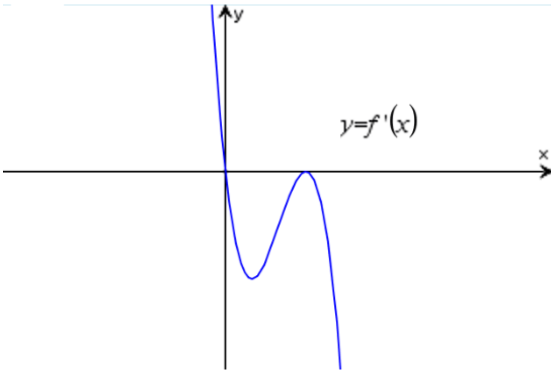
Question: 5.

The graph of $y = f'(x)$ is shown. Which one of the following statements is true for the graph of $y = f(x)$?



A. The graph has a local maximum at $x = 3$, a local minimum at $x = 1$, and a stationary point of inflection at $x = 0$

B. The graph has a stationary point of inflection at $x = 3$ and a local maximum at $x = 0$

C. The graph has a stationary point of inflection at $x = 3$ and a local minimum at $x = 0$

D. The graph has a local maximum at $x = 0$, a stationary point of inflection at $x = 1$, and a stationary point of inflection at $x = 3$

E. The graph has a local minimum at $x = 3$, a local maximum at $x = 0$, and a stationary point of inflection at $x = 1$

Question: 6.

a. Find the stationary point of the graph of $f(x) = \frac{5+x^2+2x^3}{2x}$, $x \in \mathbb{R}\backslash\{0\}$. Express you answer as coordinates.

b. Find the point of inflection of the graph given in part a. Express your answer as coordinates, giving your answer correct to two decimal places.

c. Sketch the graphs of $f(x) = \frac{5+x^2+2x^3}{2x}$ for $x \in [-3, 3]$ on the axes below, labelling the turning point and the point of inflection with their coordinates, correct to two decimal places.

Question: 7.

Find the point of inflection(s) for the function $f(x) = (x-1)^4(x+1)^3$. Express your answer as coordinates, giving your values correct to two decimal places.

Question: 8.

Let $g(x) = e^{-2x} f(x)$. There is a point of inflection on the graph of $y = g(x)$ at $\left(a, g(a)\right)$. An expression for $f''(a)$ in terms of $f'(a)$ and $f(a)$ is:

A. $f''(a) = f(a) + f'(a)$

B. $f''(a) = 4f(a)f'(a)$

C. $f''(a) = 4f(a) + 4f'(a)$

D. $f''(a) = \frac{f'(a)}{f(a)}$

E. $f''(a) = 4f'(a) - 4f(a)$

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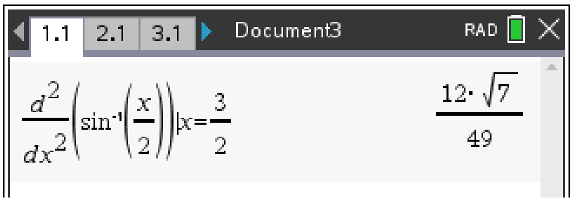
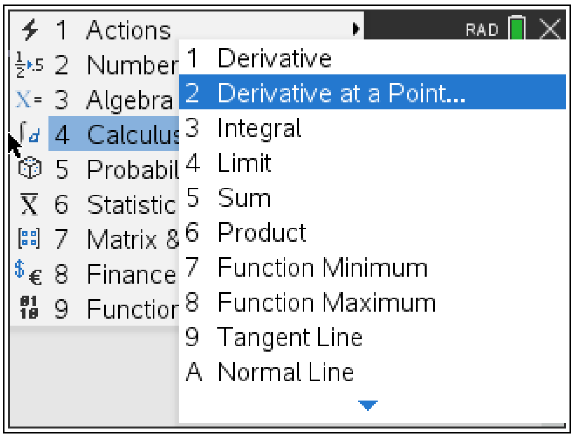
Answers

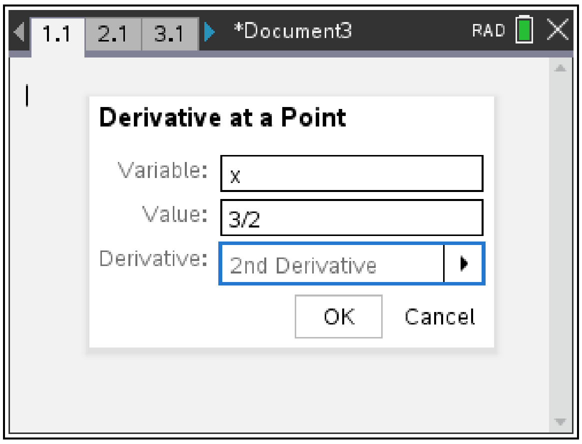
Question 1





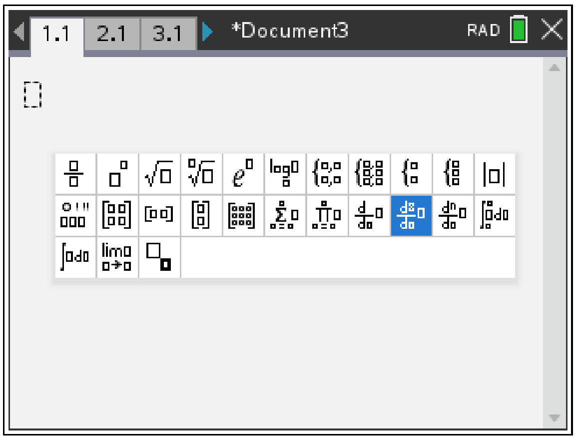
How to enter on CAS

- **Menu**
- **Calculus**
- **Derivative at a point**



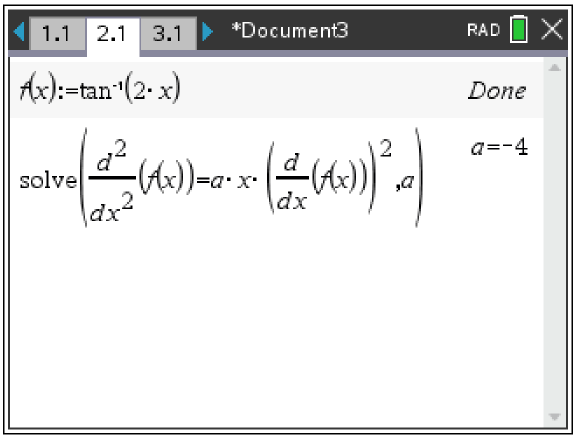
In the 'Derivative at a point' submenu, you are able to select:

- What variable you are differentiating with respect to.
- What value you would like to substitute in.
- The order of the derivative you would like to take.



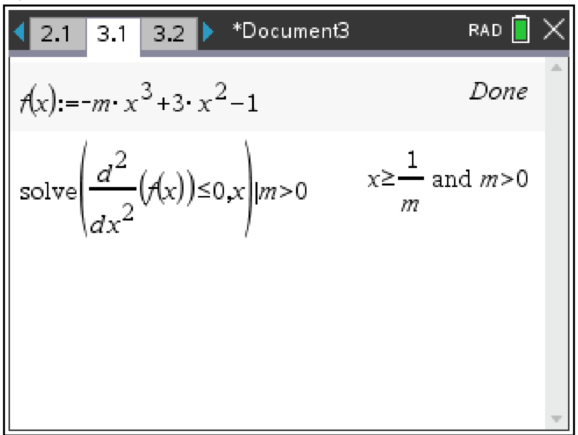
Alternatively, you can obtain the second derivative by pressing the template button

Question 2



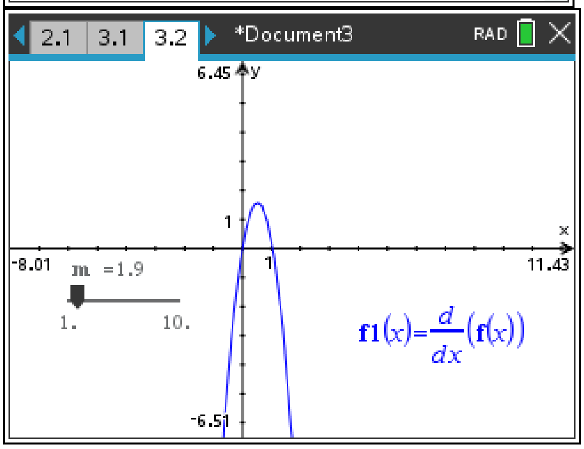
Note: You can obtain the second derivative by pressing the template button

Question 3 Answer A



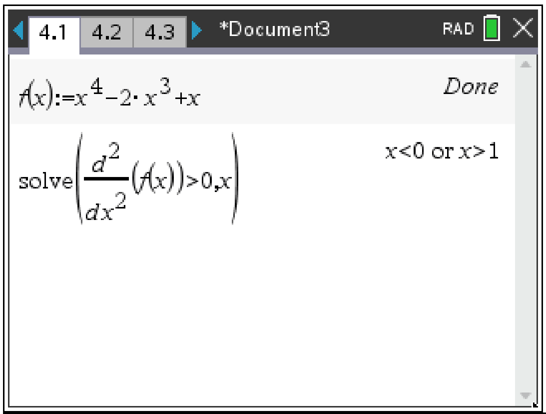
Note: The question asks for what values of m will **the gradient of f** will always be strictly decreasing.

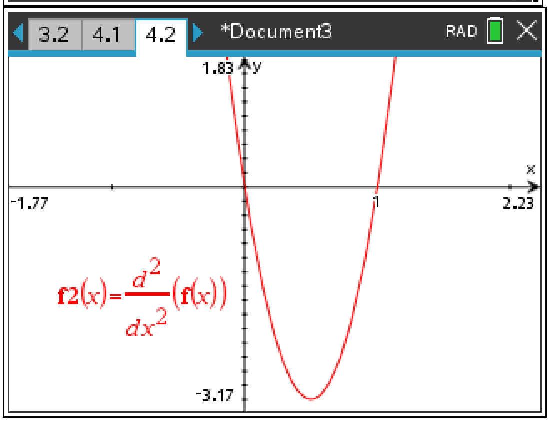
For the gradient to be strictly decreasing we can solve $f''(x) \leq 0$.It is always a good habit to sketch the graph to see how the graph behaves and to help interpret the output you obtain when solving.



Note: When sketching the derivative, you may be asked for a slider, in this case the values of m

Question 4 Answer A



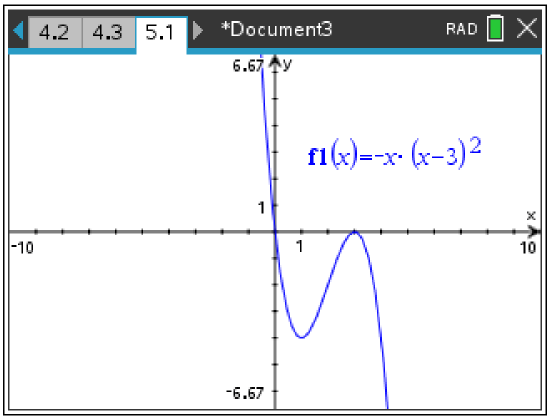


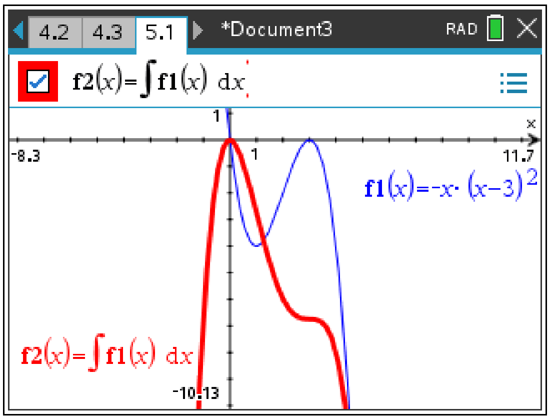
For a function to be concave up we require $f''(x) > 0$

It is always a good habit to sketch the graph to see how the graph behaves and to help interpret the output you obtain when solving.

Author: J Mott

Question 5 **Answer B**





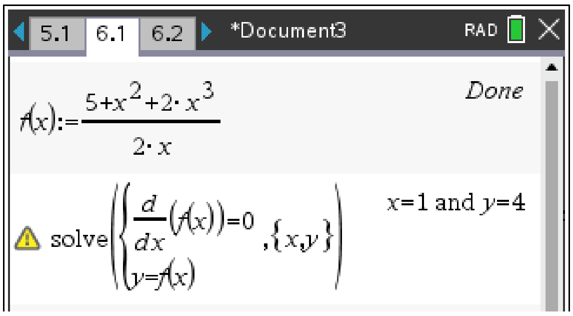
Note: When the equation to the graph is not provided one strategy is to use a curve the looks and behaves like the one given in the question.

Note: To sketch the graph of $y = f(x)$ given $y = f\,'(x)$, on CAS you can sketch $y = \int f'(x)\ dx$, as shown below.y=f(x)

It may be helpful to sketch $y = f'(x)$ and $y = f(x)$ on the same set of axes to observe their features for the corresponding x values.

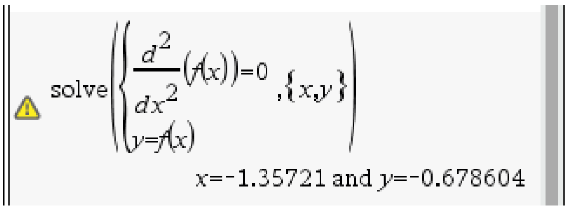
At $x = 0$, the graph of $y = f(x)$ has a local maximum stationary point. At $x = 3$, the graph of $y = f(x)$ has a stationary point of inflction.

Question 6



a. **To find the stationary points we need to solve $f'(x) = 0$.**
The stationary point is $(1, 4)$

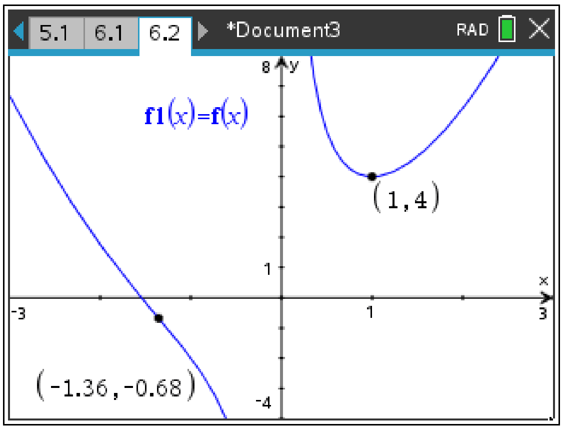
Note: A nice way to obtain 'coordinates' is to simultaneously solve for x and to also solve for $y = f(x)$, where $f(x)$ is your defined function. The CAS will output the corresponding x and y values.



b. **To find the point of inflection we can solve $f''(x) = 0$.**
The point of inflection is $(-1.35, -0.68)$

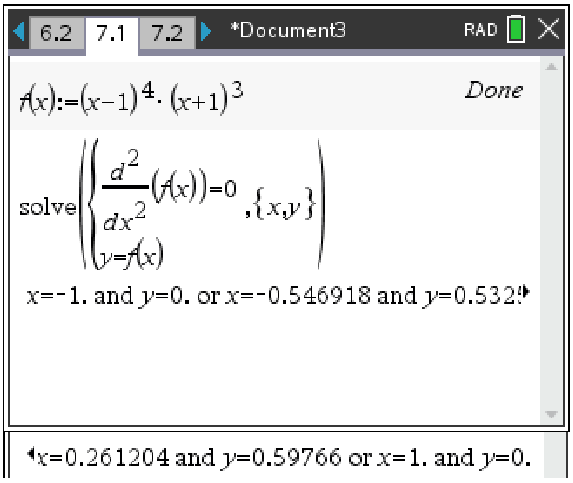
Note: It is always a good habit to sketch the graph to see how the graph behaves and to help interpret the output you obtain when solving.

Author: J Mott



c. It is a good idea to change the widow settings on a graphs page to the given interval(s) in the question or the grid provided in the question.

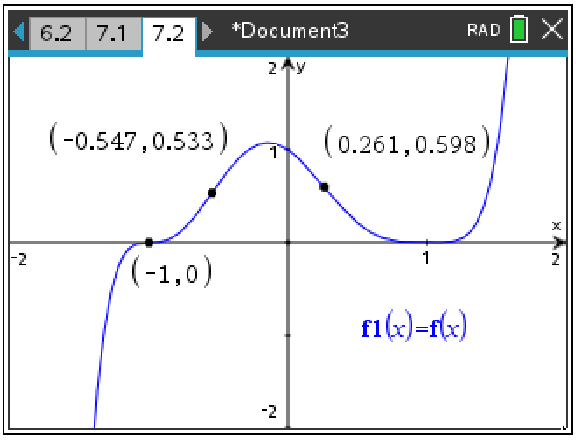
Question 7



To find the point of inflection we can solve $f''(x) = 0$.

Note: It is always a good habit to sketch the graph to see how the graph behaves and to help interpret the output you obtain when solving.

Solving $f''(x) = 0$ is not always enough. From the output we obtain the points
$(-1, 0)$
$(-0.55, 0.53)$
$(0.26, 0.60)$, and
$(1, 0)$

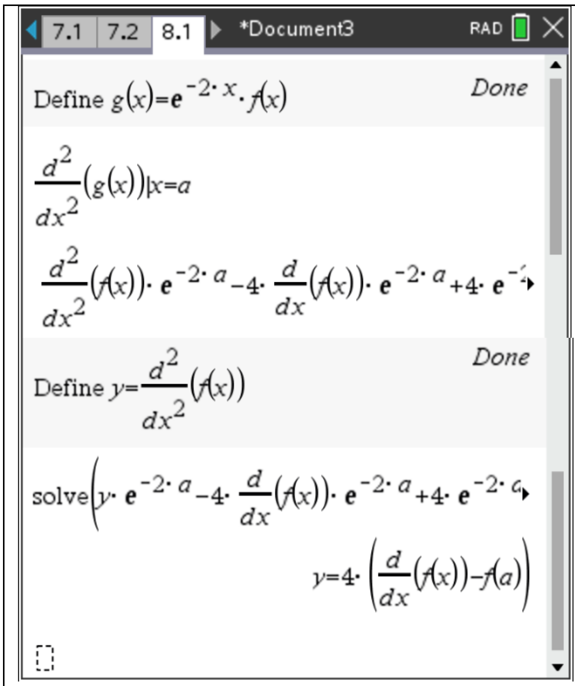


However looking at the graph, and using 'Analyze graph – Inflection', we see that only three of the four outputs are points of inflection. $(1, 0)$ is a stationary point, not a point of inflection.

The point of inflections are $(-1, 0)$, $(-0.55, 0.53)$, and $(0.26, 0.60)$.

Author: J Mott

Question 8 Answer E



Once defining $g(x)$ we can evaluate $g''(a)$ by using the 'derivative at a point' via the calculus menu.

- Menu
- Calculus
- Derivative at a point

The question asks for an expression for $f''(x)$. To make $f''(x)$ the subject, we can define $y = f''(x)$, as shown to the right.

Note: One way to efficiently obtain prior work/output is to press up ▲ and one the desired work is highlighted, press enter to bring it down to a new line.

We are told $g''(a)$ is a point of inflection, so $g''(a) = 0$.

Replacing $f''(x)$ with y in the equation $g''(a) = 0$ and solving for y we can make $f''(x)$ the subject.