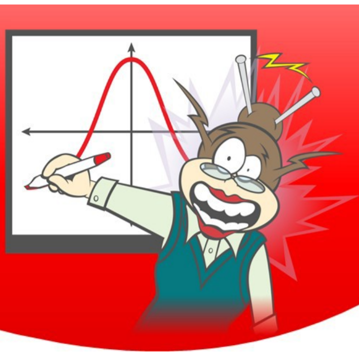
STUDENT REVISION SERIES



Complex Numbers Part 2

Question 1.
Evaluate the following: $i + i^2 + i^3 + i^4 + \cdots . + i^{2018} + i^{2019} + i^{2020}$

A. 0 B. -1 C. i D. $-i$ E. 1

Question 2.
Factorise the following quadratic into linear factors over $\mathbb{C}$: $z^2 + 16$

A. $(z - 16)(z + 16)$ B. $(z - 4)(z + 4)$ C. $(z - 4i)^2$ D. $(z + 4i)^2$ E. $(z - 4i)(z + 4i)$

Question 3.
Solve the following equation for z: $z^2 = 2z - 5$

A. $-1 + 2i\,,\ -1 - 2i$ B. $-1 + 2i\,,\ 1 + 2i$ C. $1 - 2i\,, 1 + 2i$
D. $-2 - i\,,\ -2 + i$ E. $2 - i\,,\ 2 + i$

Question 4.
Solve the following equation over $\mathbb{C}$: $z^2 + z + (1 + i) = 0$

A. $-1 + i\,,\ i$ B. $-1 + i\,, -i$ C. $-1 - i\,, -i$ D. $-1 - i\,,\ i$ E. $1 - i\,,\ i$

Question 5.
Given $P(z) = 2z^3 + 8z^2 - 20z + 24$. Which one of the following is a linear factor of *P(x)*?

A. $z + 1 + i$ B. $z - 1 + i$ C. $z + 1 - i$ D. $-z - 1 - i$ E. $-z - 1 + i$

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Question 6.
Factorise the polynomial into linear factors over $\mathbb{C}$: $p(z) = 2z^3 + 9z^2 + 14z + 5$

A. $p(z) = (2z + 1)(z + 2 + i)(z + 2 - i)$
B. $p(z) = (2z - 1)(z - 2 + i)(z - 2 - i)$
C. $p(z) = (2z - 1)(z + 2 - i)(z - 2 - i)$
D. $p(z) = (2z + 1)(z + 2 + i)(z - 2 - i)$
E. $p(z) = (2z + 1)(z - 2 + i)(z - 2 - i)$

Question 7
What is the sum of the complex roots of unity for the polynomial? $z^4 = -1$

A. $2i$ B. $2 - 2i$ C. $2 + 2i$ D. 2 E. 0

Question 8.
Which one of the following represents the sum and product of the roots for the polynomial?

$$P(z) = z^4 + z^3 + z^2 + z + 1$$

A. 0 and 1 B. 0 and -1 C. -1 and 1 D. 1 and -1 E. 0 and 0

Question 9.
Let $z = \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$ Find the exact value of: $z^1 \times z^2 \times z^3 \times z^4 \times \ldots . \times z^{98} \times z^{99} \times z^{100}$

A. 1 B. -1 C. -i D. i E. 0

Question 10.
Given $z = (1 + i)^n$, what value of n satisfies the following equation? $|z| = 16$

A. n = 4 B. n = 5 C. n = 6 D. n = 7 E. n = 8

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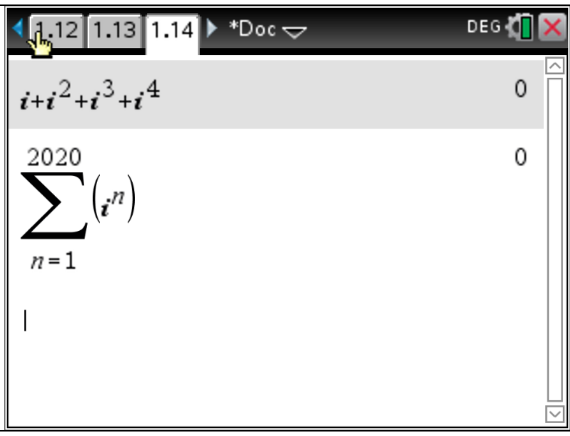
Answers

1. A	2. E	3. C	4. B	5. B	6. A	7. E	8. C	9. D	10. E

Question 1. Answer A

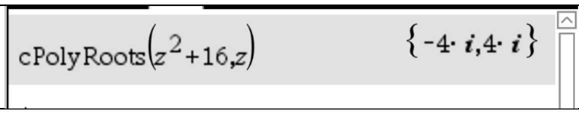
Each sum of 4 terms results in a total of 0. Since 2020 is divisible by 4, the sum of 2020 terms should also total 0.

Alternatively using sigma notation results in a sum of 0.



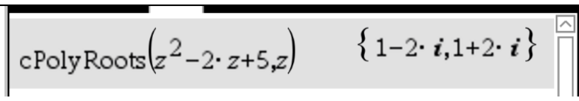
Question 2. Answer E

Use the **cPolyRoots(** tool.
Don't use "= 0"



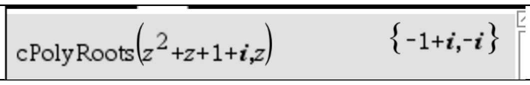
Question 3. Answer C

$z^2 = 2z - 5$ (rearrange)
$z^2 - 2z + 5 = 0$
Use **cPolyRoots(** tool



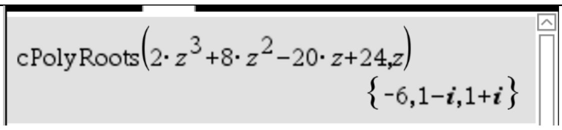
Question 4. Answer B

Zeros are $-1 + i \; and \; -i$

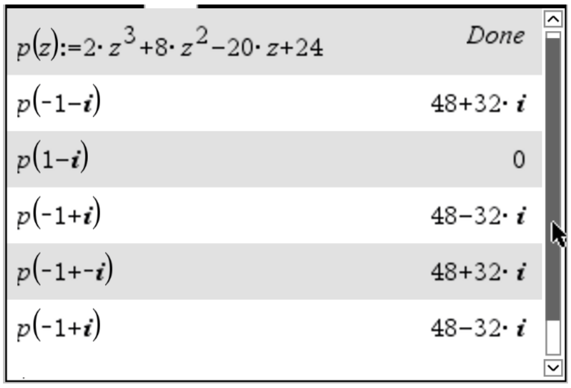


Question 5. Answer B

There are two ways of doing this. First by using the **cPolyRoots(** tool. Therefore, the factors of P(z) are (z + 6), (z - 1 + *i*) and (z - 1 - *i*).
Therefore (z - 1 - *i*) is the correct answer.



Or by defining the polynomial P(z) and testing which multi-choice answer results in a zero for P(z).
Rearranging multi-choice answers gives
(A) -1 – *i*
(B) 1- *i*
(C) -1 + *i*
(D) -1 + -*i*
(E) -1 + *i*
.



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Question 6. Answer A

Factored form is: $(z+2+i)(z+2-i)(z+\frac{1}{2})$ Which is equivalent to: $(z+2+i)(z+2-i)(2z+1)$	cPolyRoots$(2\cdot z^3+9\cdot z^2+14\cdot z+5,z)$ $\left\{-2-i,-2+i,\frac{-1}{2}\right\}$

Question 7. Answer E

$z^4=-1$ Rearrange to z^4+1, use **cPolyRoots(** tool Using **sum(** will add up the four roots. 2×10^{-14} is approximated to 0.	sum(cPolyRoots(z^4+1,z)) 2.E-14

Question 8. Answer C

Using **Sum(** and **product(** in front of the **cPolyRoots(** tool Answer is -1 and 1	sum(cPolyRoots$(z^4+z^3+z^2+z+1,z)$) -1. product(cPolyRoots$(z^4+z^3+z^2+z+1,z)$) 1.

Question 9. Answer D

There are several ways to do this calculation. Adding the powers 1 + 2 + 3 + ….+ 99 +100 gives 5050. The Sigma template can be used to determine the sum is 5050. $(\frac{1}{\sqrt{2}}+\frac{i}{\sqrt{2}})^{5050}=i$ Or using the product template tool	$\sum_{n=1}^{100}(n)$ 5050 $\left(\frac{1}{\sqrt{2}}+\frac{i}{\sqrt{2}}\right)^{5050}$ i $\prod_{n=1}^{100}\left(\left(\frac{1}{\sqrt{2}}+\frac{i}{\sqrt{2}}\right)^{n}\right)$ i

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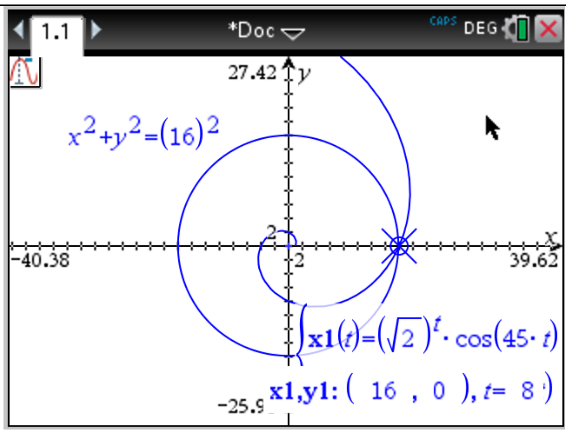
Question 10. Answer E

$r = \sqrt{1^2 + 1^2}$ $= \sqrt{2}$ $\theta = tan^{-1}(\frac{1}{1})$ $= 45°$ In parametric form: (use degree setting) $x(t) = \sqrt{2}^t \cos(45t)$ $y(t) = \sqrt{2}^t \sin(45t)$ $x^2 + y^2 = 16^2$, Use the equation template to graph the circle. Use the trace function to determine the value of n. From using the trace function, n = 8.	

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