Recurrence models for linear growth & decay

Each of the questions included here can be solved using either the TI-Nspire CX or CX CAS.

Question 1
The first five terms of an arithmetic sequence are 72, 65, 58, 51, 44, …

a  Express this sequence as a recurrence relation

b  Find the value of \( V_{20} \)

Question 2
Calculate the first 5 terms for the linear recurrence relation with rule: \( V_{n+1} = V_n + 17 \), \( V_0 = 73 \)

Question 3
An investment of $1500 is made attracting a simple interest rate of 3.5% p.a. The interest each year is added to balance of the investment account.

a  Represent this investment as a linear recurrence relation

b  What is the value of the investment after 7 years?

c  How long does it take for the initial investment to double in value?

Question 4
A simple interest investment can be modelled on the following linear recurrence rule. \( V_{n+1} = V_n + 84 \), \( V_0 = 2000 \). What is the associated annual rate of interest?
Question 5
The following plot displays the value of an asset as it depreciates in value (using flat rate depreciation method).

<table>
<thead>
<tr>
<th>Year</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>800</td>
</tr>
<tr>
<td>2</td>
<td>700</td>
</tr>
<tr>
<td>3</td>
<td>600</td>
</tr>
<tr>
<td>4</td>
<td>500</td>
</tr>
<tr>
<td>5</td>
<td>400</td>
</tr>
</tbody>
</table>

a  What is the annual rate of depreciation?

b  What is the expected value of this asset after 12 years?

Question 6
Ranjeev purchases a $10,000 coffee machine for his business which depreciates annually at a flat rate of 15% of the purchase price.

a  By how much does Ranjeev's coffee machine depreciate each year?

b  Express this flat rate depreciation as a linear recurrence relation

c  Find the annual depreciated value of the machine over its first 5 years

d  He intends to replace the machine when its depreciated value is $3000. After how many years will this occur?
Question 7
A business purchases a photocopier for $139,000. It is devalued using a unit cost method of depreciation. After 3 years, it is valued at $80,000. In that time, it has been used to make an average of 320,000 copies per year.

a  What is the unit cost (to the nearest cent)?

b  Express this depreciation as a linear recurrence relation.

c  What is the depreciated value after 5 years, based on the average usage pattern?

d  How many years will it take for the value to be less than $50,000?
Answers

Note: The file RFM_linear.tns (as shown in the webinar) can be used to help answer many of these questions.

Question 1
a  \( V_{n+1} = V_n - 7, \quad V_0 = 72 \)
b  \( V_n = V_0 - 7n = 72 - 7(20) = -68 \)

Question 2
73, 90, 107, 124, 141.

Question 3
a  Interest = \( r/100 \times V_0 = 3.5/100 \times 1500 = $52.50 \). So \( V_{n+1} = V_n + 52.5, \quad V_0 = 1500 \)
b  \( V_7 = 1500 + 7 \times 52.5 = $1867.50 \)
c  Solve \( 3000 = 1500 + 52.5n \) for \( n \). Using calculator, gives \( n = 28.5714 \) (so 29 years)

Question 4
Interest = $84, so \( 84 = r/100 \times 2000 \). Solving gives \( r = 4.2% \) p.a.

Question 5
a  From the graph, the value drops by $50 per year, so \( 50 = r/100 \times $850 \). Solving for \( r \) gives \( r = 5.88% \) (to 2 d.p.)
b  \( V_{12} = 850 - 12 \times 50 = $250 \)

Question 6
a  Annual depreciation = \( 15\% \times $10,000 = $1500 \)
b  \( V_{n+1} = V_n - 1500, \quad V_0 = 10,000 \)
c  \( V_5 = 10,000 - 5 \times 15,000 = $2500 \)
d  Solve \( 3000 = 10000 - n \times 1500 \) for \( n \). Answer is 4.667 years (so 5 years).

Question 7
a  Unit cost = \( (139,000 - 80,000)/(3 \times 320,000) = $0.06 \) per copy.
b  \( V_{n+1} = V_n - 0.06, \quad V_0 = 139,000 \)
c  In 5 years, usage would be \( 5 \times 320,000 = 1,600,000 \). So Value = \( 139,000 - (1,600,000 \times 0.06) = $43,000 \)