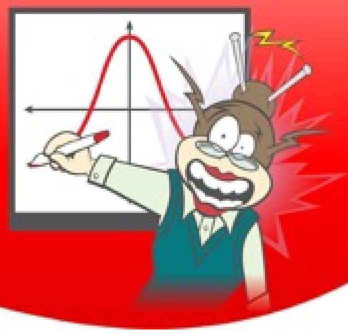
STUDENT REVISION SERIES



Matrices Part 2

Question 1.

Which one of the following represents a one-step dominance matrix which displays the results of the round robin competition below?

- A defeated B
- B defeated D
- C defeated A, B and D
- D defeated A

A.

	A	B	C	D
A	0	1	0	0
B	0	0	1	0
C	1	0	0	1
D	1	1	0	0

B.

	A	B	C	D
A	0	1	0	0
B	0	0	0	1
C	1	1	0	1
D	1	0	0	0

C.

	A	B	C	D
A	0	0	0	1
B	0	0	0	1
C	1	0	1	1
D	0	1	0	0

D.

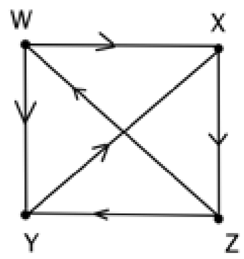
	A	B	C	D
A	0	0	0	1
B	0	0	0	1
C	1	0	1	1
D	1	1	0	0

E.

	A	B	C	D
A	0	0	0	1
B	0	0	0	1
C	1	0	1	1
D	0	1	1	0

Question 2.

Use a one-step and two-step dominance matrix ($D + D^2$) to rank the four players (W, X, Y and Z) who competed in a round-robin competition represented by the digraph below.



The ranking from best to worst is:

A. ZWXY B. ZWYX C. WZXY D. WZYX E. WYXZ

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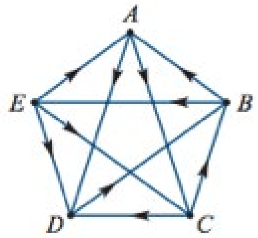
Question 3.

Five teams competed in one round of a Futsal competition, with each team playing every other team once. The digraph represents the results of the competition.



Use the weighting $D + \frac{1}{2}D^2 + \frac{1}{3}D^3$ applied to the dominance scores to determine the overall ranking of the teams. Which one of the following represents the ranking of the teams from first to last?

A. BDAEC B. BEDAC C. EBDAC D. EBDCA E. BEDCA

Question 4.

An animal population has the following characteristics described in the table below.

Age Group	0 - 5 years	5 - 10 years	10 - 15 years	15 - 20 years
Birth Rate	0	3.8	2.4	0
Death Rate	0.5	0.3	0.4	1

Which one of the following is the Leslie matrix which represents the information contained in the table?

A. $\begin{bmatrix} 0 & 3.8 & 2.4 & 0 \\ 0.5 & 0 & 0 & 0 \\ 0 & 0.3 & 0 & 0 \\ 0 & 0 & 0.4 & 0 \end{bmatrix}$

B. $\begin{bmatrix} 0 & 3.8 & 2.4 & 0 \\ 0.5 & 0 & 0 & 0 \\ 0.3 & 0 & 0 & 0 \\ 0.4 & 0 & 0 & 0 \end{bmatrix}$

C. $\begin{bmatrix} 0 & 3.8 & 2.4 & 0 \\ 0.5 & 0 & 0 & 0 \\ 0.7 & 0 & 0 & 0 \\ 0.6 & 0 & 0 & 0 \end{bmatrix}$

D. $\begin{bmatrix} 0 & 3.8 & 2.4 & 0 \\ 0.5 & 0 & 0 & 0 \\ 0 & 0.7 & 0 & 0 \\ 0 & 0 & 0.6 & 0 \end{bmatrix}$

E. $\begin{bmatrix} 0 & 3.8 & 2.4 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0.5 & 0.7 & 0.6 & 0 \end{bmatrix}$

Question 5.

Given the Leslie Matrix **L** and the initial population $\mathbf{P_o}$:

$$L = \begin{bmatrix} 0 & 0 & 20 \\ 0.2 & 0 & 0 \\ 0 & 0.4 & 0 \end{bmatrix} \text{ and } P_o = \begin{bmatrix} 250 \\ 0 \\ 0 \end{bmatrix}$$

Which one of the following represents $L^4 P_o$?

A. $\begin{bmatrix} 400 \\ 0 \\ 0 \end{bmatrix}$ B. $\begin{bmatrix} 0 \\ 0 \\ 80 \end{bmatrix}$ C. $\begin{bmatrix} 0 \\ 80 \\ 0 \end{bmatrix}$ D. $\begin{bmatrix} 0 \\ 32 \\ 0 \end{bmatrix}$ E. $\begin{bmatrix} 0 \\ 0 \\ 32 \end{bmatrix}$

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Question 6.

The Leslie Matrix **L** and the initial population $\mathbf{P_0}$ represent a female rat population which have a lifespan of 3 years.

$$L = \begin{bmatrix} 1.2 & 2.1 & 0 \\ 0.4 & 0 & 0 \\ 0 & 0.2 & 0 \end{bmatrix} \text{ and } P_0 = \begin{bmatrix} 10 \\ 6 \\ 4 \end{bmatrix}$$

The rat population is divided into 3 age groups- young, juveniles and adults with each age group representing 1 year. How many years will it take the female rat population to pass 1,000 in number?

A. 5 B. 8 C. 15 D. 24 E. 27

Question 7.

The birth rate and death rate for a certain species of moth is given in the table below.

Age (months)	0 - 3	3 - 6	6 - 9	9 - 12
Number	14	18	12	4
Birth rate	0	1.2	1.4	0
Death rate	0.4	0.1	0.2	1

Which one of the following represents the long-term behaviour of the population? In other words, what percent does the population growth rate approach?

A. 116.9% B. 16.9% C. 83.1% D. 183.1% E. 6.9%

Question 8.

A kangaroo population is described by the data below.

Age (years)	**0 - 2**	**2 - 4**	**4 - 6**	**6 - 8**	**8 – 10**
Initial population	3400	2500	2300	1750	650
Breeding rate	0	0	3.9	2.7	0.9
Death rate	0.5	0.2	0.3	0.6	1

For this changing kangaroo population, the growth factor is defined as the ratio of the total female population at a given period P_{N+1} to that of the previous period P_N for large N, that is:

$R = \frac{P_{N+1}}{P_N}$, $N \geq 0$. For an increasing population $R > 1$.

To maintain a controlled, stable population ($R = 1$), a harvesting or culling factor ***h*** is applied such that:

$$h = 1 - \frac{1}{R}$$

Which one of the following represents a suitable culling factor?

A. 76.9%. B. 130.1% C. 30.1% D. 69.9% E. 23.1%

Question 9.
Determine the unique solution for the system of equations below by using an augmented matrix and the Reduced Row Echelon command ***rref(*** **.**

$$x - y + 2z = 7$$
$$-x - 2y + 3z = 4$$
$$-2x - 2y + z = -6$$

A. (x, y, z) = (2, 3, 4)
B. (x, y, z) = (-2, -3, -4)
C. (x, y, z) = (2, -3, -4)
D. (x, y, z) = (-2, -3, 4)
E. (x, y, z) = (-2, 3, -4)

Question 10.
Use row operations to solve the following system of equations.

$$x + z = 2y - 2$$
$$3x + 5y = 20 - 2z$$
$$5x + 2y - 3z = 27$$

A. (x, y, z) = $(\frac{15}{4}, \frac{9}{4}, \frac{5}{4})$
B. (x, y, z) = $(\frac{15}{4}, \frac{-9}{4}, \frac{-5}{4})$
C. (x, y, z) = $(\frac{15}{4}, \frac{9}{4}, \frac{-5}{4})$
D. (x, y, z) = $(\frac{-15}{4}, \frac{9}{4}, \frac{-5}{4})$
E. (x, y, z) = $(\frac{-15}{4}, \frac{-9}{4}, \frac{-5}{4})$

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Answers

1. B	2. A	3. E	4. D	5. C	6. D	7. B	8. E	9. A	10. C

Question 1. Answer B

Question 2. Answer A

The matrix ***d*** represents a one-step dominance. Matrix ***e*** is used to help collate the total of the one-step dominance and two-step dominance matrices.	$d:=\begin{bmatrix}0&1&1&0\\0&0&0&1\\0&1&0&0\\1&0&1&0\end{bmatrix}$ $\begin{bmatrix}0&1&1&0\\0&0&0&1\\0&1&0&0\\1&0&1&0\end{bmatrix}$ $e:=\begin{bmatrix}1\\1\\1\\1\end{bmatrix}$ $\begin{bmatrix}1\\1\\1\\1\end{bmatrix}$
$(d + d^2) \times e$ will tally the results for ranking. Rankings are Z, W, X, and Y	$(d+d^2)\cdot e$ $\begin{bmatrix}4\\3\\2\\5\end{bmatrix}$ W X Y Z

Question 3. Answer E

First determine and define the one-step dominance matrix ***d.***	$d:=\begin{bmatrix}0&0&1&1&0\\1&0&1&0&1\\0&0&0&1&0\\0&1&0&0&0\\1&0&1&1&0\end{bmatrix}$ $\begin{bmatrix}0&0&1&1&0\\1&0&1&0&1\\0&0&0&1&0\\0&1&0&0&0\\1&0&1&1&0\end{bmatrix}$ $e:=\begin{bmatrix}1\\1\\1\\1\\1\end{bmatrix}$ $\begin{bmatrix}1\\1\\1\\1\\1\end{bmatrix}$
Apply the weighting and multiply by the column vector ***e*** to summarise the information. Ranking is B, E, D, A, C	$\left(d+0.5\cdot d^2+\frac{1}{3}\cdot d^3\right)\cdot e$ $\begin{bmatrix}4.33333\\8.33333\\2.5\\4.5\\7.\end{bmatrix}$

Question 4. Answer D

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Question 5. Answer C

$$l:=\begin{bmatrix}0 & 0 & 20\\ 0.2 & 0 & 0\\ 0 & 0.4 & 0\end{bmatrix} \quad \begin{bmatrix}0 & 0 & 20\\ 0.2 & 0 & 0\\ 0 & 0.4 & 0\end{bmatrix}$$

$$po:=\begin{bmatrix}250\\ 0\\ 0\end{bmatrix} \quad \begin{bmatrix}250\\ 0\\ 0\end{bmatrix}$$

$$l^4 \cdot po \quad \begin{bmatrix}0.\\ 80.\\ 0.\end{bmatrix}$$

Question 6. Answer D

First define the Leslie matrix (L) and initial population matrix P_o.

$$l:=\begin{bmatrix}1.2 & 2.1 & 0\\ 0.4 & 0 & 0\\ 0 & 0.2 & 0\end{bmatrix} \quad \begin{bmatrix}1.2 & 2.1 & 0\\ 0.4 & 0 & 0\\ 0 & 0.2 & 0\end{bmatrix}$$

$$po:=\begin{bmatrix}10\\ 6\\ 4\end{bmatrix} \quad \begin{bmatrix}10\\ 6\\ 4\end{bmatrix}$$

Try some values in the multi-choice answers, for example when n = 5 (15 years) and N = 8 (24 years).

$$l^5 \cdot po \quad \begin{bmatrix}189.086\\ 44.5018\\ 5.29344\end{bmatrix}$$

$$l^8 \cdot po \quad \begin{bmatrix}921.014\\ 217.305\\ 25.6286\end{bmatrix}$$

Use a row matrix to determine the total sum.

$$e:=\begin{bmatrix}1 & 1 & 1\end{bmatrix} \quad \begin{bmatrix}1 & 1 & 1\end{bmatrix}$$

$$e \cdot l^5 \cdot po \quad \begin{bmatrix}238.882\end{bmatrix}$$

$$e \cdot l^7 \cdot po \quad \begin{bmatrix}686.531\end{bmatrix}$$

$$e \cdot l^8 \cdot po \quad \begin{bmatrix}1163.95\end{bmatrix}$$

Question 7. Answer B

First determine and define the Leslie matrix (***L***) and the initial population matrix ($\boldsymbol{P_o}$)
Also define matrix ***e*** so the population in each interval can be summed to get the total population.

$$l:=\begin{bmatrix}0 & 1.2 & 1.4 & 0\\ 0.6 & 0 & 0 & 0\\ 0 & 0.9 & 0 & 0\\ 0 & 0 & 0.8 & 0\end{bmatrix} \quad \begin{bmatrix}0 & 1.2 & 1.4 & 0\\ 0.6 & 0 & 0 & 0\\ 0 & 0.9 & 0 & 0\\ 0 & 0 & 0.8 & 0\end{bmatrix}$$

$$po:=\begin{bmatrix}14\\ 18\\ 12\\ 4\end{bmatrix} \quad \begin{bmatrix}14\\ 18\\ 12\\ 4\end{bmatrix}$$

$$e:=\begin{bmatrix}1 & 1 & 1 & 1\end{bmatrix} \quad \begin{bmatrix}1 & 1 & 1 & 1\end{bmatrix}$$

Determine the ratio of a long-term distributions, say 50 and 51 cycles. Eg $\frac{L^{51}P_0}{L^{50}P_0} \times 100 = 116.905$ Therefore % increase is 16.91%	$e\cdot l^{51}\cdot po$ $\quad [160999.]$ $e\cdot l^{50}\cdot po$ $\quad [137718.]$ $\frac{188216}{160999.}\cdot 100$ $\quad 116.905$
Try another ratio of a long-term distribution. For example, 60 and 61. $\frac{L^{61}P_0}{L^{60}P_0} \times 100 = 116.905$ Therefore the % increase is 16.91%	$e\cdot l^{61}\cdot po$ $\quad [767635.]$ $e\cdot l^{60}\cdot po$ $\quad [656632.]$ $\frac{767635}{656632}\cdot 100.$ $\quad 116.905$

Question 8. **Answer E**

First define the matrices L, P_o and E.	$l:=\begin{bmatrix} 0 & 0 & 3.9 & 2.7 & 0.9 \\ 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0.8 & 0 & 0 & 0 \\ 0 & 0 & 0.7 & 0 & 0 \\ 0 & 0 & 0 & 0.4 & 0 \end{bmatrix}$ $\quad po:=\begin{bmatrix} 3400 \\ 2500 \\ 2300 \\ 1750 \\ 650 \end{bmatrix}$ $\quad e:=\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix}$
Determine long term growth rate. Eg $\frac{L^{51}P_0}{L^{50}P_0} \times 100 = 130.05$	$e\cdot l^{51}\cdot po$ $\quad [8.64593E9]$ $e\cdot l^{50}\cdot po$ $\quad [6.6481E9]$ $\frac{8.64593}{6.6481}$ $\quad 1.30051$
Eg $\frac{L^{51}P_0}{L^{50}P_0} \times 100 = 130.07$	$e\cdot l^{71}\cdot po$ $\quad [1.66224E12]$ $e\cdot l^{70}\cdot po$ $\quad [1.27792E12]$ $\frac{1.66224}{1.27792}$ $\quad 1.30074$

Now h = 1 - $\frac{1}{R}$ and R ≈1.301 h =1-$\frac{1}{1.301}$ h = 0.231 Culling factor should be 23.1%	

Question 9. Answer A

From using the **rref(** tool, x = 2, y = 3 and z = 4	$m:=\begin{bmatrix}1&-1&2&7\\-1&-2&3&4\\-2&-2&1&-6\end{bmatrix}$ $\begin{bmatrix}1&-1&2&7\\-1&-2&3&4\\-2&-2&1&-6\end{bmatrix}$ $\text{rref}(m)$ $\begin{bmatrix}1&0&0&2\\0&1&0&3\\0&0&1&4\end{bmatrix}$

Question 10. Answer C

First rearrange the equations so the variables are all on the left-hand side and the constants are on the right. $x-2y+z=-2$ $3x+5y+2z=20$ $5x+2y-3z=27$ Define an augmented matrix M	$m:=\begin{bmatrix}1&-2&1&-2\\3&5&2&20\\5&2&-3&27\end{bmatrix}$ $\begin{bmatrix}1&-2&1&-2\\3&5&2&20\\5&2&-3&27\end{bmatrix}$
Use row operations -5 × R_1 + R_3 →R_3 -3 × R_1 + R_2 →R_2	$\text{mRowAdd}(-5,m,1,3)$ $\begin{bmatrix}1&-2&1&-2\\3&5&2&20\\0&12&-8&37\end{bmatrix}$ $\text{mRowAdd}\left(-3,\begin{bmatrix}1&-2&1&-2\\3&5&2&20\\0&12&-8&37\end{bmatrix},1,2\right)$ $\begin{bmatrix}1&-2&1&-2\\0&11&-1&26\\0&12&-8&37\end{bmatrix}$
$-\frac{12}{11}\times R_2+R_3\rightarrow R_3$	$\text{mRowAdd}\left(\frac{-12}{11},\begin{bmatrix}1&-2&1&-2\\0&11&-1&26\\0&12&-8&37\end{bmatrix},2,3\right)$ $\begin{bmatrix}1&-2&1&-2\\0&11&-1&26\\0&0&\frac{-76}{11}&\frac{95}{11}\end{bmatrix}$

$\frac{-76}{11} \times R_3 \to R_3$	$\text{mRow}\left(\frac{11}{-76}, \begin{bmatrix} 1 & -2 & 1 & -2 \\ 0 & 11 & -1 & 26 \\ 0 & 0 & \frac{-76}{11} & \frac{95}{11} \end{bmatrix}, 3\right)$ $\begin{bmatrix} 1 & -2 & 1 & -2 \\ 0 & 11 & -1 & 26 \\ 0 & 0 & 1 & \frac{-5}{4} \end{bmatrix}$
$R_3 + R_2 \to R_2$	$\text{rowAdd}\left(\begin{bmatrix} 1 & -2 & 1 & -2 \\ 0 & 11 & -1 & 26 \\ 0 & 0 & 1 & \frac{-5}{4} \end{bmatrix}, 3, 2\right)$ $\begin{bmatrix} 1 & -2 & 1 & -2 \\ 0 & 11 & 0 & \frac{99}{4} \\ 0 & 0 & 1 & \frac{-5}{4} \end{bmatrix}$
$\frac{1}{11} \times R_2 \to R_2$	$\text{mRow}\left(\frac{1}{11}, \begin{bmatrix} 1 & -2 & 1 & -2 \\ 0 & 11 & 0 & \frac{99}{4} \\ 0 & 0 & 1 & \frac{-5}{4} \end{bmatrix}, 2\right)$ $\begin{bmatrix} 1 & -2 & 1 & -2 \\ 0 & 1 & 0 & \frac{9}{4} \\ 0 & 0 & 1 & \frac{-5}{4} \end{bmatrix}$

$-R_3 + R_1 \to R_1$	$\text{mRowAdd}\left(-2, \begin{bmatrix} 1 & -2 & 2 & \frac{-13}{4} \\ 0 & 1 & 0 & \frac{9}{4} \\ 0 & 0 & 1 & \frac{-5}{4} \end{bmatrix}, 3, 1\right)$ $\begin{bmatrix} 1 & -2 & 0 & \frac{-3}{4} \\ 0 & 1 & 0 & \frac{9}{4} \\ 0 & 0 & 1 & \frac{-5}{4} \end{bmatrix}$
$2R_2 + R_1 \to R_1$	$\text{mRowAdd}\left(2, \begin{bmatrix} 1 & -2 & 0 & \frac{-3}{4} \\ 0 & 1 & 0 & \frac{9}{4} \\ 0 & 0 & 1 & \frac{-5}{4} \end{bmatrix}, 2, 1\right)$ $\begin{bmatrix} 1 & 0 & 0 & \frac{15}{4} \\ 0 & 1 & 0 & \frac{9}{4} \\ 0 & 0 & 1 & \frac{-5}{4} \end{bmatrix}$
$x = \frac{15}{4}, y = \frac{9}{4}, z = \frac{-5}{4}$	

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