Translations with Matrices

Teacher Notes and Answers

7 8 9 10 11 12

TI-Nspire™ | How To | Teacher | 50 min

Introduction

This activity assumes completion of the Translations activity. In this activity you will see how to translate a set of points (shape) followed by a function using a matrix. The progression of points to function is designed to assist students by reducing the possibility of cognitive overload. Points are discrete and can therefore be handled on a unit by unit basis; functions are typically continuous and may still be relatively new in a student's understanding of mathematics. Understanding how a matrix transforms a function algebraically represents a significant leap for some students.

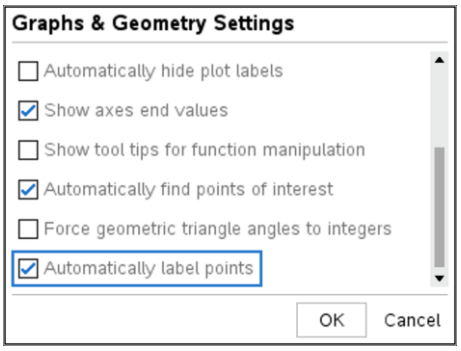
Instructions – Creating the environment

Create a New TI-Nspire file and insert a Graph Application.

To encourage students to use the correct language and referencing, it is appropriate to label geometric objects; it is also worthwhile ensuring that angles will be measured in degrees. A specific Graphs and Geometry Settings menu can customise this application:

[Menu] > Settings

Change the Graph angle to degrees, scroll down and change the default setting to 'Automatically label points'.

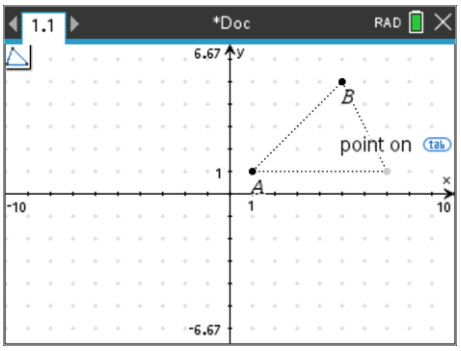


Add a background grid:

[Menu] > View > Grid > Dot Grid

Draw a triangle so that the vertices are placed on the grid.

[Menu] > Geometry > Shapes > Triangle

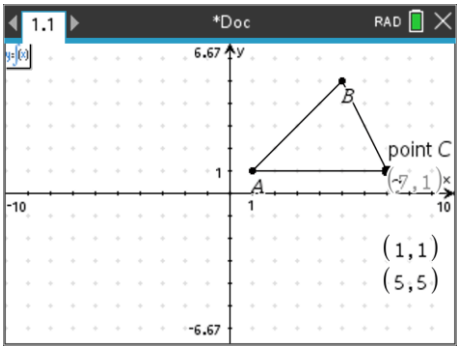


It is the coordinates of the vertices that will be operated on here. To obtain these coordinates there are two options:

- [Menu] > Actions > Coordinates & Equations

  Then click on the each coordinate and click again to locate where these coordinates will be displayed. Do this for each vertex

- Hover over the vertex then press: [Ctrl] + [Menu] to access the contextual menu and select Coordinates & Equations.
  Repeat this for each vertex.



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Authors: Peter Fox

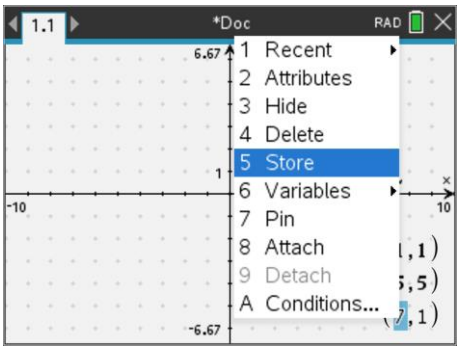
Once the vertices have been obtained, they need to be store so that the combination of triangle and matrix transformation will be completely dynamic.

Move the mouse over the x coordinate for point A. Use the contextual menu: [Ctrl] + [Menu] and select **Store**

Store the x coordinate as “ax”.

Store the corresponding y coordinate as “ay”.

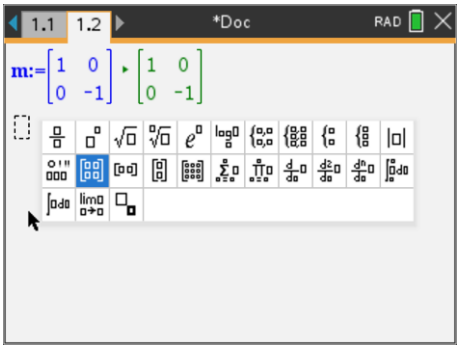
Similarly store the other points as (bx, by) and (cx, cy). Notice that all the values are now bold.



A dilation matrix can now be added in a Notes application that interacts with these stored values.

Insert a **Notes** application and a dilation matrix ‘m’ in a **Maths Box**.

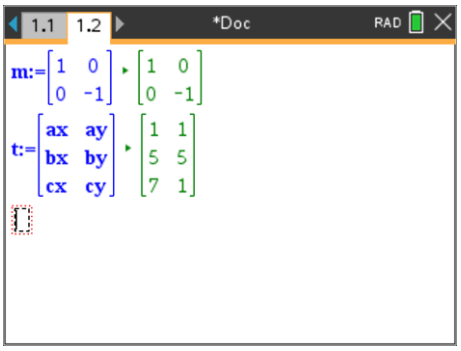
Use the templates menu to define a new matrix ‘m’ as shown opposite. This will be the dilation matrix.



The vertices of the triangle will now be written as a matrix “T”.

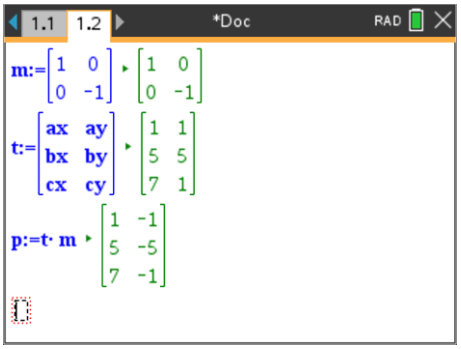
Insert another Maths Box and create a 3 x 2 matrix.

Enter the matrix values as shown opposite, note that the variable names can be typed directly or accessed from the **VAR** menu.



The set of transformed points can be created in a new matrix ‘P’.

Again, in a maths box, enter the equation shown opposite.



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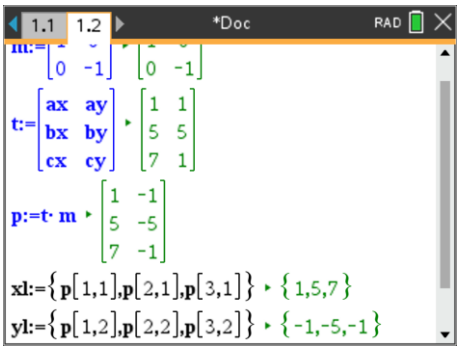
The elements of the matrix can now be turned into a list so that the points may be plotted (as a scatterplot).

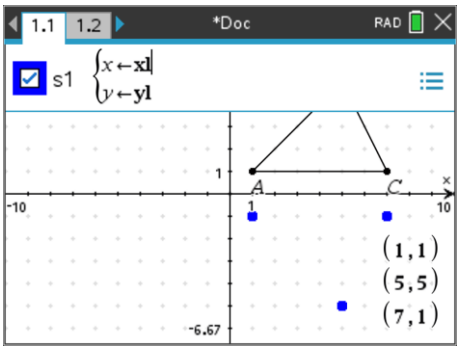
Two new Maths boxes contain the lists xl and yl.

Note that p[1, 1] identifies the element in the first row and first column of matrix p, list elements are separated by commas and the lists themselves are enclosed by { }. (As shown opposite)



Navigate back to the previous page containing the Graphs and Geometry application.

Change the graph type to a Scatterplot and plot the lists xl and yl.

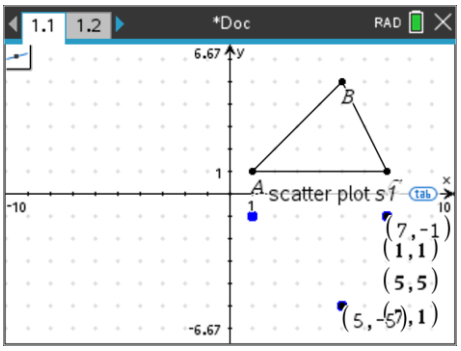


In this step points will be plotted on top of the scatterplot. It is important to ensure that the points are on the scatterplot and not the grid; otherwise they will not be dynamic. Press:

[menu] > Geometry > Points & Lines > Point On

Move the mouse over the top of each point on the scatterplot. The word 'grid' will appear and 'tab'. Press the [tab] key so that the tip shows "scatter plot s1".

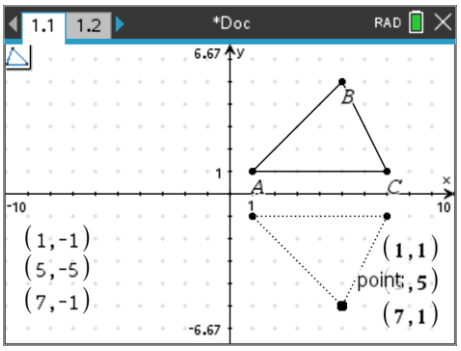
When a point is placed **on** the scatterplot it will appear a different colour. By placing points 'on' the scatterplot they will move when the scatterplot values are changed. Notice that as the points are added, so too are their coordinates.



Creating points on the scatterplot means that they can also be associated with geometrical objects, such as triangles! It is now possible to draw a triangle so that the vertices will be located on the scatterplot.

[menu] > Geometry > Shapes > Triangle

Place the vertices of the triangle 'on' each of the scatterplot points.



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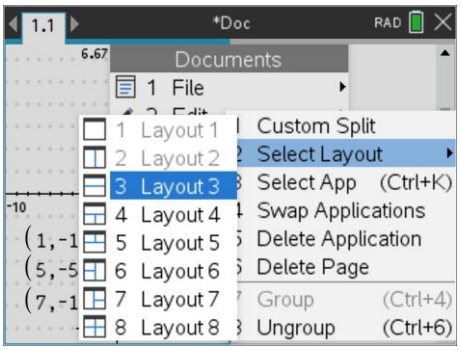
The transformation is now set and controlled by matrix (m).

To see the transformation matrix on the same page as the transformation the two pages can be merged.

On the Graph application (Page 1.1) press [Ctrl] + 4. This is a shortcut for merging two pages, also available from the page layout menu.

The Notes application will appear to the right of the Graph application, it is better positioned below. To change the orientation press:

[doc] > Page Layout > Select Layout > Layout 3

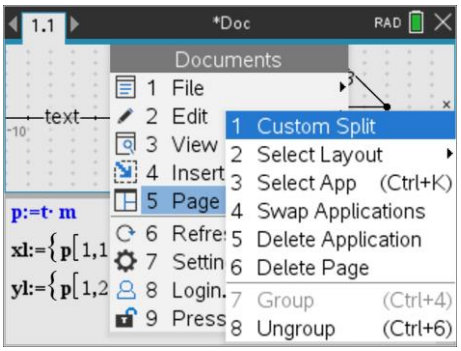


To change the layout proportions, return to the document menu:

[doc] > Page Layout > Custom Split

Arrow down to reduce the amount of space occupied by the Notes application.

Once the proportions have been set, the Window > Zoom option should be used on the Graphs and Geometry application to return this to a 'square' aspect ratio. As the content is focused very much on Geometry, it is important to have 'right angles' appearing as 90˚.



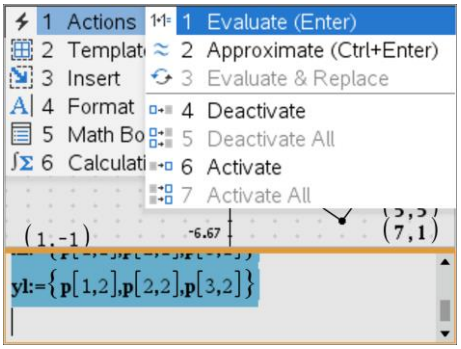
When the Notes application is moved, access to variables is halted.

To re-activate all the maths boxes, (from the Notes application) press:

[Ctrl] + A (Select all)

[menu] > Actions > Evaluate

Scroll back up through the calculations so that the transformation matrix is shown and ensure that all calculation outputs are displayed.



Calculator Tip!

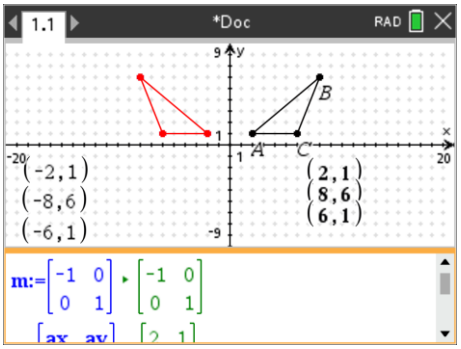
When one or more windows appear on the screen at the same time you can click on the corresponding application to change focus or press [ctrl] + [tab].

This is similar to 'Alt + Tab' on a computer keyboard.

The transformation matrix (m) can now be edited and the results will automatically be displayed in the Graph application.

The colour of the transformed triangle can be changed to make it stand out.

To edit the transformation matrix, select the 'input' (blue) and edit accordingly. The window settings may need to change if the triangle is enlarged.



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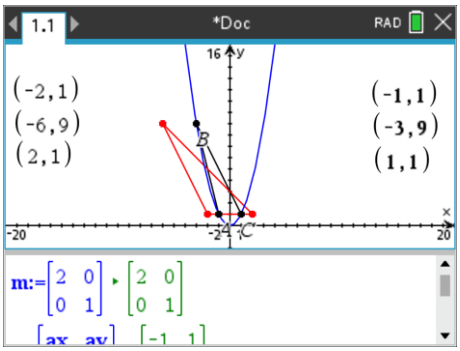
Things to explore:

- Start with the identity matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and then try some simple reflections:
  - $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$
  - $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
  - $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$
  - Or a reflection without using negatives: $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. What type of reflection is this one?
- Explore dilations, starting with: $\begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix}$
  - This approach helps students understand the 'direction' of the dilation including references such as "in the positive x direction" or "away from the y axis" and "towards the y axis".
- Explore ways to help students connect and understand dilations associated with functions.
  - The most common introduction to 'transformations' associated with functions is for quadratics. This is unfortunate as students don't see the need to differentiate between a dilation "away from the x axis" or "towards the y axis".

    Consider graphing the function $y = x^2$ and placing the points for the original triangle 'on the function' (as shown). Adjust the window settings accordingly and the location of the individual points so they appear to sit on the function. The dilation shown is "away from the y axis". This can be as the change in the x coordinates and the actual position of the dilated triangle, note that the y coordinates have not changed. Students can determine the quadratic equation that passes through all of the points in the dilated triangle.

    By working with numbers and shapes it is easier to see that the transformation aligns with: $y = \left(\frac{1}{2}x\right)^2$ which helps explain references to the standard expression: $y = ax^2$ and the value of the coefficient being $a = \frac{1}{4}$. Using the relational graphing tool students can also graph: $4y = x^2$ and see if they can produce such a transformation with the matrix.
- Translations can also be added to this document by editing the Notes application. Explore translations of the original triangle by including an additional matrix. Note that the matrix will need to be of the appropriate dimensions or combined by referencing appropriate matrix elements.

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