EXAM READINESS IB HL / SL MATHEMATICS SEPTEMBER 2018

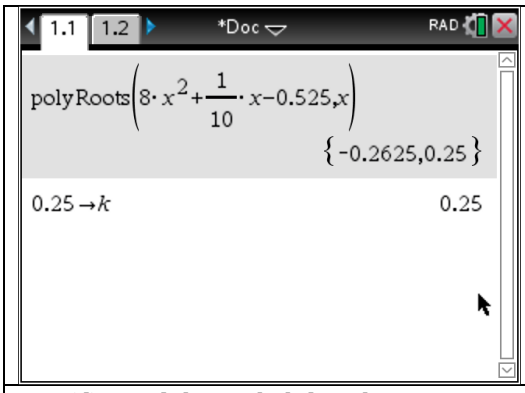
PROBABILITY

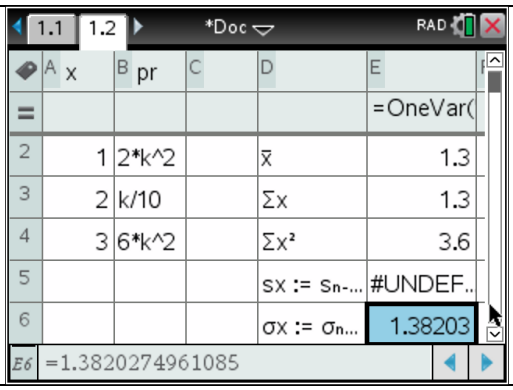
Question 1 Discrete

A discrete random variable follows the probability distribution as in the table below.

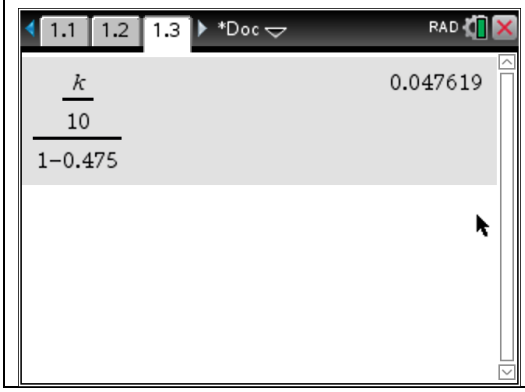
X	0	1	2	3
$P(X=x)$	0.475	$2k^2$	$\frac{k}{10}$	$6k^2$

(a) Find the value of k.
(b) Find the mean of the distribution.
(c) Find the standard deviation.





(d) Find the probability that $X = 2$ given $X > 0$.



mean = 1.3
st dev = 1.38

Question 2 Normal / Binomial

Barramundi fish are both wild-caught and farmed. Barramundi can reach up to 1.5 m and 50 kg, although the majority of fish weigh less than 6 kg. Farmed barramundi, however, average about 400 g in weight and 35 cm in length.

On a certain farm the weights of the barramundi are normally distributed with a mean of 400g and standard deviation 60 g. Barramundi are classified as oversized if they weigh more than 550 g.

A barramundi is selected at random from the farm's pond.

(a) Find the probability that this barramundi was oversized.

(a) $P(X > 550) = 0.00621$

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normCdf(550,1.E99,400,60)        0.00621
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(b) $P(X > 600 | X > 550) = \dfrac{P(X > 600)}{P(X > 550)}$

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normCdf(550,1.E99,400,60)        0.00621
normCdf(600,1.E99,400,60)        0.069104
0.0062096798534949
```

(b) Given that this barramundi is oversized, find the probability that it weighs more than 600 g.

Two barramundi are selected at random.

(c) Find the probability that they are both oversized.

$0.00621... \times 0.00621... = 0.0000386$

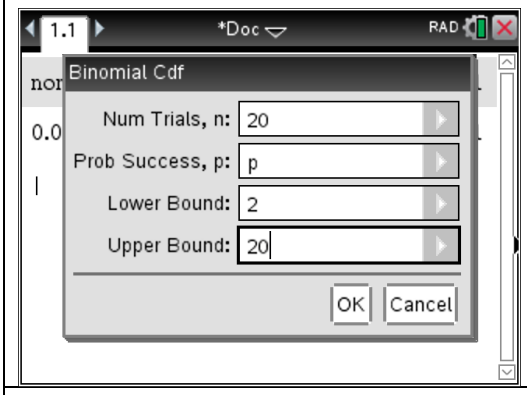
20 barramundi are selected to be sold to a restaurant.

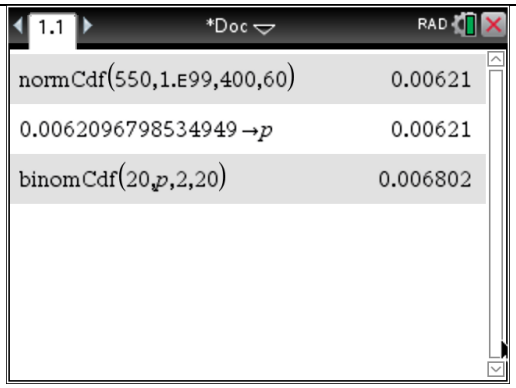
(d) Find the expected number of these barramundi to be oversized.

$20 \times 0.00621... = 0.124$

(e) Find the probability that at least two of these are oversized.

$Y \sim B(20, 00621...)$

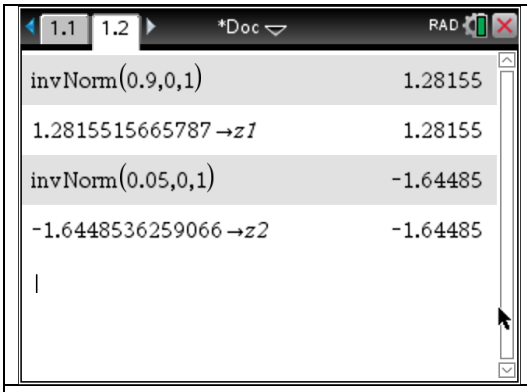


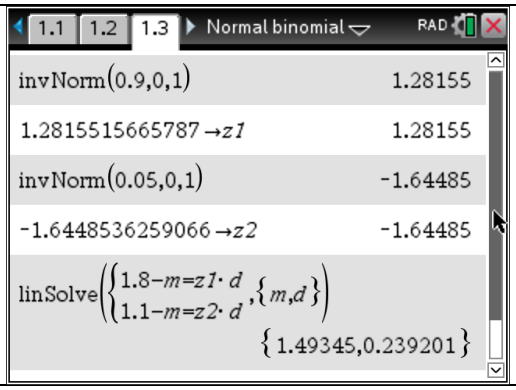


$P(Y \geq 2) = 0.00680$

Scientists from Sunshine University measured some of the wild-caught barramundi. They found that their lengths followed normal distribution with 10% being longer than 1.8 m and 5% being shorter than 1.1 m.

(f) Find the mean and standard deviation of the lengths of the wild-caught barramundi.

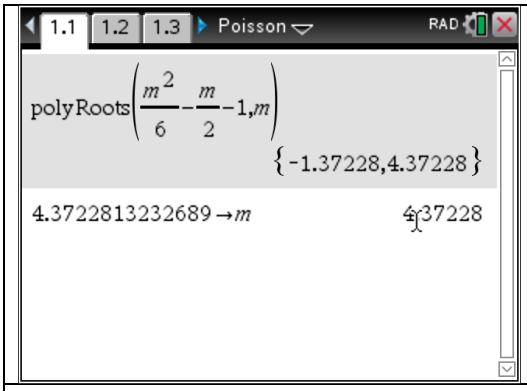


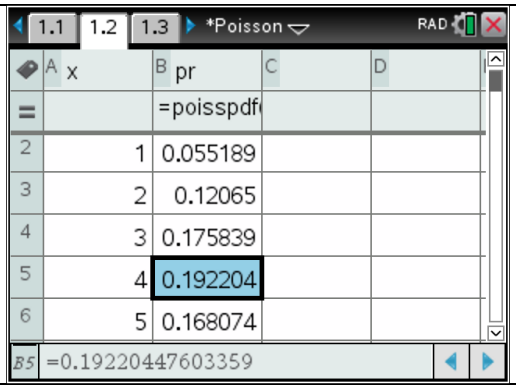


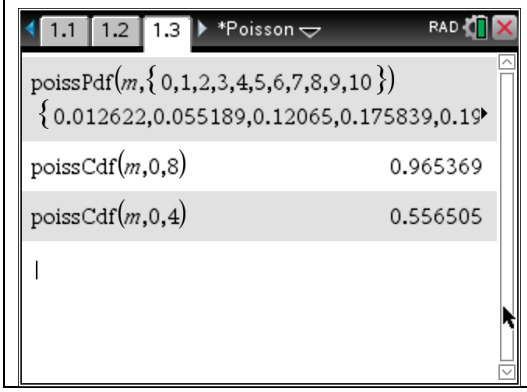
mean = 1.49, st dev = 0.239

Question 3 Poisson

For $X \sim P_o(m)$, find the value of m given that $P(X=1)+P(X=2)=P(X=3)$.







Question 4 Continuous

Jason runs a Basketball Clinics for 6-12 years old boys. The trainings often go over time and parents of the boys need to wait in their cars.

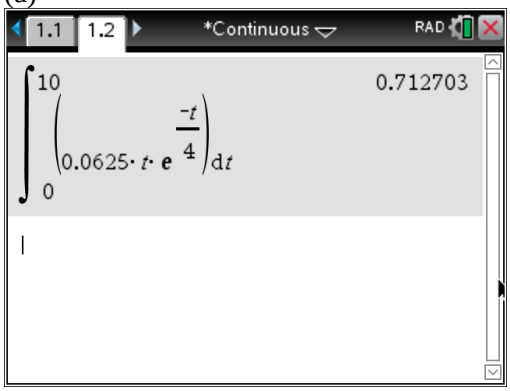
The waiting time is a continuous random variable *Y*, with a probability density function $y(t)$ defined below:

$$y(t)=\begin{cases}0.0625\,t\,e^{-\frac{t}{4}}, & t>0\\ 0, & \text{elsewhere}\end{cases}$$

where *t* is the time in minutes.

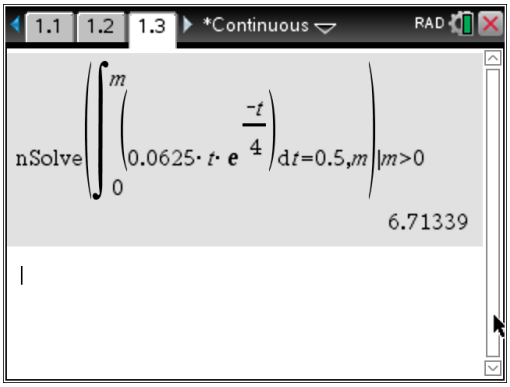
(a) Find the probability, to four decimal places, that a parent will wait no longer than 10 minutes.
(b) Find the median waiting time, correct to the nearest second.

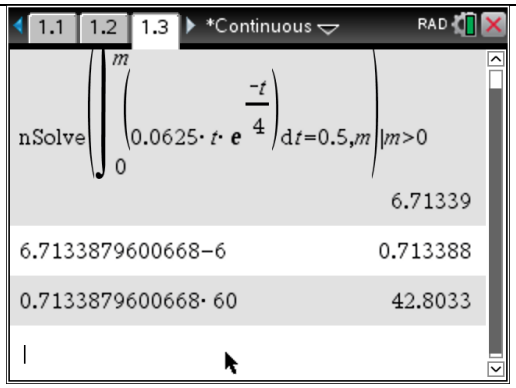
(a)



0.7127

(b)





median = 6 minutes 43 seconds

ALGEBRA

Question 5 Sequences / Sigma notation

The first term of an infinite geometric sequence is 4. The sum of the infinite sequence is 200.

(a) Find the common ratio.
(b) Find the sum of the first 8 terms.
(c) Find the least value of n for which $S_n > 163$.

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$\text{nSolve}\left(\frac{4}{1-r}=200,r\right)$ 0.98

$r = 0.98$

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$\sum_{n=1}^{8}\left(4\cdot(0.98)^{n-1}\right)$ 29.8474

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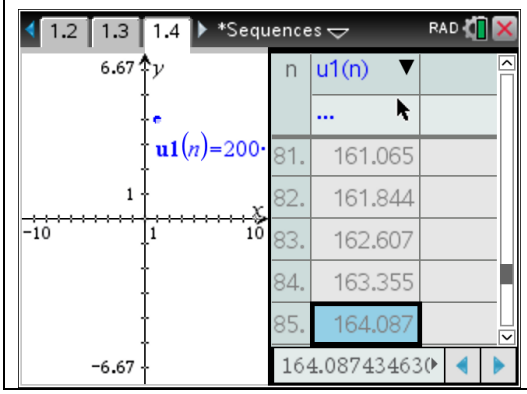
Part (c) n=85

$$S_n = \frac{4\left(1-.98^n\right)}{0.02} = 200(1-.98^n)$$

Enter in the sequence mode and insert table crtl T



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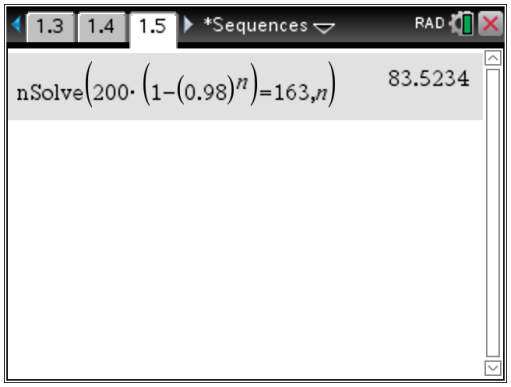
Numerical solve can only be used with equation, not inequalities.



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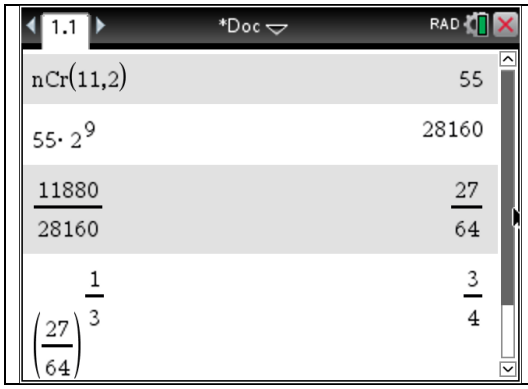
Question 6 Binomial expansion

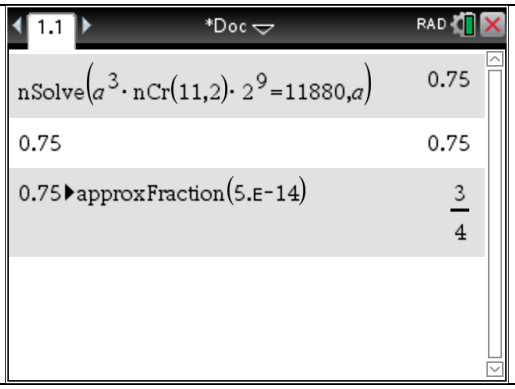
In the expansion of $ax^3(2+ax)^{11}$, the coefficient of the term in x^5 is 11880. Find the value of a.

$$ax^3 \times {}^{11}C_r \times 2^9 \times (ax)^r = 11880x^5$$

$$x^{3+r} = x^5 \Rightarrow r = 2$$

$$a \times {}^{11}C_2 \times 2^9 \times a^2 = 11880$$



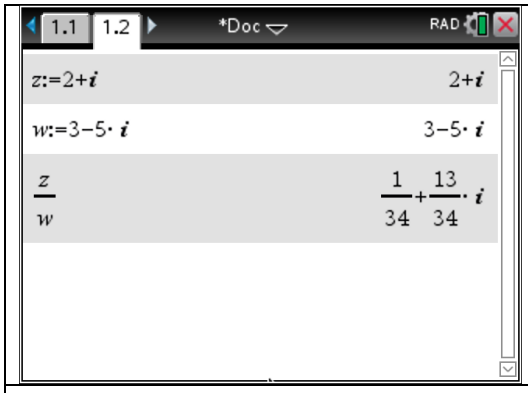


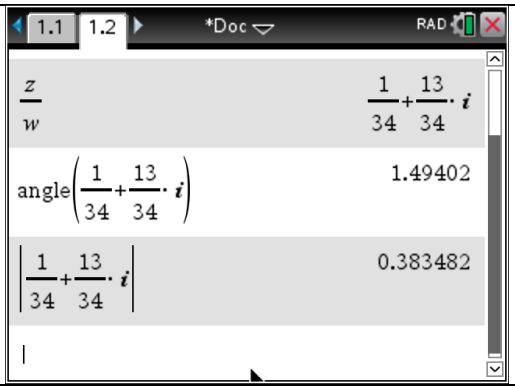
Question 7 Complex numbers

Given z=2+i and w=3−5i find

(a) modulus of $\frac{z}{w}$

(b) argument of $\frac{z}{w}$ correct to 4 decimal places.





If modulus is to be given as exact value calculate:

$$\left(\frac{1}{34}\right)^2+\left(\frac{13}{34}\right)^2 \qquad \frac{5}{34}$$

then

$$\sqrt{\frac{5}{34}}$$