Webinar IB Studies

IB Studies Topic 6 Mathematical Models

Linear models.

Quadratic models.

Exponential models.

Other models in the form

$f(x)=ax^m+bx^n+\ldots, m,n \in Z.$

Drawing accurate graphs.

Concept of a function, domain and range.

All graphs should be labelled and have some indication of scale.

Linear models.

Typical questions involve revenue versus cost modelling, profit and break-even models.

Conversion graphs.

Example.

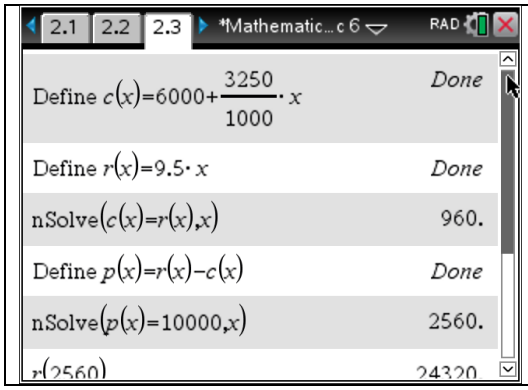
A new book is being printed. The plates required for the printing process cost $6000, after which the printing costs $3250 per thousand books. The books are to sell at $9.50 each with an unlimited market.

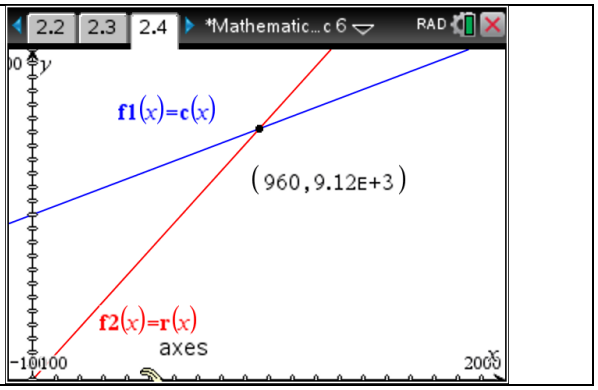
(a) Determine cost and revenue functions for the production of the book.

(b) Graph the cost and revenue functions on the same set of axes.

(c) How many books must be sold in order to break even?

(d) What level of production and cost will generate $10000 profit?





Quadratic models.

Typically just a parabola in various settings or maximising / minimising graphically with a quadratic function.

Cost functions, projectile motion.

Example 1

The quadratic function $y=ax^2+bx+c$ passes through the points (4,0) and (p,0). The vertex has coordinates (1,5).

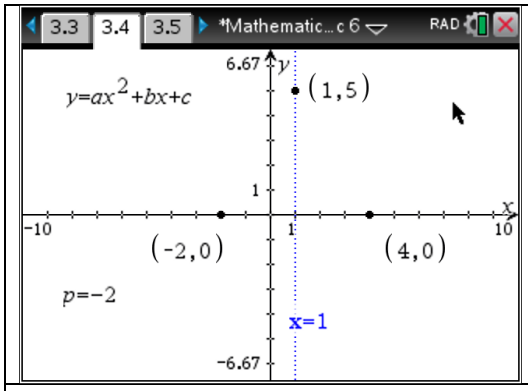
(a) Write down the equation of the axis of symmetry.

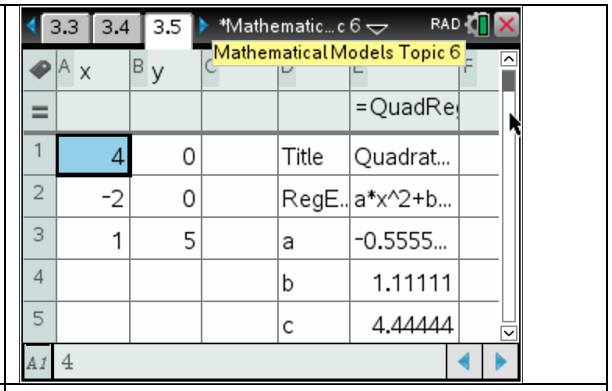
(b) Calculate the value of p.

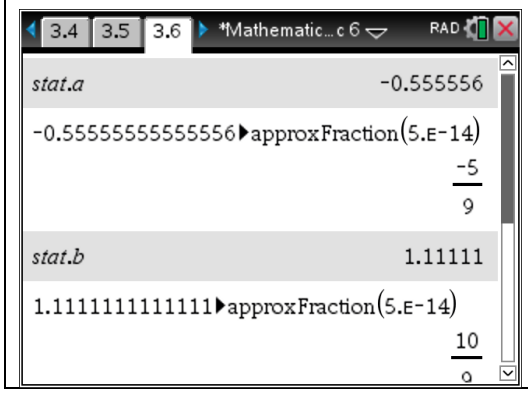
(c) Find the value of b.

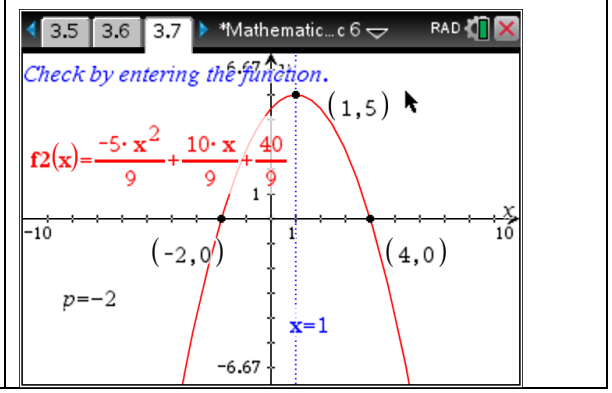
(d) Find the value of c.

(e) Find the value of a.







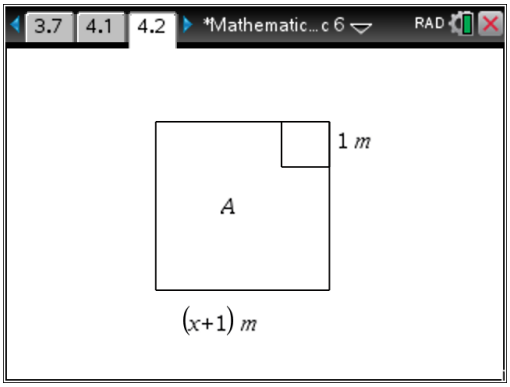


Example 2

The length of a square garden is (x+1) metres. In one of the corners a square of 1 m length is used only for grass. The rest of a garden is only for planting roses and is shaded in the diagram below.

The area of the shaded region is A.

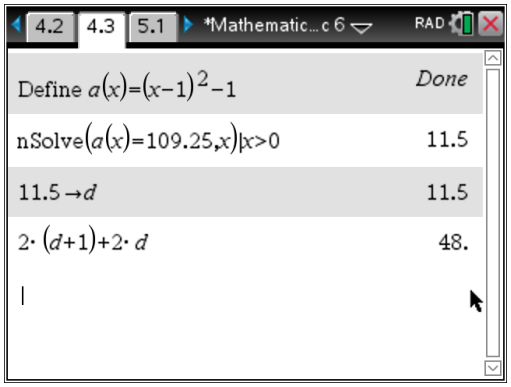
(a) Write down an expression for A in terms of x.



(b) Find the value of x given that

A=109.25 m^2

(c) The owner of the garden puts a fence around the shaded region. Find the length of this fence.



Exponential models.

Using Define, graph, table.

Population growth, radioactive decay, cooling of a liquid, spread of a virus, depreciation.

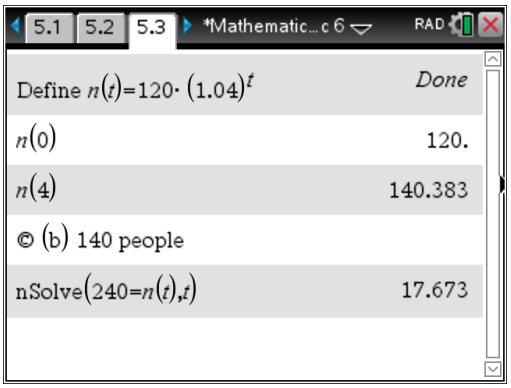
Example

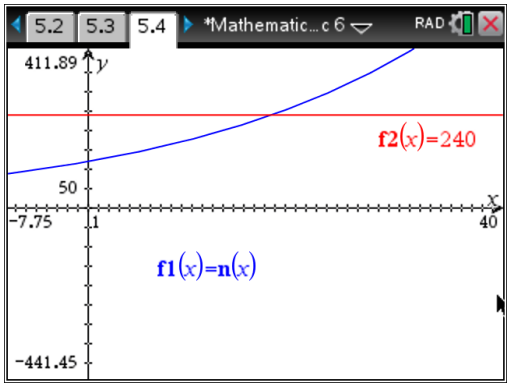
The number of people N on a small island, t years after settlement, seems to increase according to the formula $N=120\cdot 1.04^t$.

(a) Calculate the number of people present at the start of the settlement.

(b) Calculate the number of people after 4 years (to the nearest integer).

(c) How long would it take for the population to double?





Example

The concentration of a drug in the blood of a patient t hours after it was administered is given by

$C(t)=1.4\,t\cdot 3^{-0.2t}$, where C is measured in mg/L and $t\geq 0$.

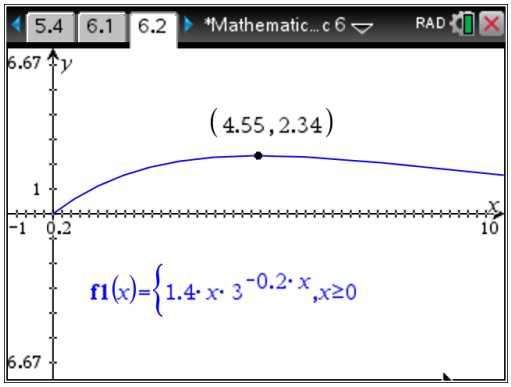
(a) Find the concentration after 3 hours.

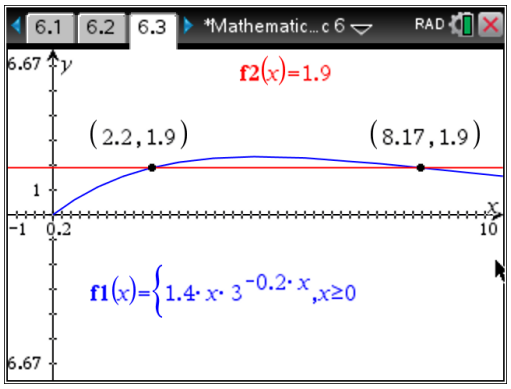
Initially the concentration increases until it reaches a maximum, and then it decreases.

(b) Find the time when this maximum occurs.

(c) What is the maximum concentration?

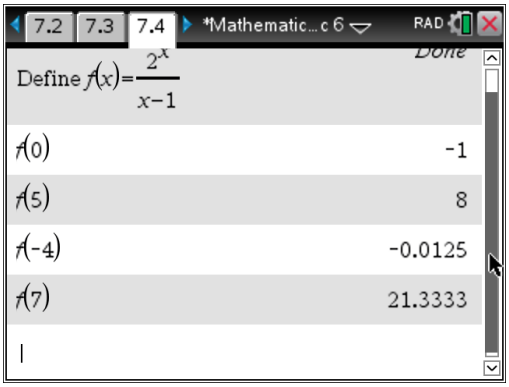
(d) Find the times when the concentration is 1.9 mg/L.





Other functions and their graphs.

Asymptotes, tables of values, define.



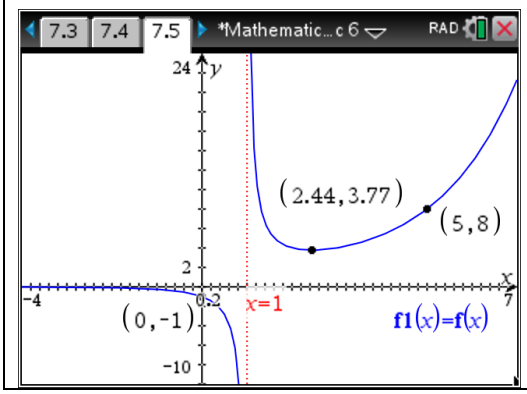
Example

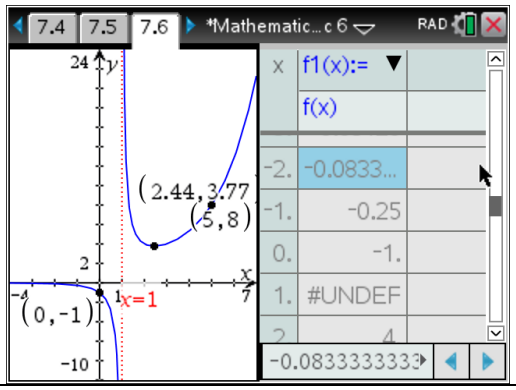
Consider the function $f(x)=\frac{2^x}{x-1}$.

(a) Find the y−intercept.

(b) Determine the minimum value of f(x) for x>1.

(c) Write down the equation of the vertical asymptote.





Example

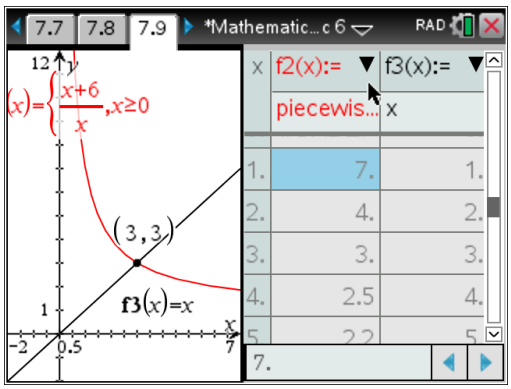
A cardboard box is to have a fixed volume of $36cm^2$. The width W (in cm) of the box is defined by the function $W(x)=\frac{x+6}{x}$.

A graph of y=W(x) is shown for 0≤x≤7.

(a) Use the graph to complete the table for the width function.

(b) The length L (in cm) of the box is defined by the function L(x)=x. Draw the graph of y=L(x) on the same set of axes above.

(c) Determine the value of x which produces a square base for the cardboard box.



Comparing models.

A population of bacteria is growing as shown in the following table.

t	0	1	2	3	4	5	6	7	8	9	10
N	300	345	396	455	522	600	689	792	909	1045	1200

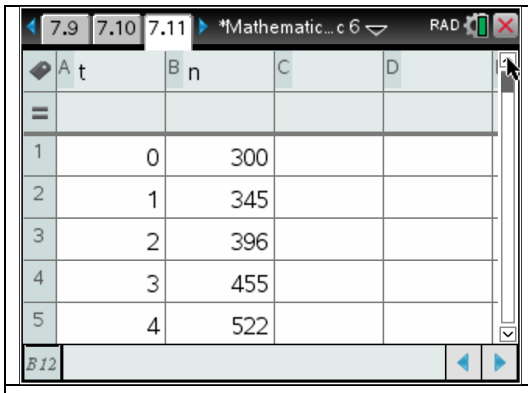
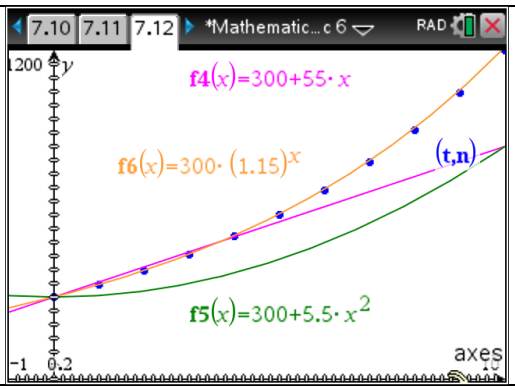
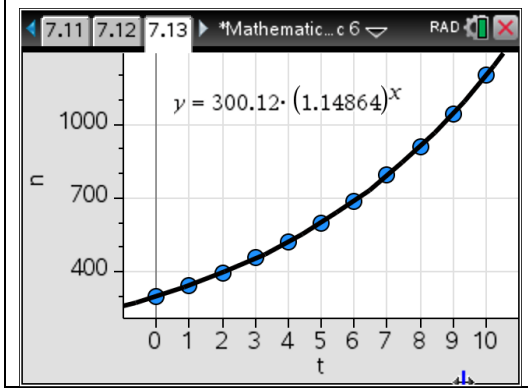
(a) Plot the graph of N versus t on your GDC.

(b) What was the initial number of bacteria?

(c) How long did it take for the population to double in number?

(d)A linear model could be N=300+55t. Add this line to your graph.

(e) A quadratic model could be $\mathbf{N}=300+5.5\mathbf{t}^2$. Add this line to your graph.

(f) An exponential model could be $\mathbf{N}=300\cdot 1.15^{\mathbf{t}}$. Add this curve to your graph.

(g) Which is the best model for the growth in population of bacteria?