Solving equations effectively with TI Nspire.

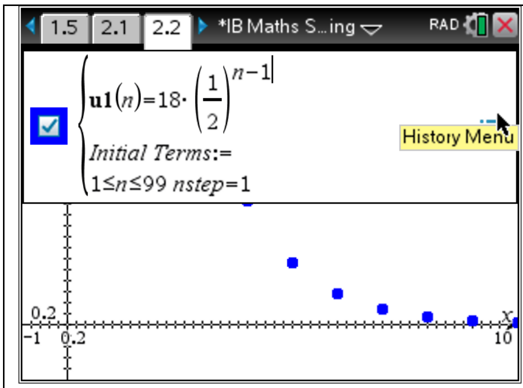
Sequences and Series.

Example 1

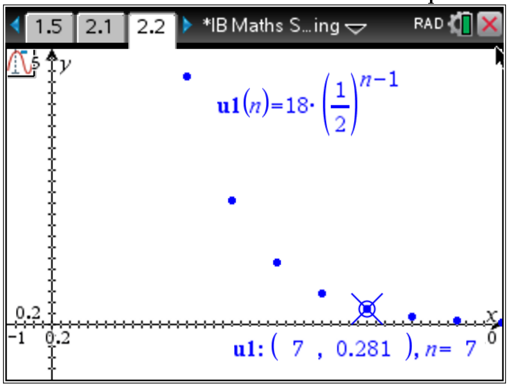
Consider a geometric sequence $u_n = 18 \times \left(\frac{1}{2}\right)^{n-1}$.

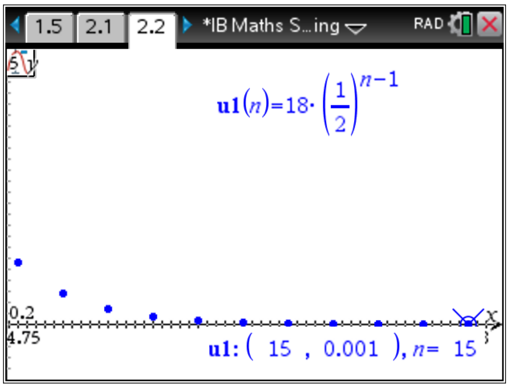
(a) Find the smallest value of n for which u_n is less than 10^{-3}.

We can graph the sequence in Sequence mode (available in PTT).

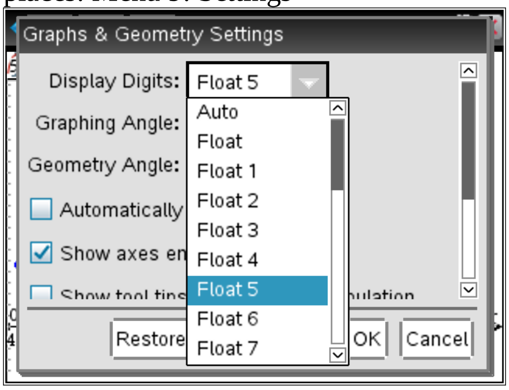


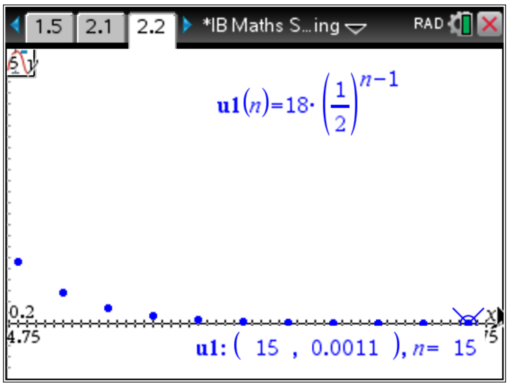
Use Trace to see the terms of the sequence.

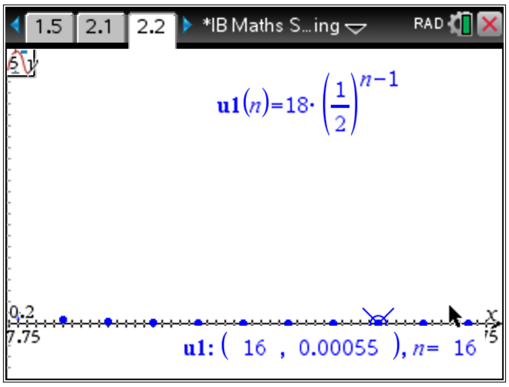




Change the Float Setting to see more decimal places: Menu 9: Settings

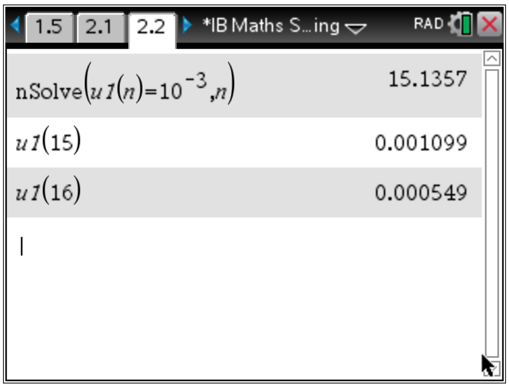






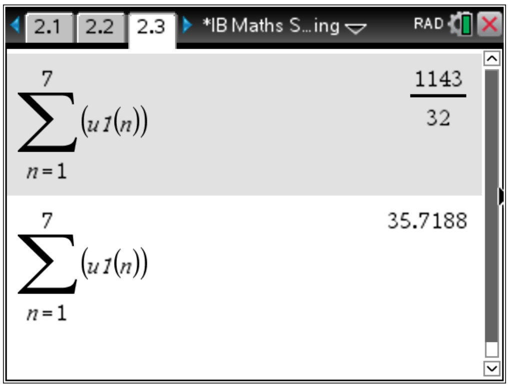
We need 16 terms to have the term less than 0.001.

Alternatively we can use nSolve and then check the answer. Having the sequence defined saves time.

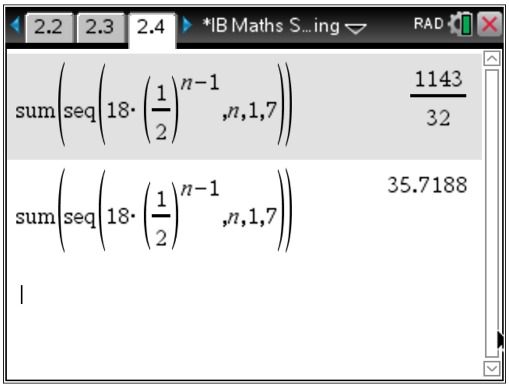


(b) Find the sum of the first 7 terms.

HL /SL



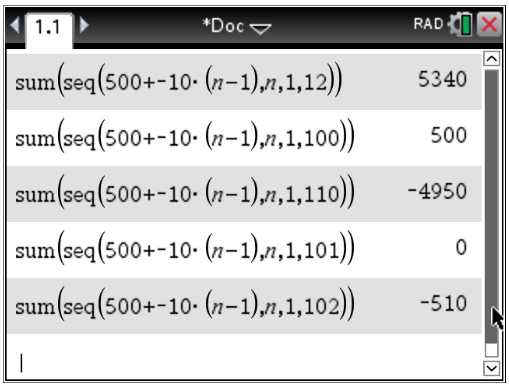
Studies



Example 2

An arithmetic sequence has the first term 500 and the common difference -10. How many terms need to be added for the sum of the terms to become negative?

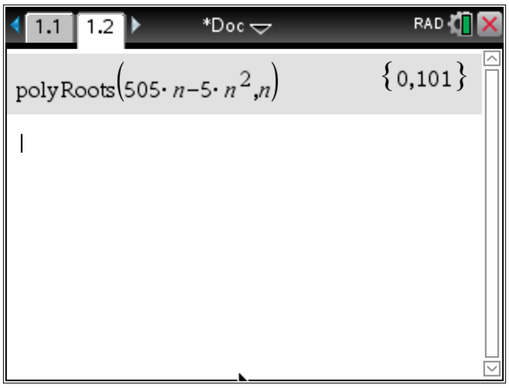
Trial and error approach:



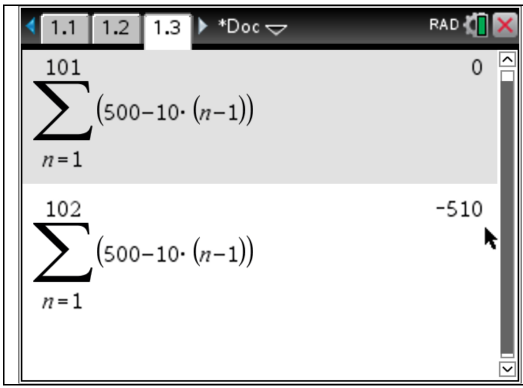
Algebraic approach:

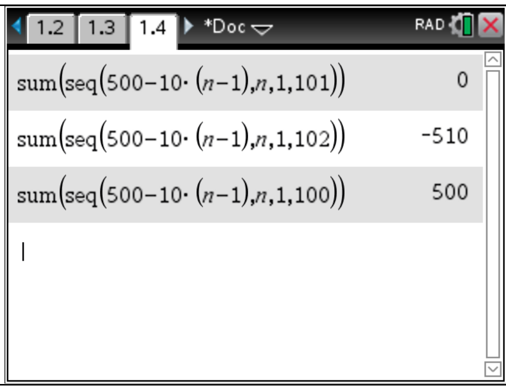
$$S_n = \frac{n}{2}\left[2\times 500 - 10(n-1)\right]$$
$$= 505n - 5n^2$$

Use polyRoots



Check using sigma notation (HL/SL) or sum(seq command (Studies)





Example 3 (HL/SL)

The arithmetic sequence has first term $u_1 = 1.6$ and common difference $d = 1.5$.

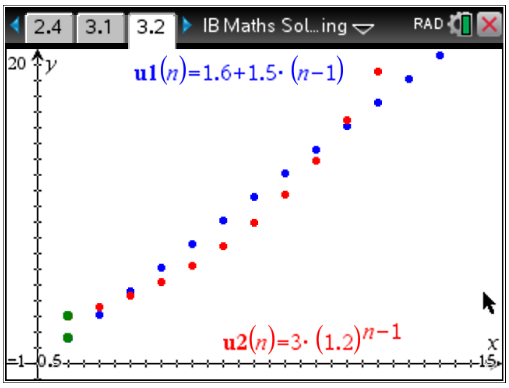
The geometric sequence has first term $v_1 = 3$ and common ratio $r = 1.2$.

(a) Find an expression for $u_n - v_n$ in terms of *n*.

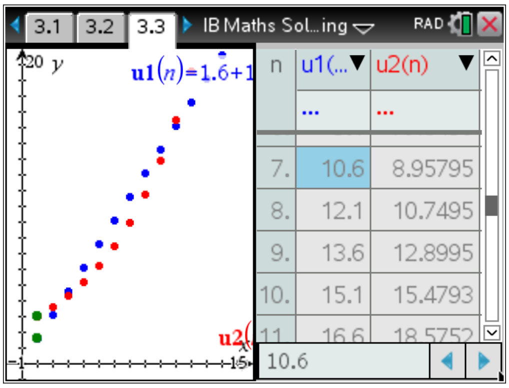
(b) Determine the set of values of *n* for which $u_n > v_n$.

(c) Determine the greatest value of $u_n - v_n$.

Draw both sequences using Sequence Graph Entry:

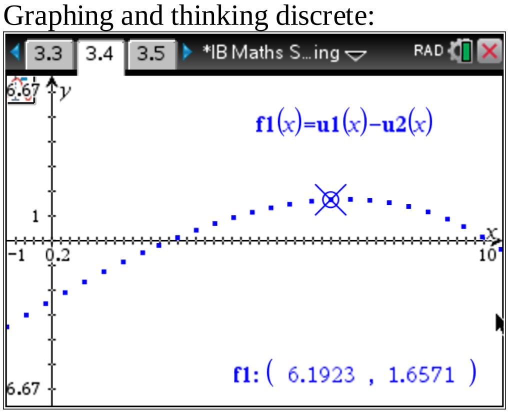


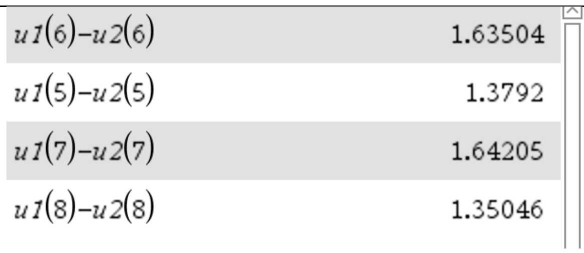
For part (b) Look up Graph/Table (crtlT) **Answer:** $3 \le n \le 9$



For part (c): The greatest value occurs for *n* between 6 and 7

Graphing and thinking discrete:





The greatest value of $u_n - v_n$ is 1.64

Probability.

Example 1

A random variable X has the following probability distribution, where a and b are real constants.

x	0	1	2	3
$\Pr(X=x)$	0.1	a	b	0.1

Given that $Var(X)$=0.56, find all possible values of a and b.

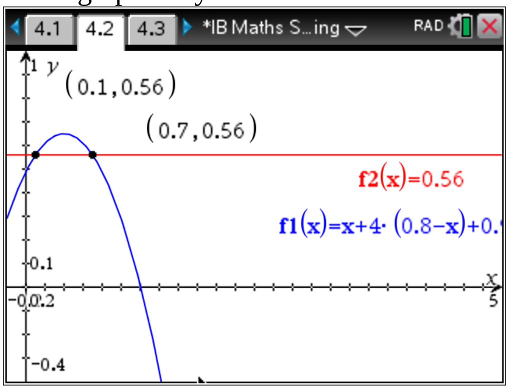
$a+b=0.8,\quad b=0.8-a$

$E(X)=a+2b+.3$

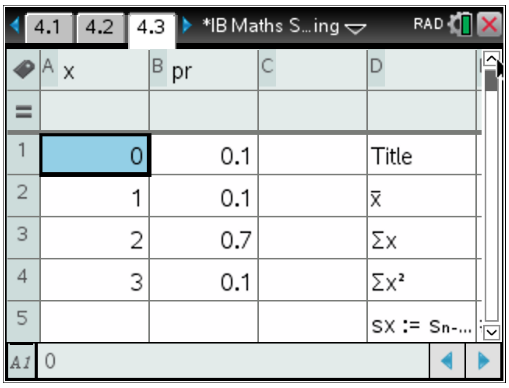
Using Var(x)= $E(X^2)-(E(X))^2$ and the above we arrive at the following equation:

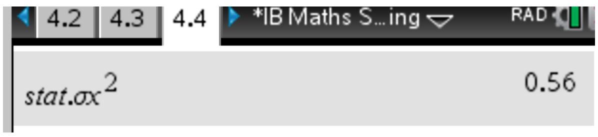
$a+4(.8-a)+0.9-(1.9-a)^2=0.56$

Solve graphically:



Check





Example 2 (SL/HL)

Binomial distribution

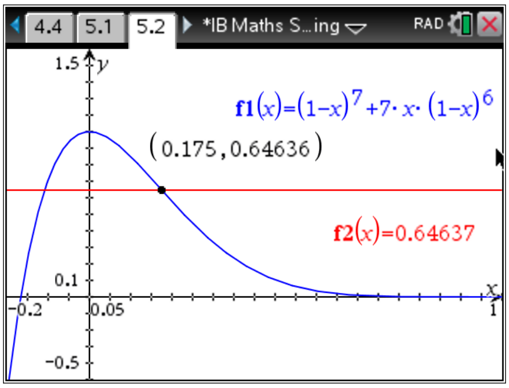
The random variable X has a binomial distribution with parameters n and p.

Given that $n=7$ and $P(X\leq 1)=0.646365$ correct to 6 significant figures, determine the value of p.

Using the formula we can write: $\binom{7}{0}p^0(1-p)^7+\binom{7}{1}p^1(1-p)^6$

Or simplified $(1-p)^7+7p(1-p)^6=0.646365$

Solving graphically:



Numerically with the restricted domain:

$$\text{nSolve}\left((1-p)^7+7\cdot p\cdot(1-p)^6=0.646365,p\right)|0<p<1$$

$$\text{nSolve}\left((1-p)^7+7\cdot p\cdot(1-p)^6=0.646365,p\right)|0<$$

0.175

Example 3 (HL)

Continuous probability distribution.

The random variable, X, has probability density function

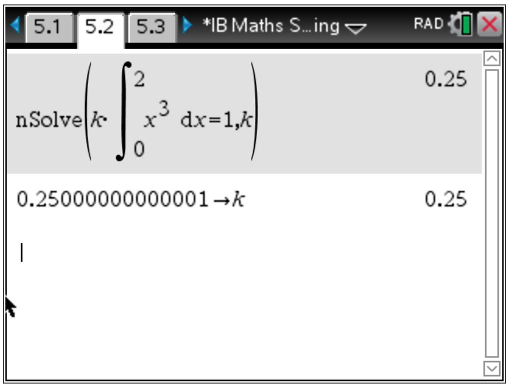
$$f(x)=\begin{cases} kx^3 & 0\le x\le 2, \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the value of k.
(b) Find $E(X)$.
(c) Find $VAR(X)$.
(d) Find the median of the distribution.
(e) Find the interquartile range.
(f) Find the probability that an observation lies within one standard deviation of the mean.
(g) Find the probability that out of 4 observations, two observations lie within one standard deviation from the mean.

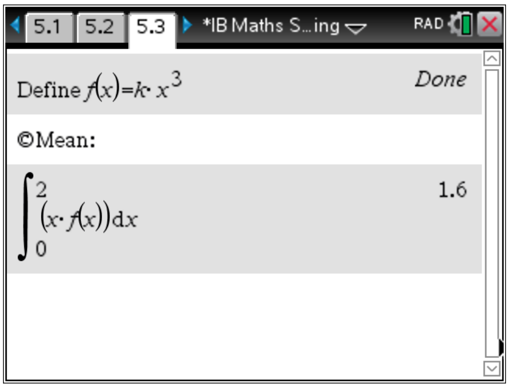
Solving for k. Hint: k needs to be in front of the integral:

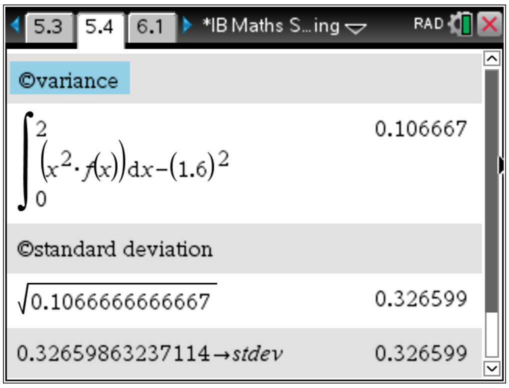
$$\text{nSolve}\left(k\cdot\int_0^2 x^3\,dx=1,k\right) \qquad 0.25$$

Store the value of k, sometimes it is more complicated:

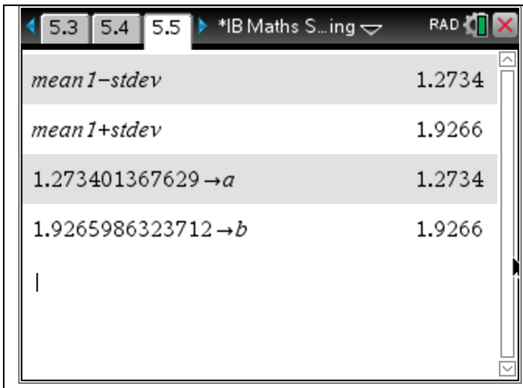


Define the function now for further calculations:





One standard deviation from the mean:



Store as a and b to use as the limits for the integral:

Store this value as p for the binomial distribution calculations:

$0.69674374905849 \rightarrow p$	0.696744
$\text{binomPdf}(4,p,2)$	0.267866

Mathematical models.

Example 1 Exponential models (Studies)

The following equation show the temperature in degrees Celsius of Ms Brown’s cup of coffee, t minutes after pouring it out.

$f(t) = 16 + 74 \times 2.8^{-0.2t}$ where $f(t)$ is the temperature and t is the time in minutes.

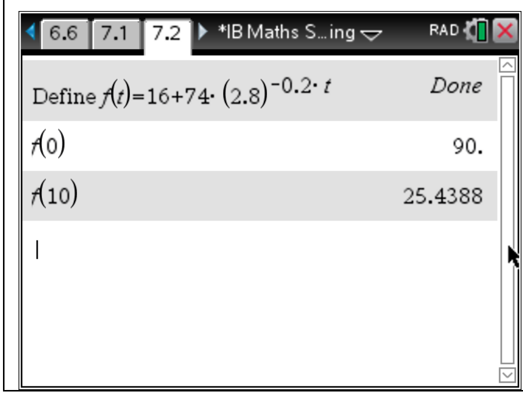
(a) Find the initial temperature of the coffee.
(b) Sketch the graph of f showing clearly the equation of the horizontal asymptote.
(c) State the room temperature.
(d) Find the temperature of the coffee after 10 minutes.

If the coffee is not hot enough it is reheated in a microwave. The temperature of the coffee in the microwave varies with time according to the formula

$T = A \times 2^{1.5t}$

where T is the final temperature of the liquid, A is the initial temperature of coffee in the microwave and t is the time in minutes after switching the microwave on.

(e) Find the temperature of Ms Brown’s coffee after being heated in the microwave for 30 seconds after it has reached the temperature in part (d).
(f) Calculate the length of time it would take a similar cup of coffee, initially at 20°C, to be heated in the microwave to reach 100°C.

<table>
<tr>
<td>It is recommended to define the function in the calculator screen.

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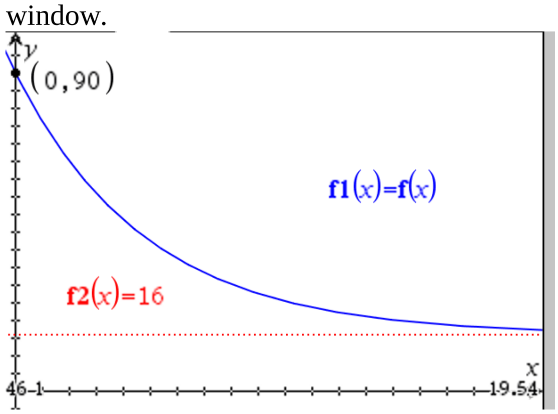
<td>Enter f(x) in the Graph screen and set the window.

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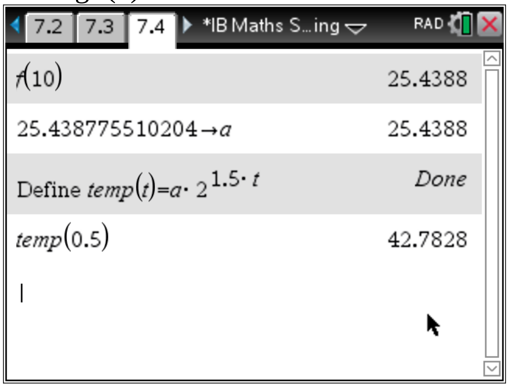
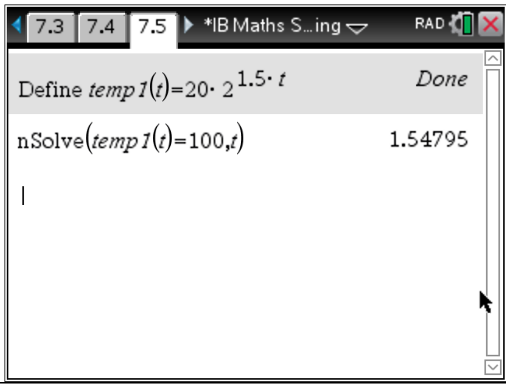
<tr>
<td>Storing f(1) as a to use with the second formula:

</td>
<td>Since the initial temperature of the coffee changed, define a new function and then use nSolve to find time.

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