AP Calculus – Mixed Six – Using Technology Calculator-Active

Connecting a Function, Its First Derivative, and Its Second Derivative (Topic 5.9)

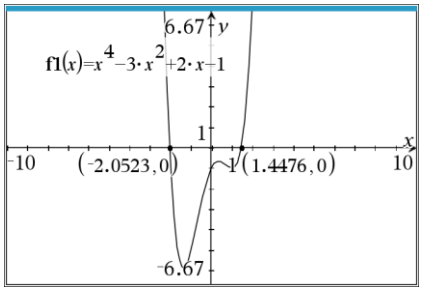
1. Factual Recall

The function *f* has first derivative given by $f'(x)=x^4-3x^2+2x-1$. Use the graph of the derivative to determine the intervals on the graph of *f* is decreasing.

$f'(x)=0 \quad \Rightarrow \quad x=-2.0523, 1.4476$

f decreases when $f'(x)<0$

$[-2.053, 1.448]$



2. Carry out a Procedure

Use the graph of $f'(x)$ given in #1 to find the *x*-coordinate of the relative maximum and the relative minimum of the function *f*.

The relative maximum is at *x* = –2.052 because $f'(x)$ changes from positive to negative.

The relative mimimum is at *x* = 1.448 because $f'(x)$ changes from negative to positive.

3. Classify a Mathematical Object

The first derivative of a function *f* is defined by $f'(x)=\left(\frac{x^2+1}{3}\right)\cos(2x-1)$ for $-2.5 \le x \le 2.5$.

Various values of *x* are given below. Classify each as the location of a relative minimum, a relative maximum, a point of inflection on the graph of *f*, or none of these. See graph on pg. 2.

x	Classify
–1.856	Relative Maximum
–1.285	Point of Inflection
–0.762	None of these
–0.285	Relative Minimum
0.722	Point of Inflection
1.285	Relative Maximum
1.681	None of these
2.249	Point of Inflection

4. Prove, Show, Justify

The function *h* has first derivative given by $h'(x)=\frac{x^3-7x^2+3}{\sqrt{x^3+4x+6}}$. At what values of *x* does the graph of *h* have points of inflection? Give a reason for your answer. See graph on page 2.

The points of inflection are located at $x=-0.0733, 3.4267$

The graph of $h'(x)$ changes from increasing to decreasing at *x* = –0.073 and the graph of $h'(x)$ changes from decreasing to increasing at *x* = 3.427.

5. Extend a Concept

The first derivative of a function *g* is defined by $g'(x)=x^2\left(\sin^{-1}(0.6x+0.5)+1\right)$.

On what open interval is the graph of *g* increasing and concave down?

(–1.584 , 0)

$g'(x)$ is positive and decreasing on this interval

6. Critique a Fallacy

The second derivative of a function *h* is defined by $h''(x)=\frac{7(x-1.2)}{x^2+5}$. Paige says that the graph of *h* has points of inflection at *x* = –1.338 and *x* = 3.738 because the graph of $h''(x)$ changes from increasing to decreasing or vice versa. Do you agree or disagree? Why?

Disagree, Paige needs to find the zeros of $h''(x)$ in order to determine the points of inflection of the graph of *h*. The point of inflection for the graph of *h* is at *x* = 1.2. This is where the 2nd derivative changes from negative to positive. See graph on pg. 2.

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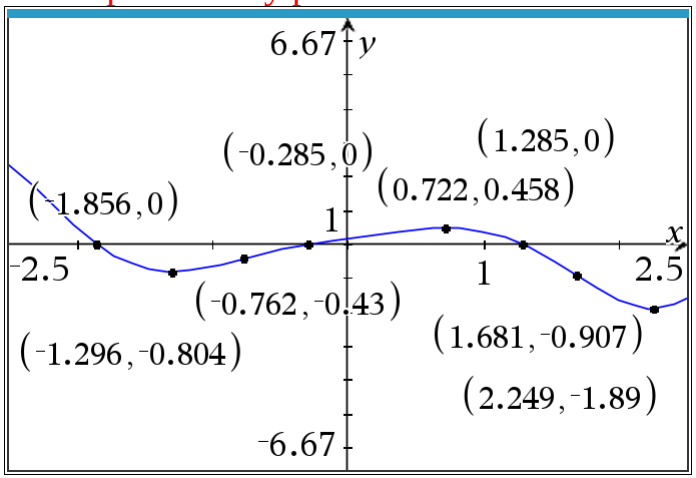
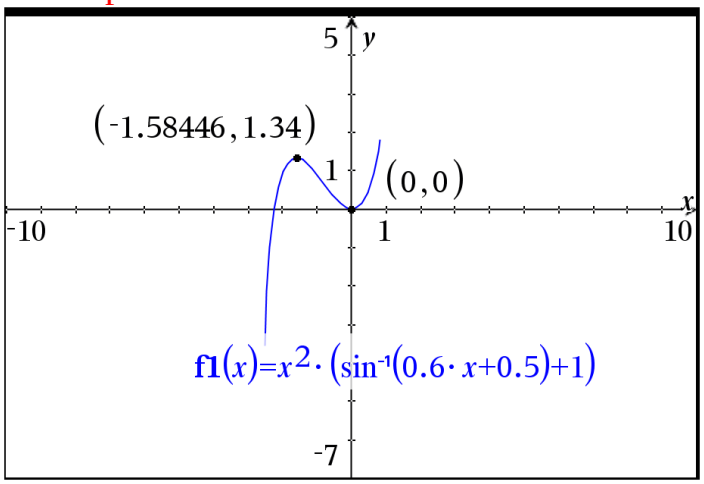
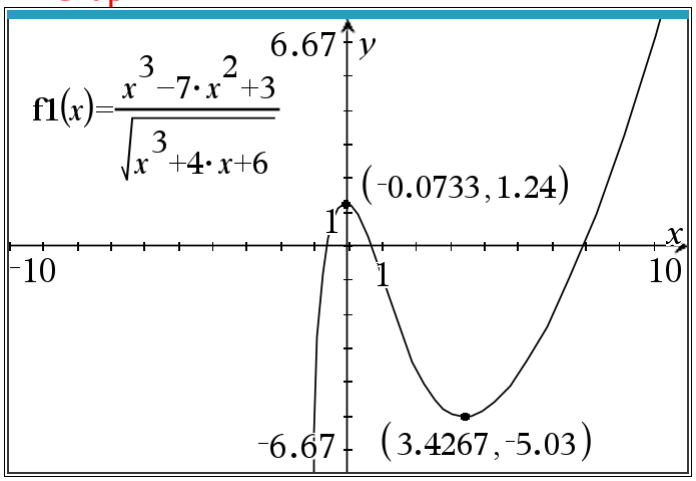
#3 Graph with key points



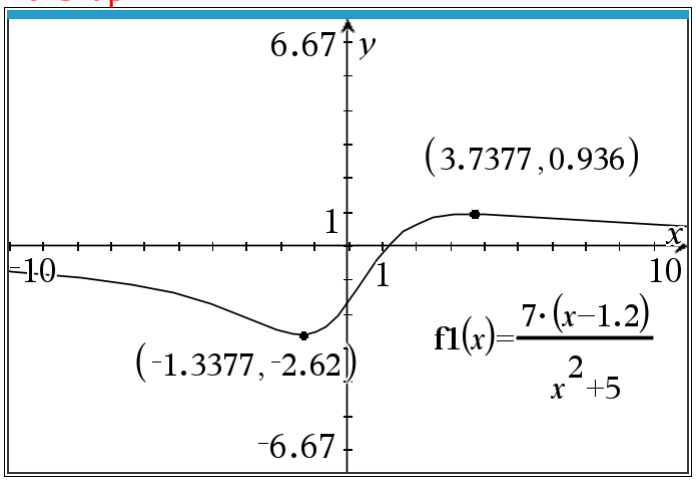
#5 Graph



#4 Graph



#6 Graph



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