KEY AP Calculus – Mixed Six – Limits with Technology

Calculator-Active

Topics 1.3 and 1.4 Estimating Limits from Tables and Graphs

1. Factual Recall

Consider the function $f(x)=\dfrac{x^2+0.6x-2.8}{x-1.4}$ and the limit as x approaches 1.4.

Explore the graph to determine the limit, then complete the table to support your limit approximation from the graph.

x	$f(x)$
1.38	3.38
1.39	3.39
1.41	3.41
1.42	3.42

$\lim\limits_{x\to 1.4} f(x)=3.4$

See page 2 for the graph.

2. Carry out a Procedure

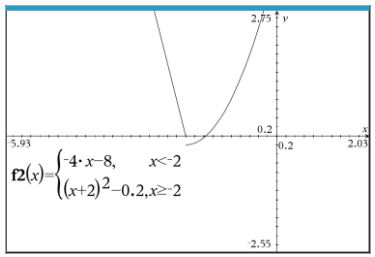
Consider the piecewise function

$$f(x)=\begin{cases}-4x-8, & x<-2\\ (x+2)^2-0.2, & x\ge -2\end{cases}.$$

Explore the graph to determine the one-sided limits.

$\lim\limits_{x\to -2^-} f(x)=0$

$\lim\limits_{x\to -2^+} f(x)=-0.2$



3. Classify a Mathematical Object

Explore the graph of each function to determine whether the limit is true or false.

T/F	Function	Limit
T	$f(x)=\dfrac{\sin(3x)}{x}$	$\lim\limits_{x\to 0} f(x)=3$
F	$g(x)=\dfrac{\ln(x^2)}{\ln x}$	$\lim\limits_{x\to \infty} g(x)=1$
T	$h(x)=\dfrac{1-\cos^2(2x)}{x^2}$	$\lim\limits_{x\to 0} h(x)=4$
F	$j(x)=\dfrac{\sin(x-5)}{2(x-5)}$	$\lim\limits_{x\to 0} j(x)=1$

4. Prove, Show, Justify

Which table of values would be the best to use to determine that $\lim\limits_{x\to 3} g(x)=11$? Give a reason for your answer.

Table 1	
x	$g(x)$
2.5	11.211
2.9	11.208
2.99	11.201
3	4
3.01	11.202
3.1	11.207
3.5	11.212

Table 2	
x	$g(x)$
2.5	4.211
2.9	4.208
2.99	4.201
3	11
3.01	4.202
3.1	4.207
3.5	4.212

Table 1. In Table 2 $g(3)=11$, but $\lim\limits_{x\to 3} g(x)=4.2$.

5. Extend a Concept

Consider the function $f(x)=\dfrac{\cos\left(\dfrac{3}{x}\right)}{2+\dfrac{3}{x}}$. Use either a table or graph (or both), to determine the following limits.

$\lim\limits_{x\to -\infty} f(x)=\dfrac{1}{2}$

$\lim\limits_{x\to \infty} f(x)=\dfrac{1}{2}$

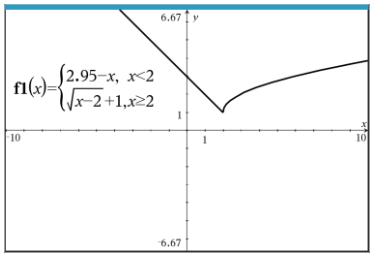
$\lim\limits_{x\to 0} f(x)=0$

See page 2 for the graph and possible tables to use.

6. Critique a Fallacy

Consider the graph of the piecewise function

$$f(x)=\begin{cases}2.95-x, & x<2\\ \sqrt{x-2}+1, & x\ge 2\end{cases}.$$



Cathy observes the graph and states that $\lim\limits_{x\to 2} f(x)=1$.

What is wrong with her statement? Write a correct statement.

She needs to use a different window or look at a table of values. $\lim\limits_{x\to 2} f(x)$ does not exist because the left- and right-hand limits are not equal. See page 2 for the graph and possible tables to use.

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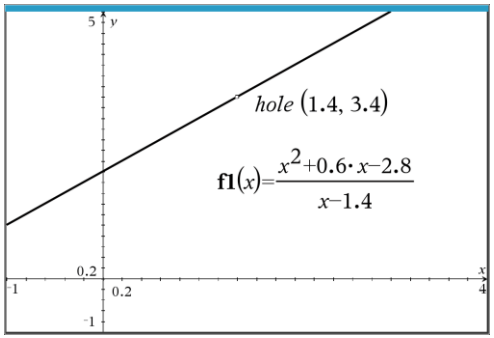
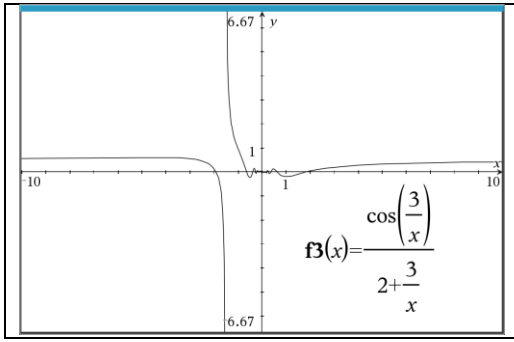
#1 Graph



#5 Graph and Tables

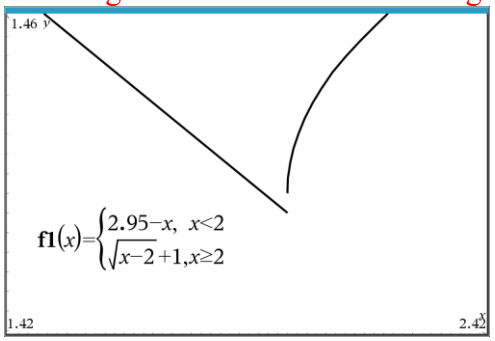


A xcord	B ycord
	=f3('xcord)
-0.1	-0.005509
-0.01	0.000074
-0.001	0.000325
0.001	-0.000325
0.01	-0.000073
0.1	0.00482

A xcord	B ycord
	=f3('xcord)
10.	0.415364
100.	0.492389
10000.	0.499925
-10.	0.561963
-100.	0.507386
-10000.	0.500075

#6 Graph and Tables

Zooming in we see a "break" in the graph.



x	f1(x):= piecewise.
1.9	1.05
1.99	0.96
1.999	0.951
1.9999	0.9501
1.99999	0.95001

$\lim_{x\to 2^-} f(x)=0.95$

x	f1(x):= piecewise.
2.1	1.31623
2.01	1.1
2.001	1.03162
2.0001	1.01
2.	1.001

$\lim_{x\to 2^+} f(x)=1$

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