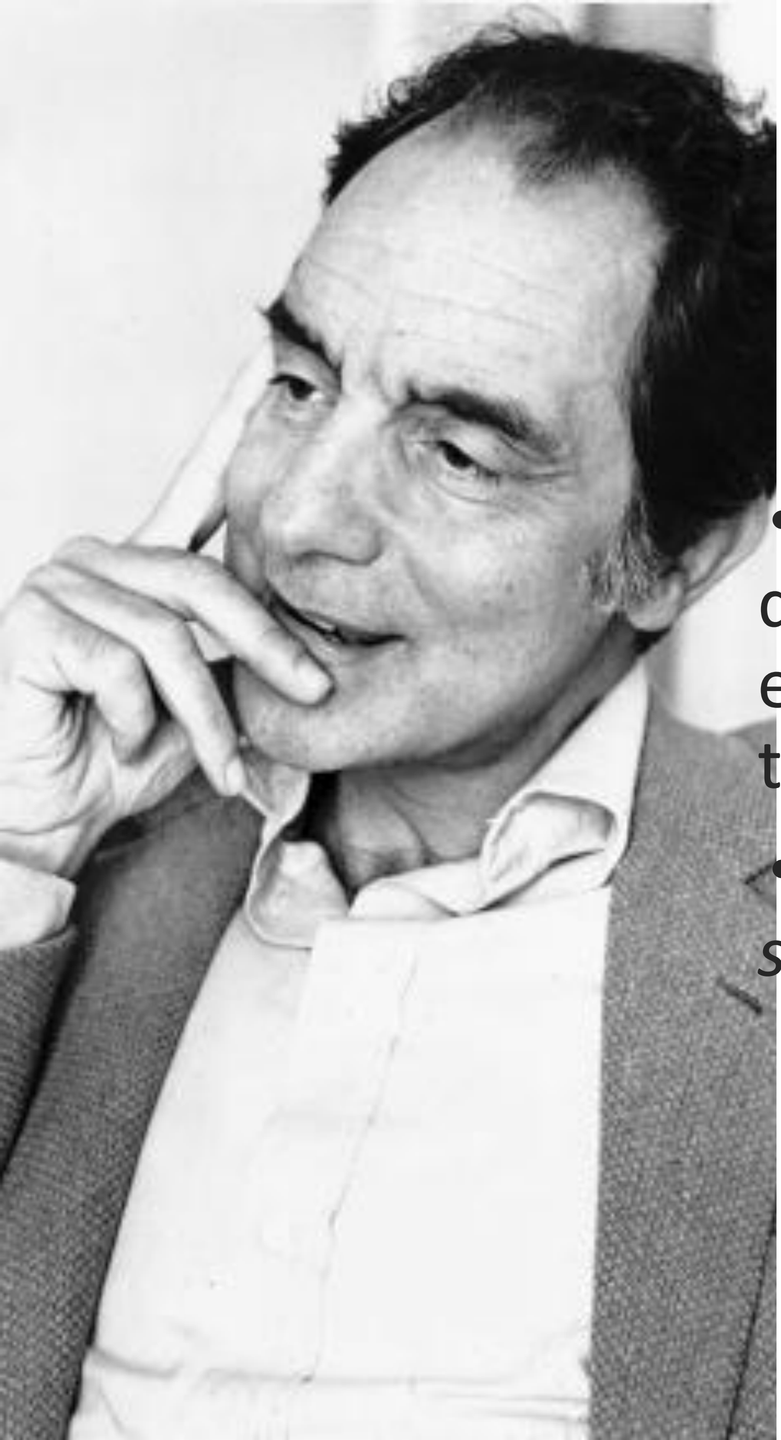
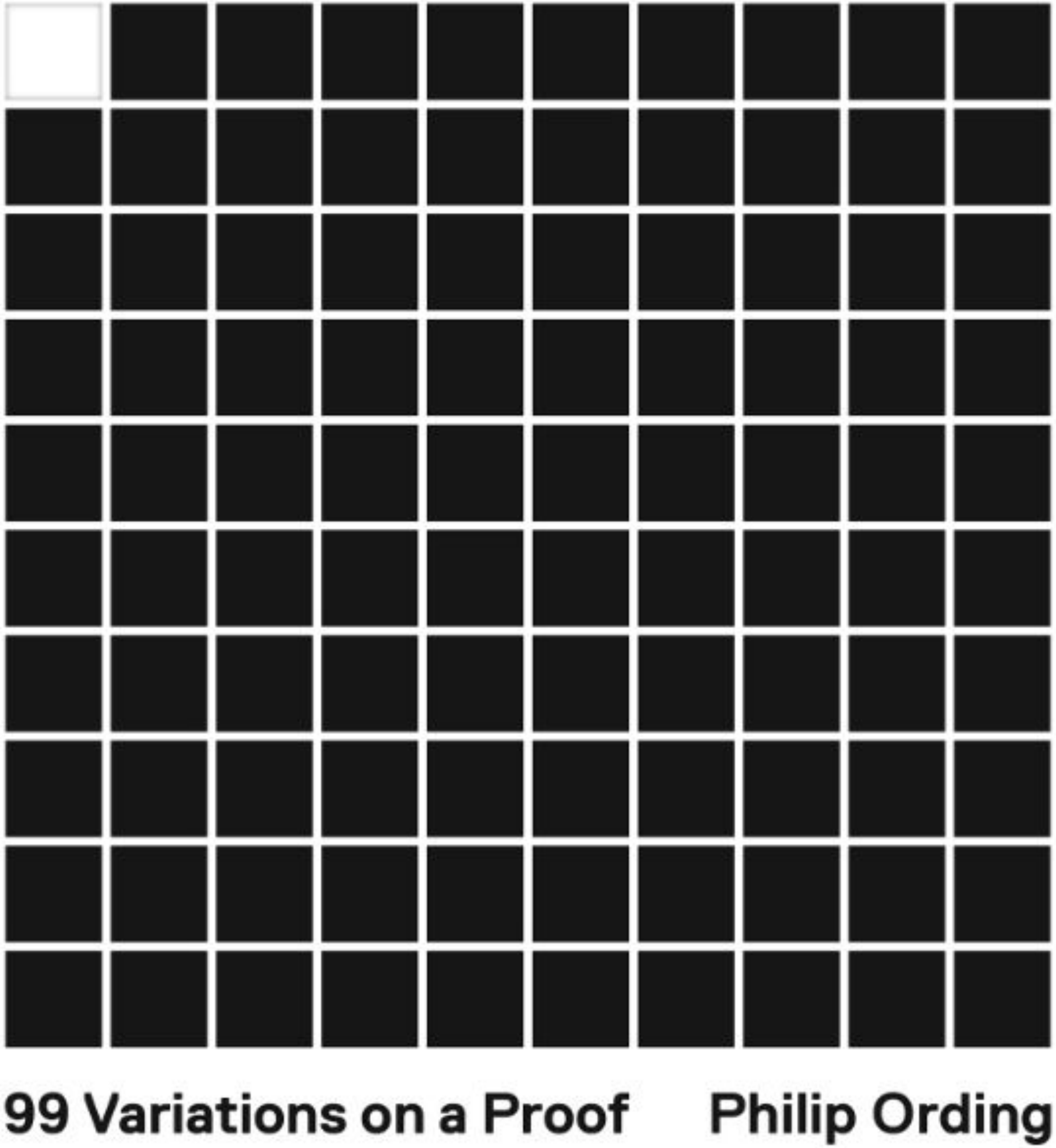
Creative Solutions for Solving Equations



Mathematical Style

- “For him, good thinking means quickness, agility in reasoning, economy in argument, but also the use of imaginative examples.”
- *Italo Calvino commenting on the style of Galileo*



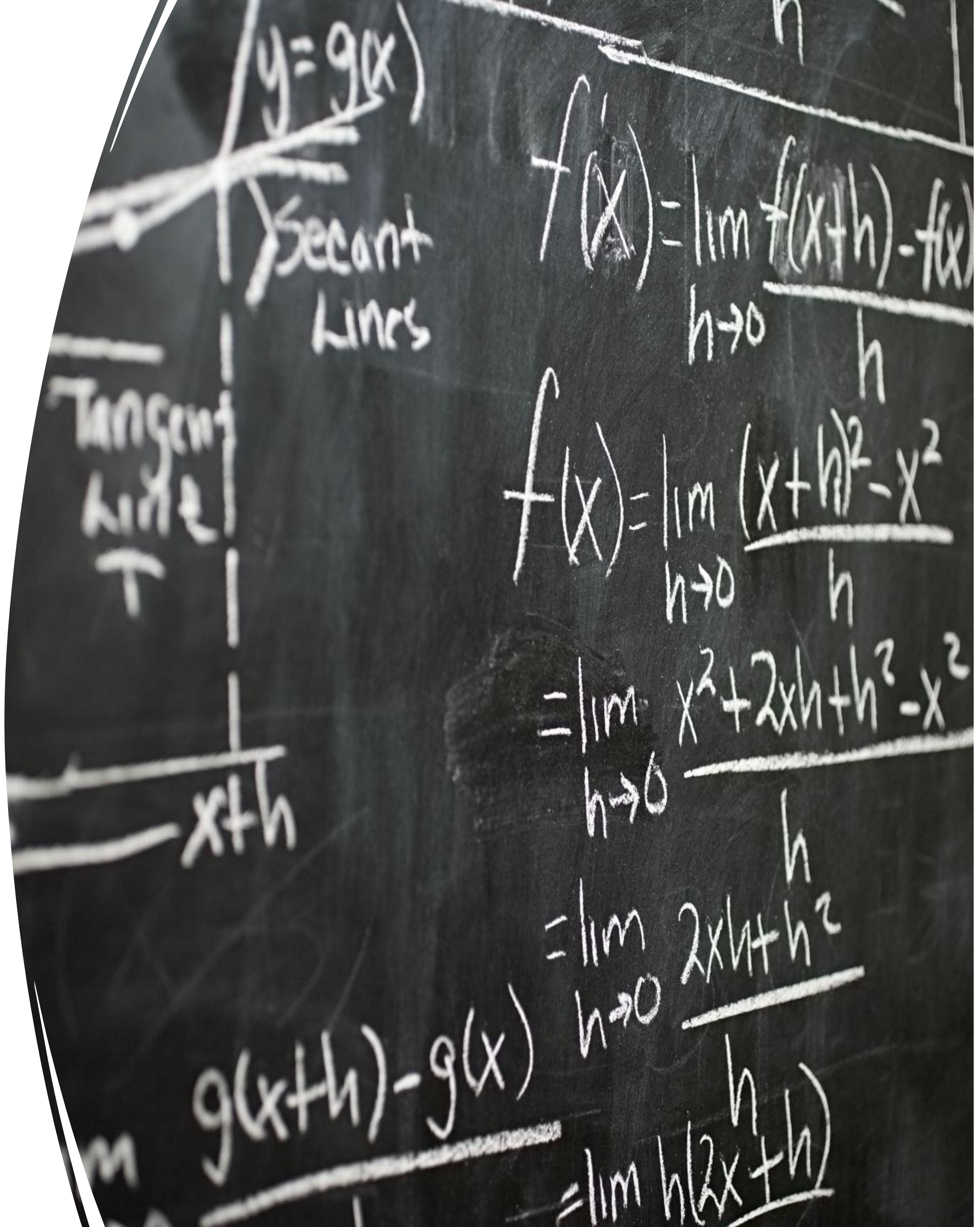
The Inspiration

Contents

The Equation

- Solve the cubic equation for all of its REAL solutions:

$$x^3 - 6x^2 + 11x - 6 = 2x - 2$$



0

Omitted

Theorem. If $x^3 - 6x^2 + 11x - 6 = 2x - 2$, then $x = 1 \; or \; x = 4$

Since the solution is trivial . . . The proof is left to the reader.

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Calculator


TEXAS INSTRUMENTS
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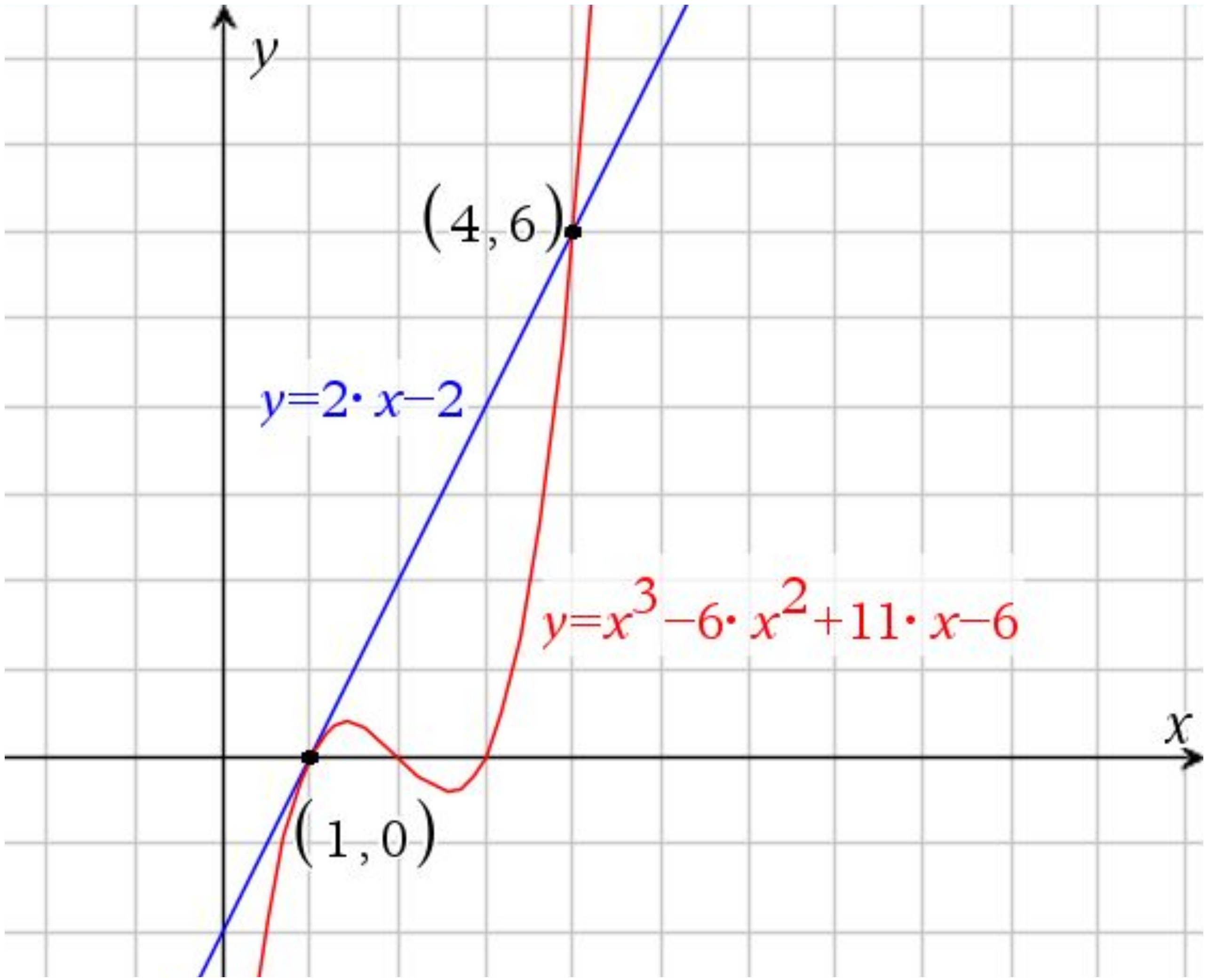
Illustrated



The two points of intersection of the cubic $y = x^3 - 6x^2 + 11x - 6$ and the line $y = 2x - 2$ occur at (1,0) and (4,6).

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Substitution

Theorem. *If* $x^3 - 6x^2 + 11x - 6 = 2x - 2$, *then* $x = 1$ *or* $x = 4$.

Proof:

$$x^3 - 6x^2 + 11x - 6 = 2x - 2$$

$$x^3 - 6x^2 + 9x - 4 = 0$$

Let $x = m + 1$

$$(m + 1)^3 - 6(m + 1)^2 + 9(m + 1) - 4 = 0$$

$$(m^3 + 3m^2 + 3m + 1) - 6(m^2 + 2m + 1) + 9(m + 1) - 4 = 0$$

$$m^3 + 3m^2 + 3m + 1 - 6m^2 - 12m - 6 + 9m + 9 - 4 = 0$$

$$m^3 - 3m^2 = 0$$

$$m^2(m - 3) = 0$$

$$m = 0 \text{ or } m = 3$$

$$x = 0 + 1 \text{ or } x = 3 + 1$$

$$x = 1 \text{ or } x = 4$$

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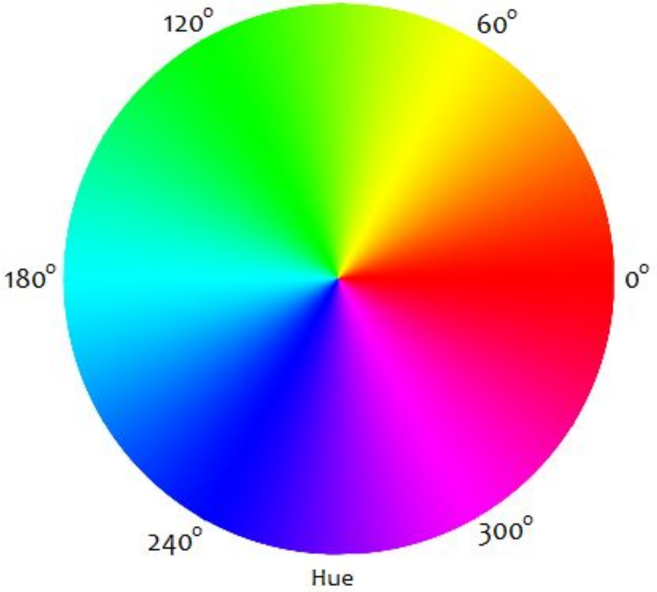
Chromatic

Chromatic

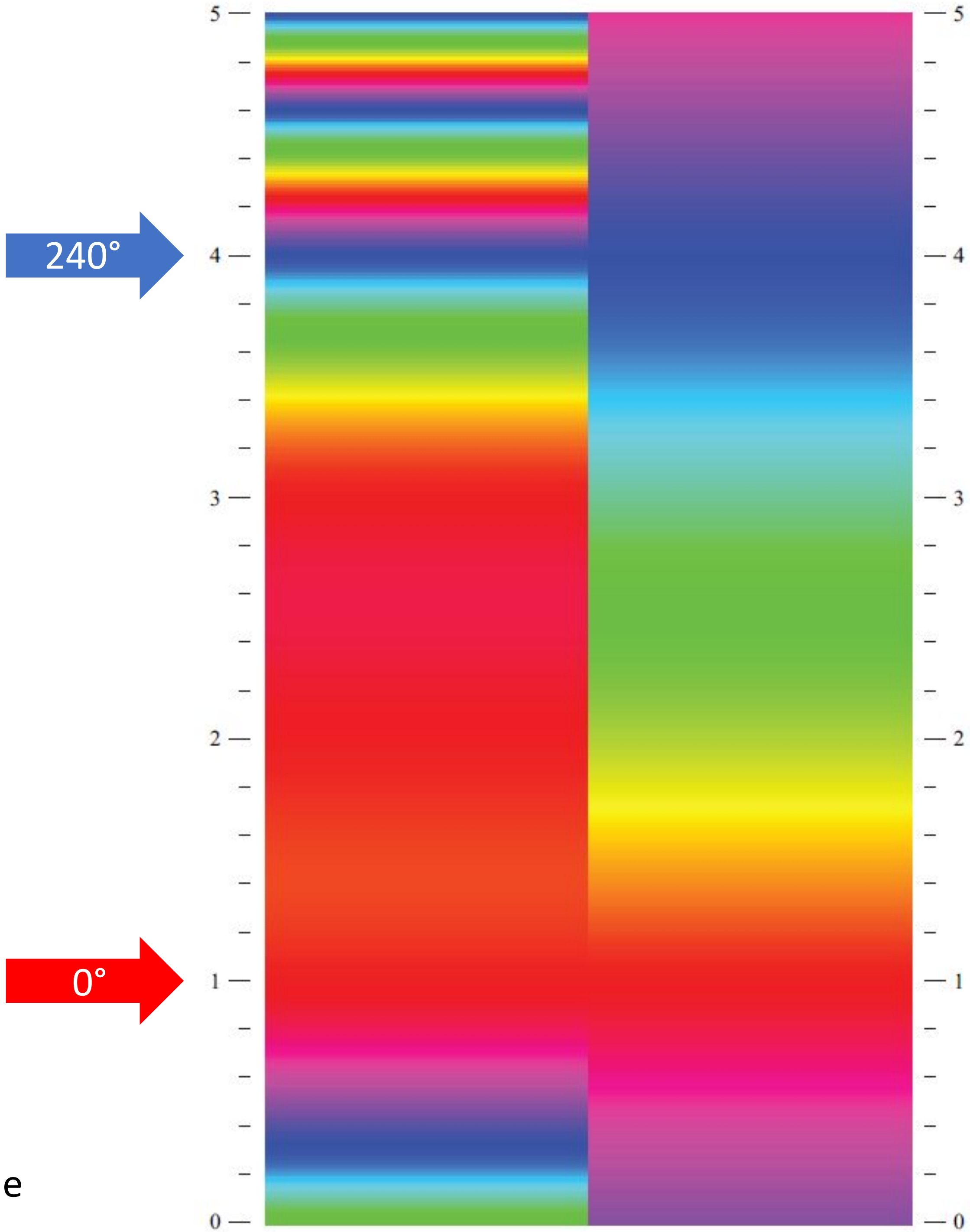
- The two spectra represent both sides of the equation

$$x^3 - 6x^2 + 11x - 6 = 2x - 2$$

- At height x the hue on the left is proportional to the left side of the equation and the hue on the right to the right side of the equation.
- The red band (0°) at x = 1 and the blue band (240°) at x = 4 are the two solutions.



1 unit = 80° in hue



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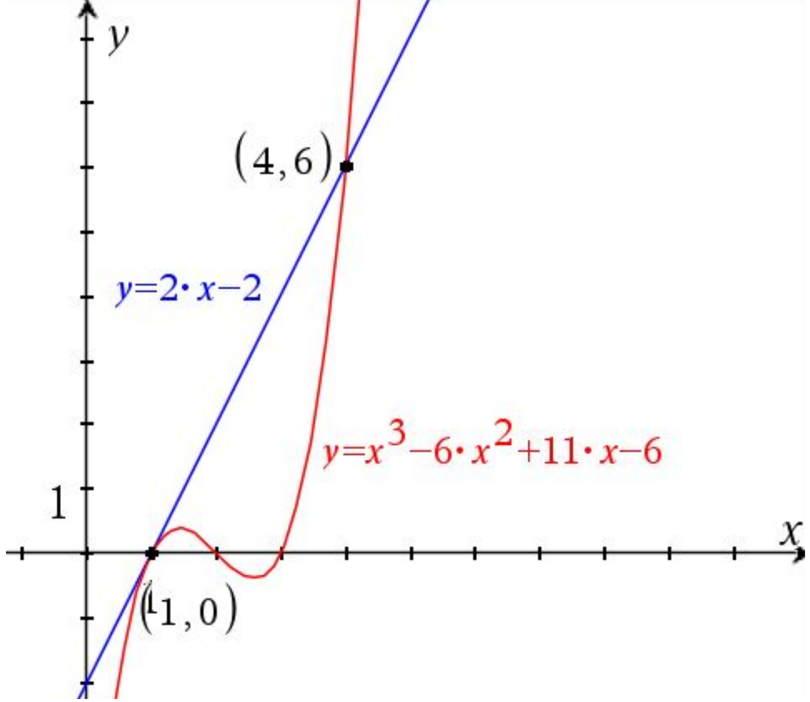
Auditory


♩ = 52
Linear
Function

♩ = 52
Cubic Function

♩ = 52
Cubic Function
Linear Function


y
(4, 6)
y=2·x−2
y=x^3−6·x^2+11·x−6
1
x
(1, 0)


= 52
Cubic Function
Linear Function

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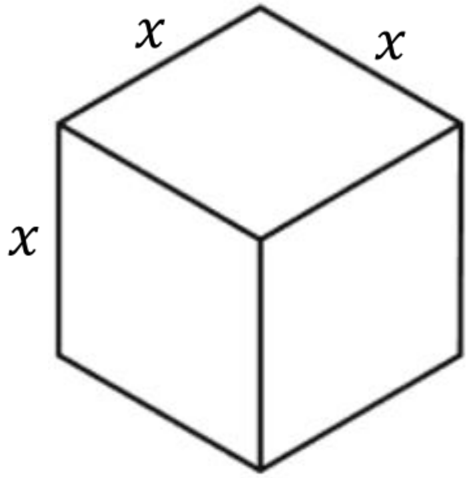
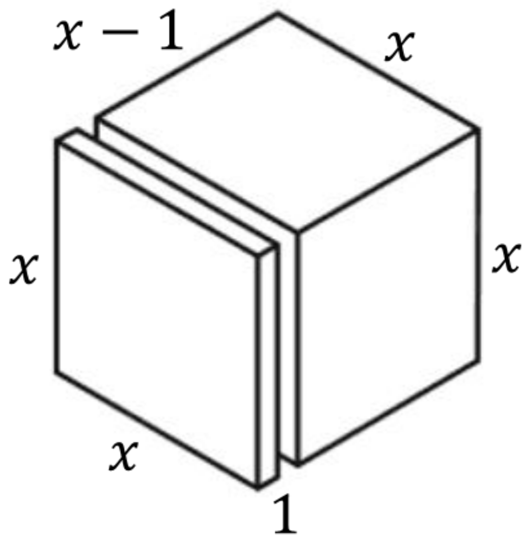
Wordless

$$x^3 - 6x^2 + 9x - 4$$

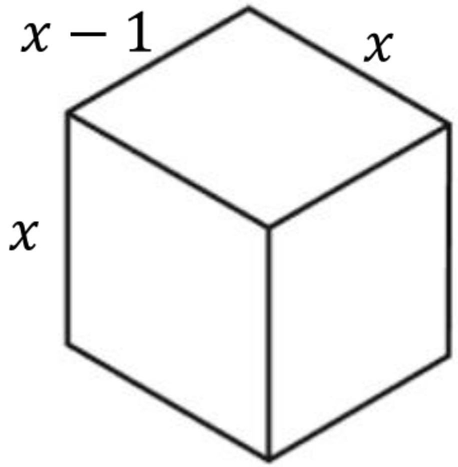
x^3



$(x^3) - x^2$

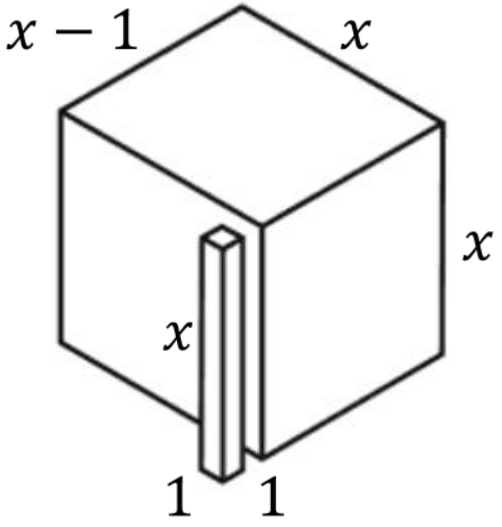


$x^3 - x^2$

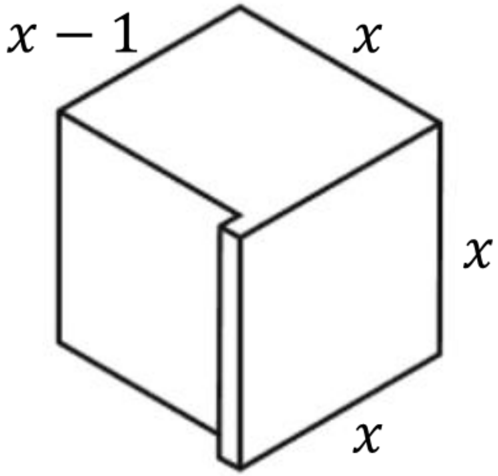


$$x^3 - 6x^2 + 9x - 4$$

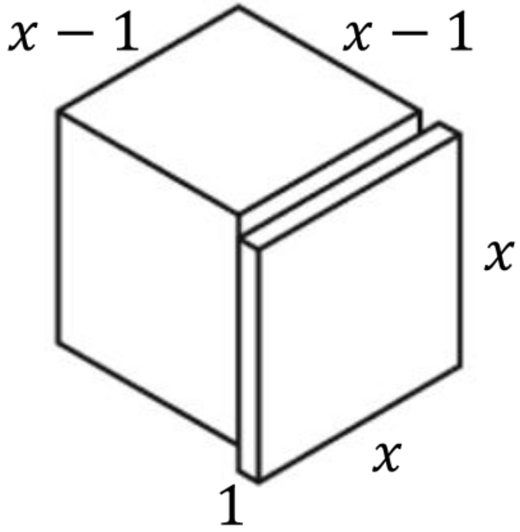
$(x^3-x^2)+x$



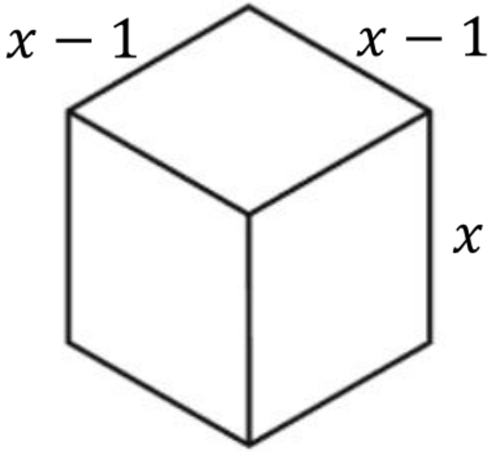
x^3-x^2+x



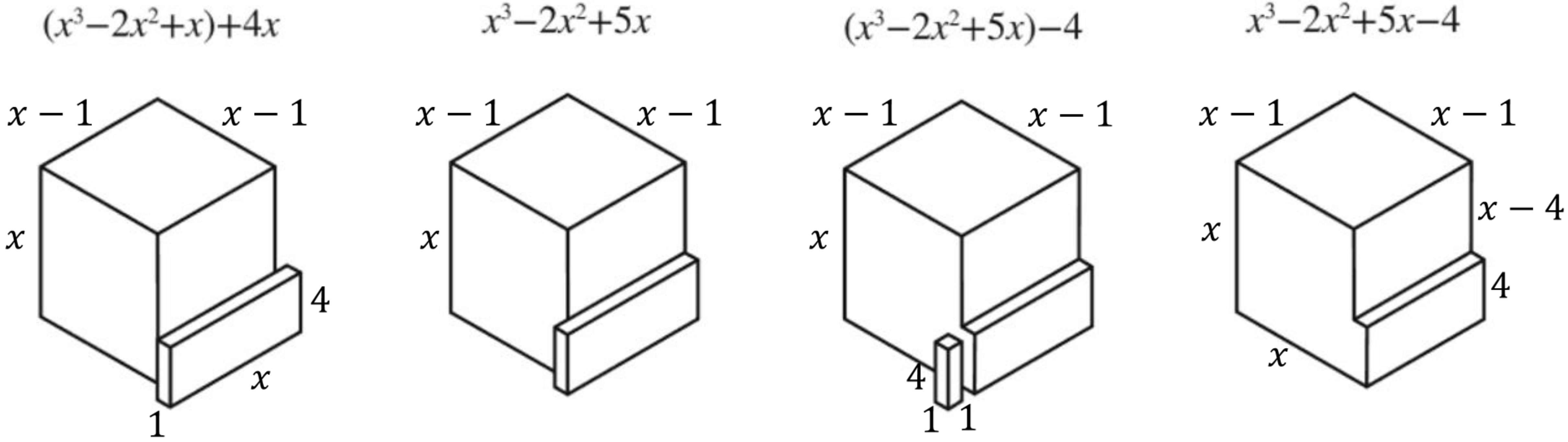
$(x^3-x^2+x)-x^2$



x^3-2x^2+x

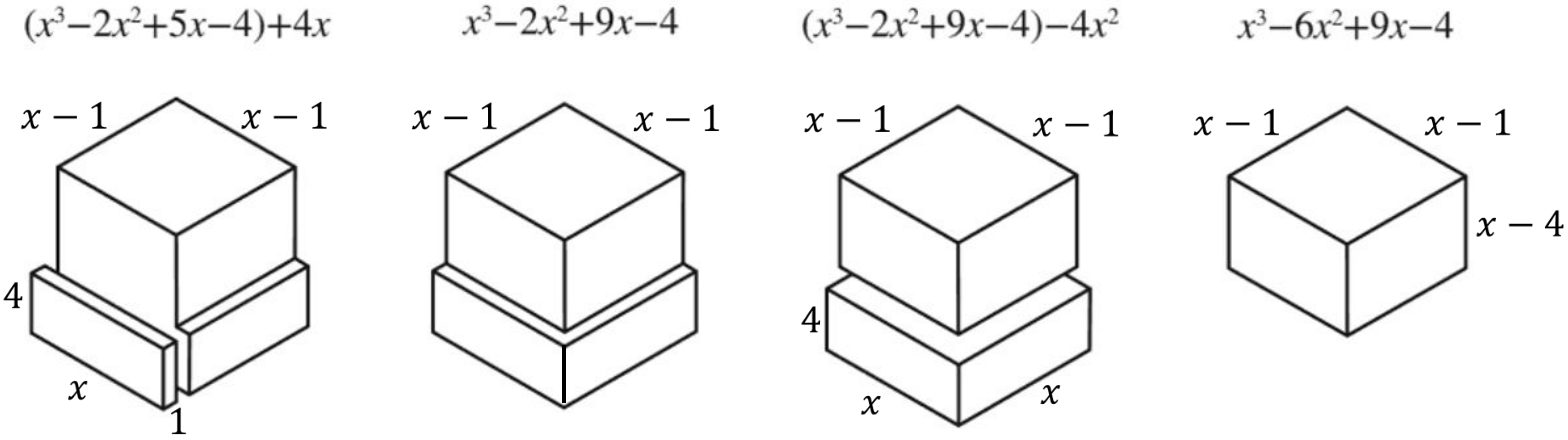


$$x^3 - 6x^2 + 9x - 4$$



(x³−2x²+x)+4x
x³−2x²+5x
(x³−2x²+5x)−4
x³−2x²+5x−4
x − 1
x − 1
x
4
x
1
x − 1
x − 1
x
x − 1
x − 1
x
4
1 1
x − 1
x − 1
x
x − 4
4
x

$$x^3 - 6x^2 + 9x - 4$$



(x³−2x²+5x−4)+4x
x − 1
x − 1
4
x
1
x³−2x²+9x−4
x − 1
x − 1
(x³−2x²+9x−4)−4x²
x − 1
x − 1
4
x
x
x³−6x²+9x−4
x − 1
x − 1
x − 4

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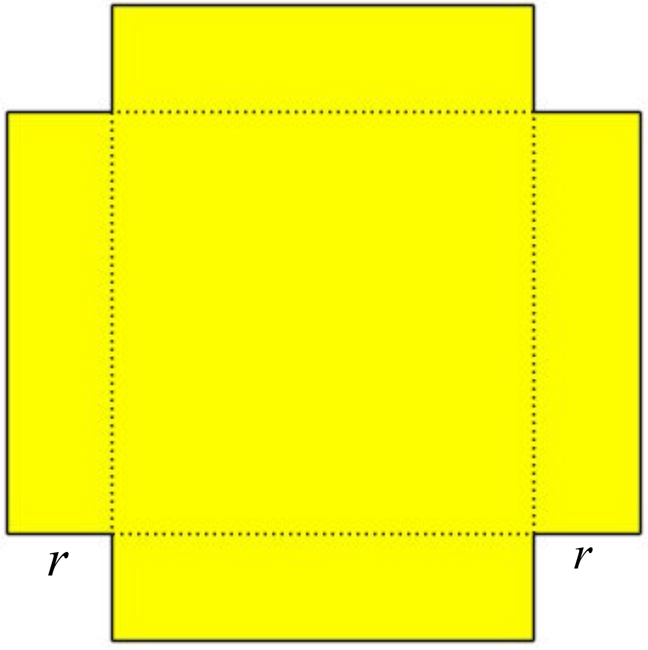
Word Problem

We want to make a box from a 6”× 6” piece of paper by cutting out four equal squares from the corners and folding up the resulting flaps. If the volume of the box is to be 16 in^3, how many inches should we cut out of each corner?



Proof:

Let r be the number of inches cut from each corner. Since the volume = length × width × height, and we want the volume to be 16 in^3, we need to solve the equation:

$$(6-2r)(6-2r)r = 16$$

$$2(3-r)2(3-r)r = 16$$

$$(3-r)(3-r)r = 4$$

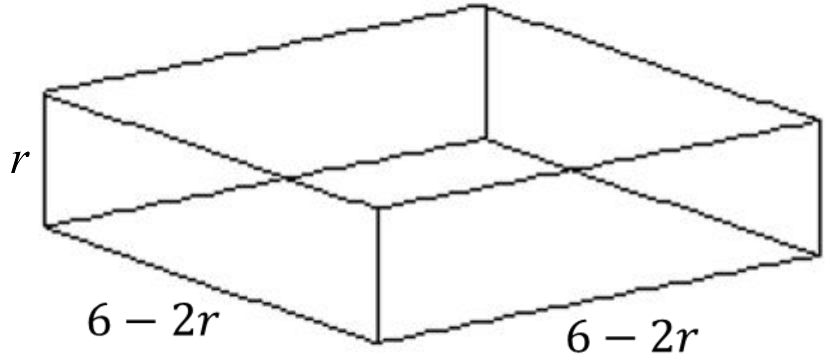
$$(9-6r+r^2)r = 4$$

$$r^3-6r^2+9r = 4$$

$$r^3-6r^2+9r-4 = 0$$

$$(r-1)(r-1)(r-4) = 0$$

$$r = 1, r \neq 4$$



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Student Exam

Instructions: Write your solution on this page. Clearly show all appropriate algebraic work. The use of any communications device is strictly prohibited.

Solve: $x^3 - 6x^2 + 11x - 6 = 2x - 2$

Please refer to my scratch paper for the solution.

$$x^3 - 6x^2 + 11x - 6 = 2x - 2$$

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Reductio ad Absurdum

Theorem. *If* $x^3 - 6x^2 + 11x - 6 = 2x - 2$, *then* $x = 1$ *or* $x = 4$.

Proof:

Suppose there is a third solution x such that $x \neq 1$ *and* $x \neq 4$. This means that $x - 1 \neq 0$ and $x - 4 \neq 0$.

$$x^3 - 6x^2 + 11x - 6 = 2x - 2$$

$$(x - 1)(x^2 - 5x + 6) = 2(x - 1)$$

$$x^2 - 5x + 6 = 2$$

$$x^2 - 5x + 4 = 0$$

$$(x - 1)(x - 4) = 0$$

$$x - 1 = 0$$

$$1 = 0$$

Based on our assumption that there was a third solution, we reached a conclusion which is absurd.

So our assumption that there was a third solution other than 1 and 4 must be false. Hence $x = 1$ *or* $x = 4$.

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Contrapositive

A *conditional* statement and its *contrapositive* are logically equivalent.

i.e. $p \to q \equiv {\sim}q \to {\sim}p$

or (*if p, then q* $\equiv$ *if* ${\sim}q$*, then* ${\sim}p$)

Theorem: *If* $x^3 - 6x^2 + 11x - 6 = 2x - 2$, *then* $x = 1$ *or* $x = 4$.

Contrapositive: *If* $x - 1 \neq 0$ and $x - 4 \neq 0$, *then* $x^3 - 6x^2 + 11x - 6 \neq 2x - 2$

Proof:

Assume the hypothesis of the contrapositive: $x - 1 \neq 0$ and $x - 4 \neq 0$

Then

$$(x-1)(x-1)(x-4) \neq 0$$

$$x^3 - 6x^2 + 9x - 4 \neq 0$$

$$x^3 - 6x^2 + (11x - 2x) - (6 - 2) \neq 0$$

$$(x^3 - 6x^2 + 11x - 6) - (2x - 2) \neq 0$$

$$x^3 - 6x^2 + 11x - 6 \neq 2x - 2$$

Based on our assumption, we reached the conclusion stated in the contrapositive, which proves that the contrapositive statement is true, which also means that it’s logically equivalent statement, the original conditional statement must be true, so the roots must be 1 and 4.

2
Two-Column

Prove: The function $x^3 - 6x^2 + 11x - 6 = 2x - 2$

where *x* is a real number, has the solutions *x* = 1 or 4

Statement	Reason
1. $x^3 - 6x^2 + 11x - 6 = 2x - 2$	Given
2. $x^3 - 6x^2 + 9x - 4 = 0$	Addition Property
3. $x^3 - (1+5)x^2 + (5+4)x - 4 = 0$	Addition
4. $x^3 - x^2 - 5x^2 + 5x + 4x - 4 = 0$	Distributive Property
5. $x^2(x-1) - 5x(x-1) + 4(x-1) = 0$	Factor (Distributive Property)
6. $(x^2 - 5x + 4)(x-1) = 0$	Distributive Property
7. $[(x-4)(x-1)](x-1) = 0$	Factor
8. $x = 1 \; or \; x = 4$	QED

4
Elementary

Proposition: If *x* is a real number and $x^3 - 6x^2 + 11x - 6 = 2x - 2$ then *x* = 1 or x = 4

$$x^3 - 6x^2 + 11x - 6 = 2x - 2$$

$$x^3 - 6x^2 + 9x - 4 = 0$$

$$(x^3 - 6x^2 + 5x) + (4x - 4) = 0$$

$$(x^2 - 5x)(x - 1) + 4(x - 1) = 0$$

$$(x^2 - 5x + 4)(x - 1) = 0$$

$$[(x - 4)(x - 1)](x - 1) = 0$$

Thus *x* = 1 or *x* = 4

5
Puzzle

Suppose that among four consecutive integers, the product of the first three equals twice the third. What's the fourth integer?

Let the 4th integer = x.

Then the 1st integer = $x-3$, the 2nd integer = $x-2$, and the 3rd integer = $x-1$

$$(x-3)(x-2)(x-1)=2(x-1)$$

$$(x-3)(x-2)(x-1)-2(x-1)=0$$

$$(x-1)[(x-3)(x-2)-2]=0$$

$$(x-1)(x^2-5x+6-2)=0$$

$$(x-1)(x^2-5x+4)=0$$

$$(x-1)\ (x-1)(x-4)=0$$

$$x=1\ or\ x=4$$

If the 4th integer is 1, then the four consecutive integers are -2, -1, 0, 1, and $(-2)(-1)(0)=2(0)$.✓

If the 4th integer is 4, then the four consecutive integers are 1, 2, 3, 4, and $(1)(2)(3)=2(3)$.✓

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Formulaic

$$x^3-6x^2+11x-6=2x-2$$
$$x^3-6x^2+9x-4=0$$

The root of a cubic polynomial of the form ax^3+bx^2+cx+d is given by Cardano's formula:

$$x=\frac{-b}{3a}+\sqrt[3]{\left(\frac{-b^3}{27a^3}+\frac{bc}{6a^2}-\frac{d}{2a}\right)+\sqrt{\left(\frac{-b^3}{27a^3}+\frac{bc}{6a^2}-\frac{d}{2a}\right)^2+\left(\frac{c}{3a}-\frac{b^2}{9a^2}\right)^3}}$$
$$+\sqrt[3]{\left(\frac{-b^3}{27a^3}+\frac{bc}{6a^2}-\frac{d}{2a}\right)-\sqrt{\left(\frac{-b^3}{27a^3}+\frac{bc}{6a^2}-\frac{d}{2a}\right)^2+\left(\frac{c}{3a}-\frac{b^2}{9a^2}\right)^3}}$$

$$x=\frac{-(-6)}{3(1)}+\sqrt[3]{\left(\frac{-(-6)^3}{27(1)^3}+\frac{(-6)(9)}{6(1)^2}-\frac{(-4)}{2(1)}\right)+\sqrt{\left(\frac{-(-6)^3}{27(1)^3}+\frac{(-6)(9)}{6(1)^2}-\frac{(-4)}{2(1)}\right)^2+\left(\frac{(9)}{3(1)}-\frac{(-6)^2}{9(1)^2}\right)^3}}$$
$$+\sqrt[3]{\left(\frac{-(-6)^3}{27(1)^3}+\frac{(-6)(9)}{6(1)^2}-\frac{(-4)}{2(1)}\right)-\sqrt{\left(\frac{-(-6)^3}{27(1)^3}+\frac{(-6)(9)}{6(1)^2}-\frac{(-4)}{2(1)}\right)^2+\left(\frac{(9)}{3(1)}-\frac{(-6)^2}{9(1)^2}\right)^3}}$$
$$=2+\sqrt[3]{1}+\sqrt[3]{1}$$
$$=2+1+1$$
$$=4$$

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Counterexample

Cardano's formula returns the following equation:

$$x = 2 + \sqrt[3]{1} + \sqrt[3]{1}$$

However, 1 is not the only number whose cube equals 1. There are additional solutions resulting from different *complex* cube roots. The other two complex cube roots of 1 are

$$\frac{-1 + i\sqrt{3}}{2} \; and \; \frac{-1 - i\sqrt{3}}{2}$$

Combining these two complex cube roots of unity yields a second solution

$$x = 2 + \sqrt[3]{1} + \sqrt[3]{1} = 2 + \frac{-1 + i\sqrt{3}}{2} + \frac{-1 - i\sqrt{3}}{2} = 1$$

And a third solution

$$x = 2 + \sqrt[3]{1} + \sqrt[3]{1} = 2 + \frac{-1 - i\sqrt{3}}{2} + \frac{-1 + i\sqrt{3}}{2} = 1$$

Here is a complete formula for each root x_n of the general cubic equation, where $n = 0{,}1{,}2$:

$$x_n = \frac{-b}{3a} + \left(\frac{-1 + i\sqrt{3}}{2}\right)^n \sqrt[3]{\left(\frac{-b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right) + \sqrt{\left(\frac{-b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right)^2 + \left(\frac{c}{3a} - \frac{b^2}{9a^2}\right)^3}}$$

$$+ \left(\frac{-1 + i\sqrt{3}}{2}\right)^{2n} \sqrt[3]{\left(\frac{-b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right) - \sqrt{\left(\frac{-b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right)^2 + \left(\frac{c}{3a} - \frac{b^2}{9a^2}\right)^3}}$$

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Symmetry

Theorem. *If* $x^3 - 6x^2 + 11x - 6 = 2x - 2$, *then* $x = 1$ *or* $x = 4$.

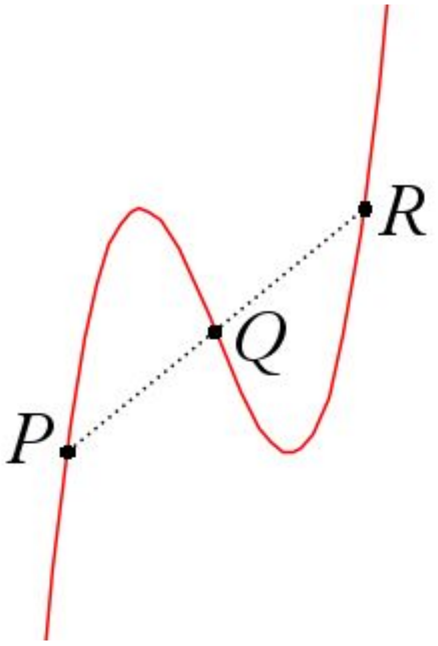
Proof:

$$x^3 - 6x^2 + 11x - 6 = 2x - 2$$

$$x^3 - 6x^2 + 9x - 4 = 0$$

The solutions of the equation are the roots of the cubic curve $y = x^3 - 6x^2 + 9x - 4$.

Every cubic curve is symmetric about its point of inflection Q, so if P is on the curve, so is its reflection R thru Q.



$$\frac{P + R}{2} = Q$$

$$P + R = 2Q$$

$$R = 2Q - P$$

A root P of the cubic curve may be realized as the reflection $R = 2Q - P$.

The point of inflection occurs where the second derivative is equal to zero.

$$y = x^3 - 6x^2 + 9x - 4$$

$$y' = 3x^2 - 12x + 9$$

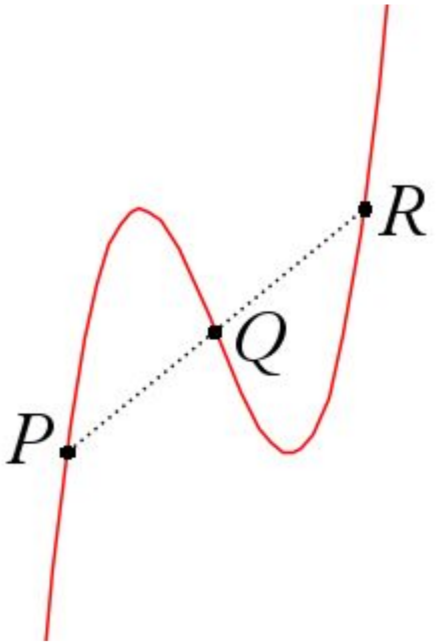
$$y'' = 6x - 12$$

$$6x - 12 = 0$$

$$x = 2$$

$$y = (2)^3 - 6(2)^2 + 9(2) - 4 = -2$$

Therefore, the point of inflection is $Q(2, -2)$.



The root $P(x, 0)$ reflected through $Q(2, -2)$ yields $R = 2Q - P$.

$$R = 2(2, -2) - (x, 0)$$

$$R = (4, -4) - (x, 0)$$

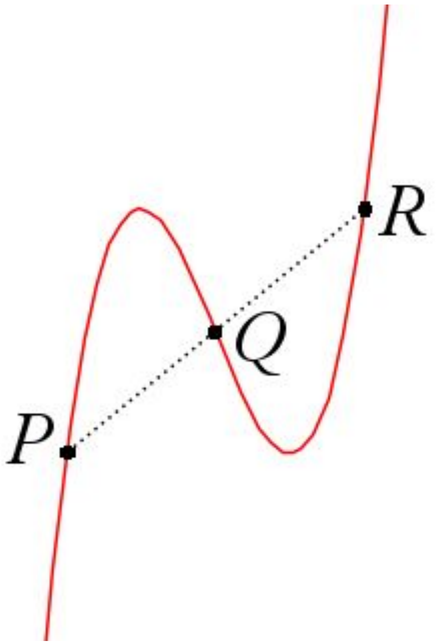
$$R = (4 - x, -4)$$

$$R = (4 - x, -4)$$

$$x^3 - 6x^2 + 9x - 4 = y$$

$$(4 - x)^3 - 6(4 - x)^2 + 9(4 - x) - 4 = -4$$

Make the substitution $4 - x = r$ for the x-coordinate of R in the cubic equation:



$$r^3 - 6r^2 + 9r - 4 = -4$$

$$r^3 - 6r^2 + 9r = 0$$

$$r(r^2 - 6r + 9) = 0$$

$$r(r - 3)^2 = 0$$

$$r = 0 \; or \; r = 3$$

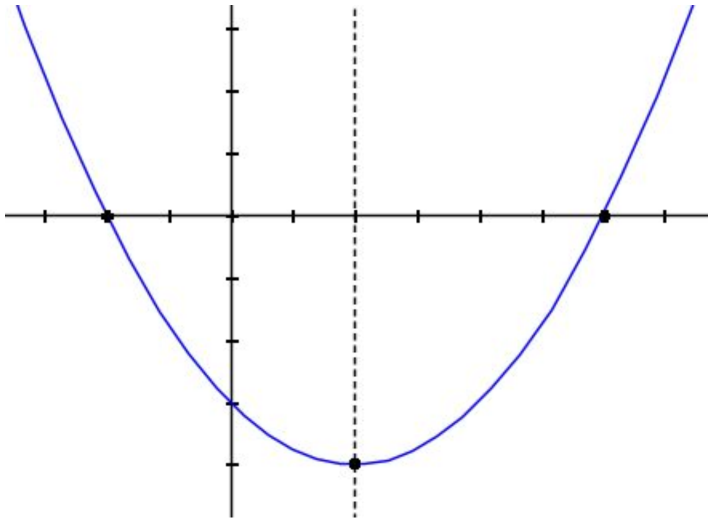
But $4 - x = r$

$$4 - x = 0 \; or \; 4 - x = 3$$

$$x = 4 \; or \; x = 1$$

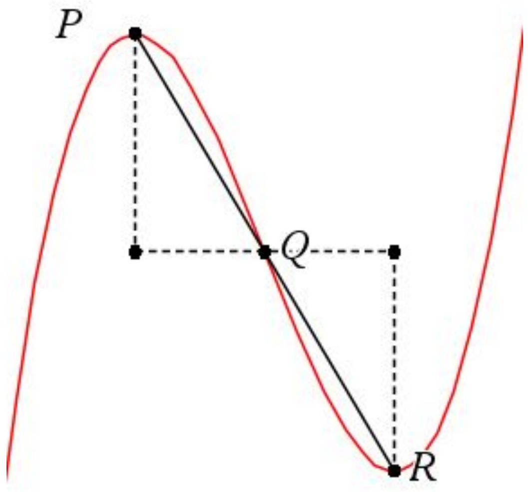
So by *symmetry* the roots of the cubic curve are $P(4,0)$ or $P(1,0)$

$$y = ax^2 + bx + c$$
$$y' = 2ax + b$$
$$2ax + b = 0$$
$$x = \frac{-b}{2a}$$
$$ax^2 + bx + c = 0$$
$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$



The line of symmetry of a parabola goes through the vertex, and the zeros are equally spaced on either side of the axis of symmetry.

$$y = ax^3 + bx^2 + cx + d$$
$$y' = 3ax^2 + 2bx + c$$
$$y'' = 6ax + 2b$$
$$6ax + 2b = 0$$
$$x = \frac{-b}{3a}$$
$$3ax^2 + 2bx + c = 0$$
$$x = \frac{-b}{3a} \pm \frac{\sqrt{b^2 - 3ac}}{3a}$$



The minimum and maximum values of a cubic function are equally spaced on either side of the point of inflection.

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Rational Zeros Theorem

Rational Zeros Theorem

Demonstration

Slide Rule

Solve $x^3 - 6x^2 + 11x - 6 = 2x - 2$

Write in standard form: $x^3 - 6x^2 + 9x - 4 = 0$

Substitute $y + 2$ for x to get

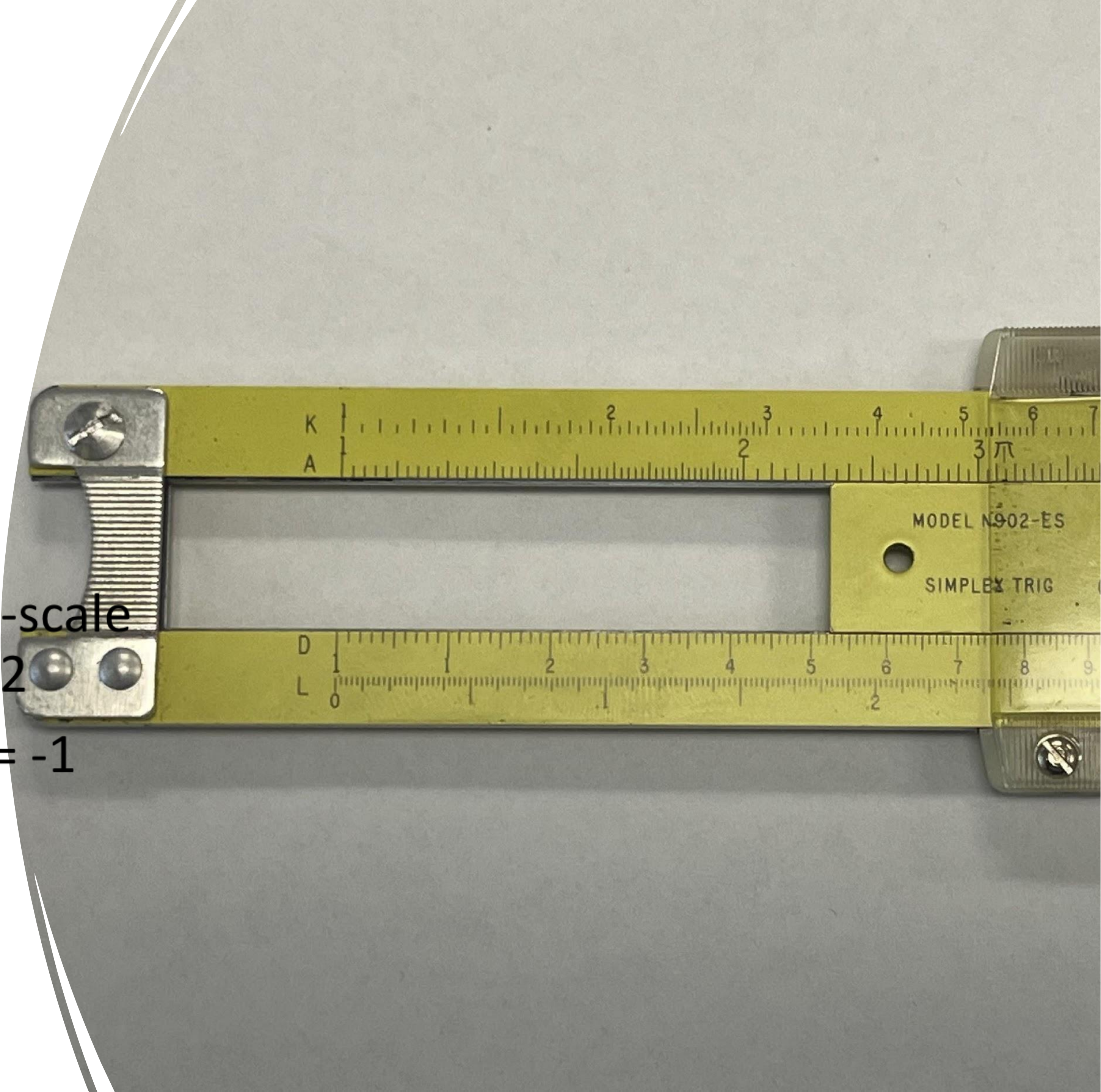
$$(y+2)^3 - 6(y+2)^2 + 9(y+2) - 4 = 0$$
$$y^3 + 6y^2 + 12y + 8 - 6y^2 - 24y - 24 + 9y + 18 - 4 = 0$$
$$y^3 - 3y - 2 = 0$$

Slide Rule

$y^3 - 3y - 2 = 0$

For roots y_1, y_2, and y_3

- The sum of the roots is 0
- The product of the roots is 2
- Difference of the A-scale and C-scale of 3 yields D-scale of 2, so $y_1 = 2$
- Then $y_2 y_3 = 1$, so $y_2 = -1$ and $y_3 = -1$



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Matrices

Matrices

We wish to show that if **λ** is an eigenvalue of the linear operator of the matrix A

$$\begin{bmatrix} 3 & 0 & 1 \\ 0 & 1 & 0 \\ 2 & 0 & 2 \end{bmatrix}$$

then **λ**=1 or **λ**=4 .

We use the matrix since the Characteristic Polynomial can be found on the Nspire as

$$\text{charPoly}\left(\begin{bmatrix} 3 & 0 & 1 \\ 0 & 1 & 0 \\ 2 & 0 & 2 \end{bmatrix}, x\right) \quad -x^3+6\cdot x^2-9\cdot x+4$$

Matrices (continued)

- A number λ is an eigenvalue of the matrix **A** if there is a non-zero vector **x** such that $\mathbf{A}\mathbf{x} = \lambda\mathbf{x}$
- This implies the homogeneous set of simultaneous equations $(\mathbf{A} - \lambda I)\, x = 0$, which have a nontrivial solution when the determinant $(\mathbf{A} - \lambda I)$ vanishes.
- Expanding the determinant of $(\mathbf{A} - \lambda I)$ gives:
- The roots of this equation are $\lambda_1 = 1$, $\lambda_2 = 1$, $\lambda_3 = 4$

$$0 = \begin{vmatrix} 3-\lambda & 0 & 1 \\ 0 & 1-\lambda & 0 \\ 2 & 0 & 2-\lambda \end{vmatrix}$$

$$= (3-\lambda)(1-\lambda)(2-\lambda) - 2(1-\lambda)$$

$$= (1-\lambda)^2(4-\lambda)$$

Matrices (continued)

- The eigenvalues can also be shown on the Nspire calculator:

$$\operatorname{eigVl}\left(\begin{bmatrix} 3 & 0 & 1 \\ 0 & 1 & 0 \\ 2 & 0 & 2 \end{bmatrix}\right) \qquad \{4.,1.,1.\}$$

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Calculus

- Define $\Re \to \Re$ *to be the function* $f(x) = x^3 - 6x^2 + 9x - 4.$

If $f(x) = 0$, *then* $x = 1$ *or* $x = 4.$

Construct the Taylor series of *f* about the first of the two purported roots, $x = 1$.

Derivatives of *f*:

$$f'(x) = 3x^2 - 12x + 9$$

$$f''(x) = 6x - 12$$

$$f'''(x) = 6$$

$$f^{(n)} = 0 \; for \; n \geq 4$$

- $$f(x) = f(1) + \frac{f'(1)}{1!}(x-1) + \frac{f''(1)}{2!}(x-1)^2 + \frac{f'''(1)}{3!}(x-1)^3 + \cdots$$
$$= 0 + 0 - 3(x-1)^2 + (x-1)^3 + 0$$
$$= (x-1)^2(-3 + x - 1)$$
$$= (x-1)^2(x-4)$$

Since $f(x) = 0$, the roots of f are 1 and 4.

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Geometric

Theorem. *If* $x^3 - 6x^2 + 11x - 6 = 2x - 2$, *then* $x = 1$ *or* $x = 4$.

Proof: $$x^3 - 6x^2 + 9x - 4 = 0$$

Since $\Delta PBQ \sim \Delta OAP$

$$\frac{QB}{6-x} = \frac{x}{1}$$

$$QB = x(6-x)$$

Since $\Delta OAP \sim \Delta QCR$

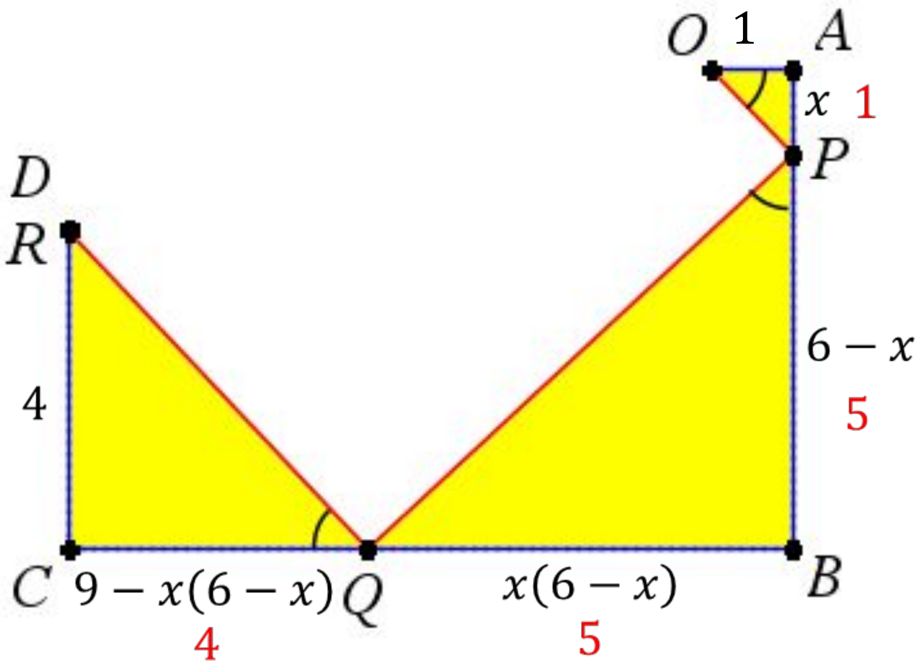
$$\frac{x}{1} = \frac{4}{9 - x(6-x)}$$

$$x\big(9 - x(6-x)\big) = 4$$

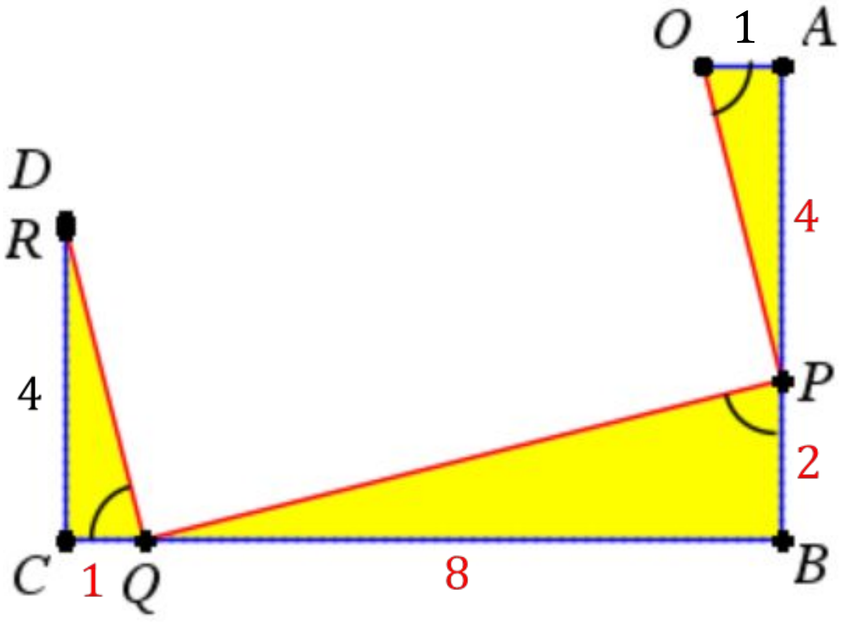
$$-4 + x\big(9 - x(6-x)\big) = 0$$

Let

$$p(x) = -4 + x\big(9 - x(6-x)\big)$$

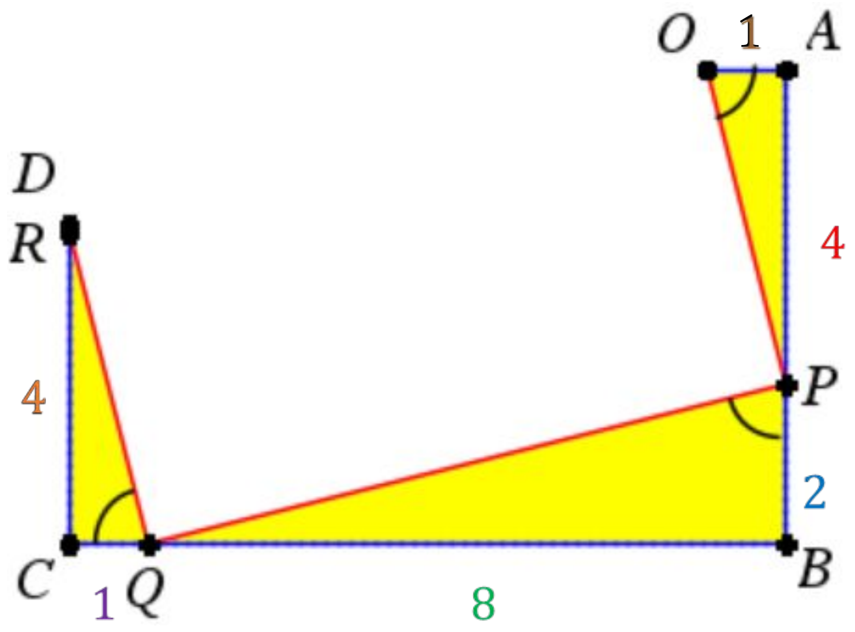


$$p(1) = -4 + 1\big(9 - 1(6-1)\big)$$
$$p(1) = -4 + 1\big(9 - 1(5)\big)$$
$$p(1) = -4 + 1(9 - 5)$$
$$p(1) = -4 + 1(4)$$
$$p(1) = -4 + 4$$
$$p(1) = 0$$



$$p(4) = -4 + 4\big(9 - 4(6-4)\big)$$
$$p(4) = -4 + 4\big(9 - 4(2)\big)$$
$$p(4) = -4 + 4(9 - 8)$$
$$p(4) = -4 + 4(1)$$
$$p(4) = -4 + 4$$
$$p(4) = 0$$

Comments on Lill's Method



$$p(x) = -4 + x\big(9 - x(6 - x)\big)$$
$$p(x) = -4 + x\big(9 - x(-x + 6)\big)$$
$$p(x) = -4 + x\big(9 + x(x - 6)\big)$$
$$\boxed{p(x) = \big((x - 6)x + 9\big)x - 4}$$

Horner's Form

Horner's Method

$$p(x) = \big((1x - 6)x + 9\big)x - 4$$
$$p(4) = \big((1 \cdot 4 - 6)4 + 9\big)4 - 4$$
$$p(4) = \big((4 - 6)4 + 9\big)4 - 4$$
$$p(4) = \big((-2)4 + 9\big)4 - 4$$
$$p(4) = (-8 + 9)4 - 4$$
$$p(4) = (1)4 - 4$$
$$p(4) = 4 - 4$$
$$p(4) = 0$$

Synthetic Division

$$\frac{x^3 - 6x^2 + 9x - 4}{x - 4}$$

$$\begin{array}{r|rrrr} 4 & 1 & -6 & 9 & -4 \\ & & 4 & -8 & 4 \\ \hline & 1 & -2 & 1 & \boxed{0} \end{array}$$

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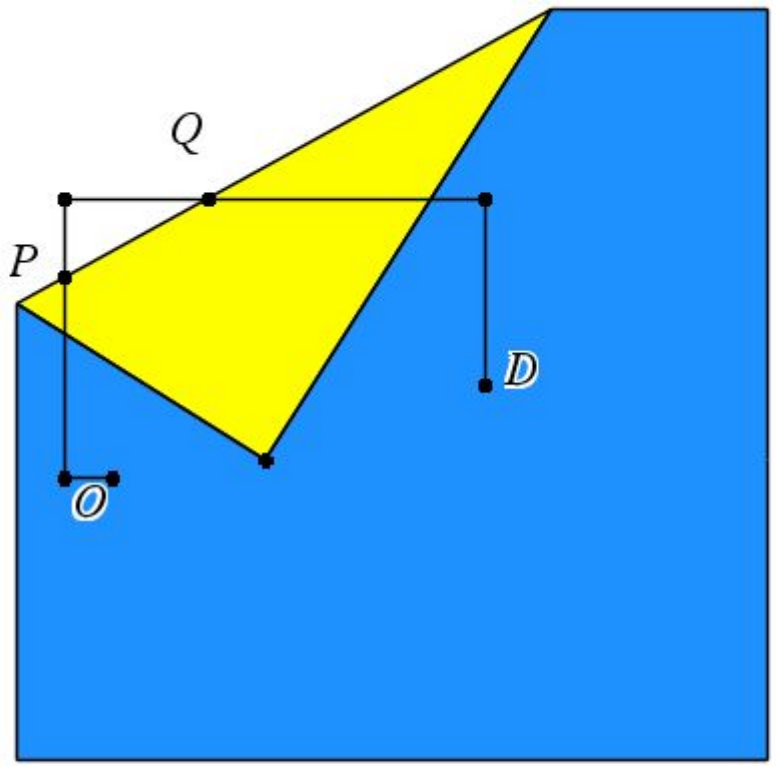
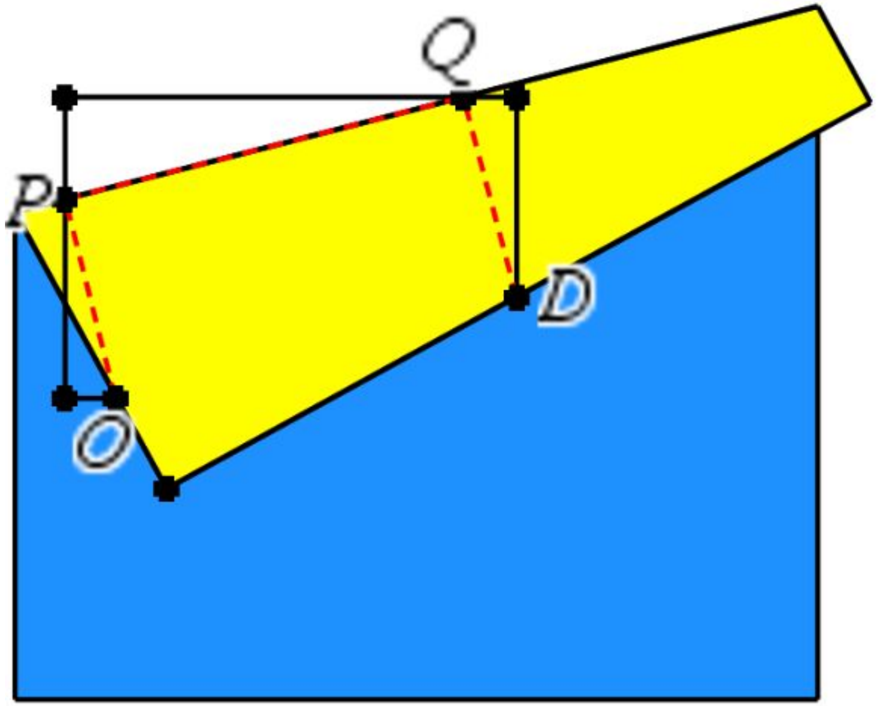
Origami

Beloch Fold



A single fold which places two distinct points onto two distinct lines simultaneously (or vice-versa).

$\overleftrightarrow{PQ}$ is a Beloch Fold



$\overleftrightarrow{PQ}$ creates a right-angled path from O to D.

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Authority

Authority

If $x^3 - 6x^2 + 11x - 6 = 2x - 2$, then it follows from Euler that the real number in question must be 1 or 4.

How do we recognize mathematical truth? If a theorem has a short complete proof, we can check it. But if the proof is deep, difficult, and already fills 100 journal pages, if no one has the time and energy to fill in the details, if a "complete" proof would be 100,000 pages long, then we rely on the judgements of the bosses in the field. In mathematics, a theorem is true, or it is not a theorem. But even in mathematics, truth can be political. (from *Notices of the AMS,* Melvyn Nathanson.)

When the authority of reference is oneself, a proof by authority becomes a proof by *intimidation.*