A magical mystery tour of fascinating sequences

Live Webinar for Teachers Teaching with Technologies
March 28, 2023

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Description

In this session, our presenter will lead participants on an excursion into a world beyond arithmetic and geometric sequences where the patterns are full of mystery and intrigue.

The rules for these sequences are simple and easy to state, yet the patterns that emerge are full of surprises, moments of joy and baffling results.

These sequences can be used to provide students with an opportunity to practice basic skills, pose questions, make conjectures, test them using coding and prove them using mathematics. These sequences are well-suited for class warmups and they can be weaved into many lessons.

1

The Borisoff Sequence

The Borisoff Sequence

31,415,926,535,897,932,384,626,433,832,795,028,841,971,693,993,751

The Borisoff Sequence

31,415,926,535,897,932,384,626,433,832,795,028,841,971,693,993,751

The Borisoff Sequence

31415926535897932384626433832795028841971693993751

18 even numbers, 32 odd numbers, 50 numbers
183250

The Borisoff Sequence

31415926535897932384626433832795028841971693993751

18 even numbers, 32 odd numbers, 50 numbers

183250

3 even numbers, 3 odd numbers, 6 numbers

336

The Borisoff Sequence

31415926535897932384626433832795028841971693993751

18 even numbers, 32 odd numbers, 50 numbers
183250
3 even numbers, 3 odd numbers, 6 numbers
336
1 even number, 2 odd numbers, 3 numbers
123
1 even number, 2 odd numbers, 3 numbers
123
123
123
...
You may be wondering why this is called the Borisoff Sequence...

Leonard Warren **Borisoff** (June 12, 1942–November 5, 2020) known professionally as **Len Barry**, was an American recording star, vocalist, songwriter, lyricist, record producer, author, and poet.

Born on June 12, 1942 and raised in Philadelphia, Barry had little thought of a show business career while still in school. Instead, he aspired to become a professional basketball player upon his graduation. It was not until he entered military service and had occasion to sing with the US Coast Guard band at Cape May, New Jersey, and was so encouraged by the response of his military audiences, that he decided to make music a career.

https://en.wikipedia.org/wiki/Len_Barry

1 - 2 - 3
(John Madara-David White-Leonard Borisoff)
Vocal With Chorus And Instrumental
Accompaniment Arranged By
JIMMY WISNER
DECCA
REG. U.S. PAT. OFF. MARCA REGISTRADA
M'F'R'D BY DECCA RECORDS INC. NEW YORK • U.S.A.
31827
(116,297)
Champion Music Corp.
And Double Diamond
Music (BMI)
(2:21)
LEN BARRY
PRODUCED BY MADARA - WHITE

The Sisyphus String

Caution: Black Holes at Work by Michael Ecker
New Scientist (Dec. 1992)

Mathemagical Black Holes by Michael Ecker
The Mathemagician and Pied Puzzler: A Collection in Tribute to Martin Gardner
Edited By Elwyn R. Berlekamp, Tom Rodgers
Published March 8, 1999 by A K Peters/CRC Press

Dr. Michael W. Ecker, Ph.D., a mathematician and computer hobbyist, is a retired Pennsylvania State University math professor and computer journalist. He was editor and publisher of **Recreational and Educational Computing** (REC) for 21 years.

https://dr-michael-ecker.weebly.com

Why does this example work, and why do most mathemagical black holes occur? My argument is to show that **large inputs** have **smaller outputs**, thus reducing an infinite universe to a manageable finite one. At that point, an argument by cases, or a computer check of the finitely many cases, suffices. In the case of the 123 hole, we can argue as follows: If $n > 999$, then $f(n) < n$. In other words, the new number that counts the digits is smaller than the original number. (It's intuitively obvious, but try induction if you would like rigor.) Thus, starting at 1000 or above eventually pulls one down to under 1000. For $n < 1000$, I've personally checked the iterates of $f(n)$ for $n = 1$ to 999 by a computer program such as the one above. The direct proof is actually faster and easier, as a three-digit string for a number must have one of these possibilities for (# even digits, # odd digits, total # digits):

(0, 3, 3)
(1, 2, 3)
(2, 1, 3)
(3, 0, 3)

The Sisyphus String
On-Line Encyclopedia of Integer Sequences
https://oeis.org/search?q=Michael+Ecker&sort=&language=&go=Search

A073053 Apply DENEAT operator (or the Sisyphus function) to n. +20 14

101, 11, 101, 11, 101, 11, 101, 11, 101, 11, 112, 22, 112, 22, 112, 22, 112, 22, 112, 22, 202, 112, 202, 112, 202, 112, 202, 112, 202, 112, 112, 22, 112, 22, 112, 22, 112, 22, 112, 22, 202, 112, 202, 112, 202, 112, 202, 112, 202, 112, 112, 22, 112, 22 (list; graph; refs; listen; history; text; internal format)

OFFSET 0,1

COMMENTS DENEAT(n): concatenate number of even digits in n, number of odd digits and total number of digits. E.g., 25 -> 1.1.2 = 112 (Digits: Even, Not Even, And Total). Leading zeros are then omitted.
This is also known as the Sisyphus function. - N. J. A. Sloane, Jun 25 2018
Repeated application of the DENEAT operator reduces all numbers to 123. This is easy to prove. Compare A073054, A100961. - N. J. A. Sloane Jun 18 2005

REFERENCES M. E. Coppenbarger, Iterations of a modified Sisyphus function, Fib. Q., 56 (No. 2, 2018), 130-141.
M. **Ecker**, Caution: Black Holes at Work, New Scientist (Dec. 1992)
M. J. Halm, Blackholing, Mpossibilities 69, (Jan 01 1999), p. 2.
J. Schram, The Sisyphus string, J. Rec. Math., 19:1 (1987), 43-44.
M. Zeger, Fatal attraction, Mathematics and Computer Education, 27:2 (1993), 118-123.

LINKS N. J. A. Sloane, Table of n, a(n) for n = 0..20000

EXAMPLE a(1) = 0.1.1 -> 11.
a(10000000000) = 10111 because 10000000000 has 10 even digits, 1 odd digit and 11 total digits
DENEAT(1356) -> "1" even (6), "3" odd (1,3,5), "4" total digits -> 134.
DENEAT(1490) -> "2" even (0 is considered even,4), "2" odd (1,9), "4" total digits -> 224. - Paolo P. Lava, Mar 05 2010

2

What's going on here? Suggest a **rule** that can be used to determine the four numbers in the next row. Use your rule to find the numbers in the next ten rows. What do you notice? What questions do you have?

11	5	42	33
6	37	9	22

What's going on here? Suggest a **rule** that can be used to determine the four numbers in the next row. Use your rule to find the numbers in the next ten rows. What do you notice? What questions do you have?

largest - smallest

11	5	42	33
6	37	9	22
11 – 5 = 6	**42 – 5 = 37**	**42 – 33 = 9**	**33 – 11 = 22**

What's going on here? Suggest a **rule** that can be used to determine the four numbers in the next row. Use your rule to find the numbers in the next ten rows. What do you notice? What questions do you have?

a	b	c	d
$\text{abs}(b-a)$	$\text{abs}(c-b)$	$\text{abs}(d-c)$	$\text{abs}(a-d)$

What do you notice? What questions do you have?

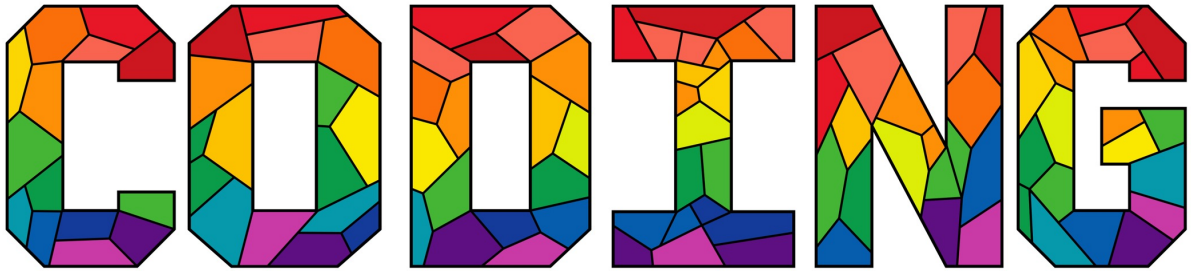
11	5	42	33
6	37	9	22
31	28	13	16
3	15	3	15
12	12	12	12
0	**0**	**0**	**0**
0	0	0	0
0	0	0	0
0	0	0	0

Some questions:

Will we **always** end up with a row of 0s, no matter what four numbers we start with? How would we **prove** this?

If it is true that we always obtain a row of four 0s, what is the **relationship** between the number of steps to reach the 0s and the four initial numbers?

What happens if we start with **five** numbers or **six** numbers or **seven** numbers and so on?



Investigation

For each of the following cases, how **many steps** does it take to reach a row of four 0s? How are these cases **related to each other** and what can we learn from the results of this investigation?

1	2	4	8

1	3	9	27

1	4	16	64

1	5	25	125

1	6	36	216

1	7	49	343

1, 2, 4, 8 takes **7** steps
1, 3, 9, 27 takes **6** steps
1, 4, 16, 64 takes **6** steps
1, 5, 25, 125 takes **6** steps
1, 6, 36, 216 takes **6** steps
1, 7, 49, 343 takes **6** steps

Hmmm...

1	2	4	8

1	3	9	27

1	4	16	64

1	5	25	125

1	6	36	216

1	7	49	343

Let x be a positive integer greater than or equal to 2. For the four numbers given below, determine the number of steps that it takes to reach a row of four 0's as a function of x.

1	x	x^2	x^3

Let x be a positive integer greater than or equal to 2. For the four numbers given below, determine the number of steps that it takes to reach a row of four 0's as a function of x.

1	x	x^2	x^3
$x - 1$	$x^2 - 1$	$x^3 - x^2$	$x^3 - 1$

Starting with the four numbers given below, how many steps does it take to reach a row of four 0s?

$$\frac{11}{15} \qquad \frac{13}{30} \qquad \frac{7}{12} \qquad \frac{2}{55}$$

Starting with the four numbers given below, how many steps does it take to reach a row of four 0s?

$$\frac{11}{15} \qquad \frac{13}{30} \qquad \frac{7}{12} \qquad \frac{2}{55}$$

least common multiple of 15, 30, 12 and 55 is equal to 660.

$$484 \qquad 286 \qquad 385 \qquad 24$$

Starting with the four numbers given below, how many steps does it take to reach a row of four 0s?

$$\sqrt{2} \qquad log_{10}2 \qquad e \qquad \pi$$

Starting with the four numbers given below, how many steps does it take to reach a row of four 0s?

$$3 + 2i, i, 2 - i, 7i$$

COMPLEX
HOTEL

GRANGE ACADEMY MATHEMATICS DEPARTMENT NEWSLETTER

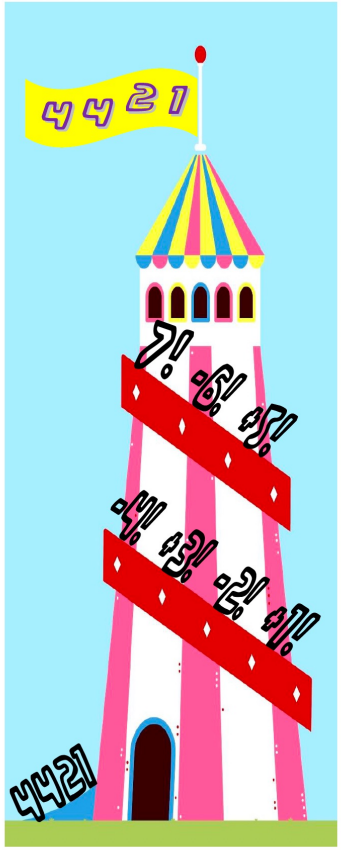
ISSUE 590
1/3/2022

4421 subscribers and counting...

@aap03102

WELCOME

Another few subscribers since last week so we're now at 4421 which I'm sure you already know is equal to this nifty factorial countdown:



THOUGHT OF THE WEEK

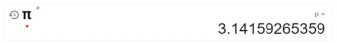
TECHNICALLY AREN'T ALL NATIONAL ANTHEMS COUNTRY MUSIC?

PI MEMORISING PRACTICE?

Looking to hone your π-reciting skills before PiDay?

Google "pi":

You'll see this basic wee "game":



It'll show you more and more digits of pi. All you have to do is repeat them back without any mistakes!

MILK ROTA

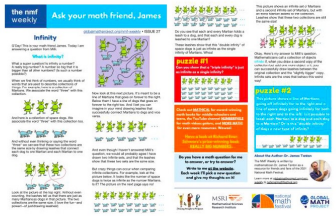
The next four people on milk:

7th March	Alysanne
14th March	Lynne
21st March	Jamie
28th March	Ward

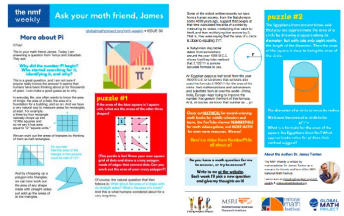
NMF NEWS

You all obviously know I'm partial to a wee Maths Newsletter and the National Math Festival have the fabulously talented James Tanton (@jamestanton) writing their weekly newsletter. Each week James tackles topics inspired by brilliant questions from young Mathematicians. Topics include...

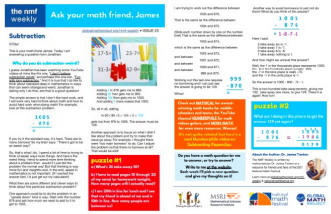
infinity



pi



crazy subtraction



...and there's loads more to catch up on in previous newsletters. It's free to sign up and here's where: https://bit.ly/3s8ZPFN

WEEKLY PUZZLE

Last week's was a classic puzzle: It's house numbers on sale, costing £2 per digit so 314 would be £6.

THIS WEEK'S PUZZLE:

Newsletter reader Ron Lancaster sent me this photo of him outside the Mathematically named "Complex Hotel" (which, confusingly, is a real place... it's not imaginary).

The challenge this week is to figure out where this hotel is...and the closest answer will get a prize courtesy of Ron*!

Please send your answers to Ron (ron2718@nas.net) or me!

*assuming we get answers that are deemed by me and Ron to be close enough to the hotel's actual vicinity- this is vague so it might be the correct country or adjacent country even!

FOR NEXT NEWSLETTER...

Let me know of any news/items/lesson ideas to share by Friday lunchtime. Chris

To enter the contest, send me an email message at ron**2718**@nas.net

To receive Chris Smith's amazing newsletter, send him an email message at chris@piwire.co.uk

By the way, my ***e***mail address is a true ***e***mail address, especially when you realize that 2718 is not a random number. I chose the 2718 because

e = 2.718...

It is also a play on words for my Aunt Marlene, who has always called me Ronnie (Ron-***e***).

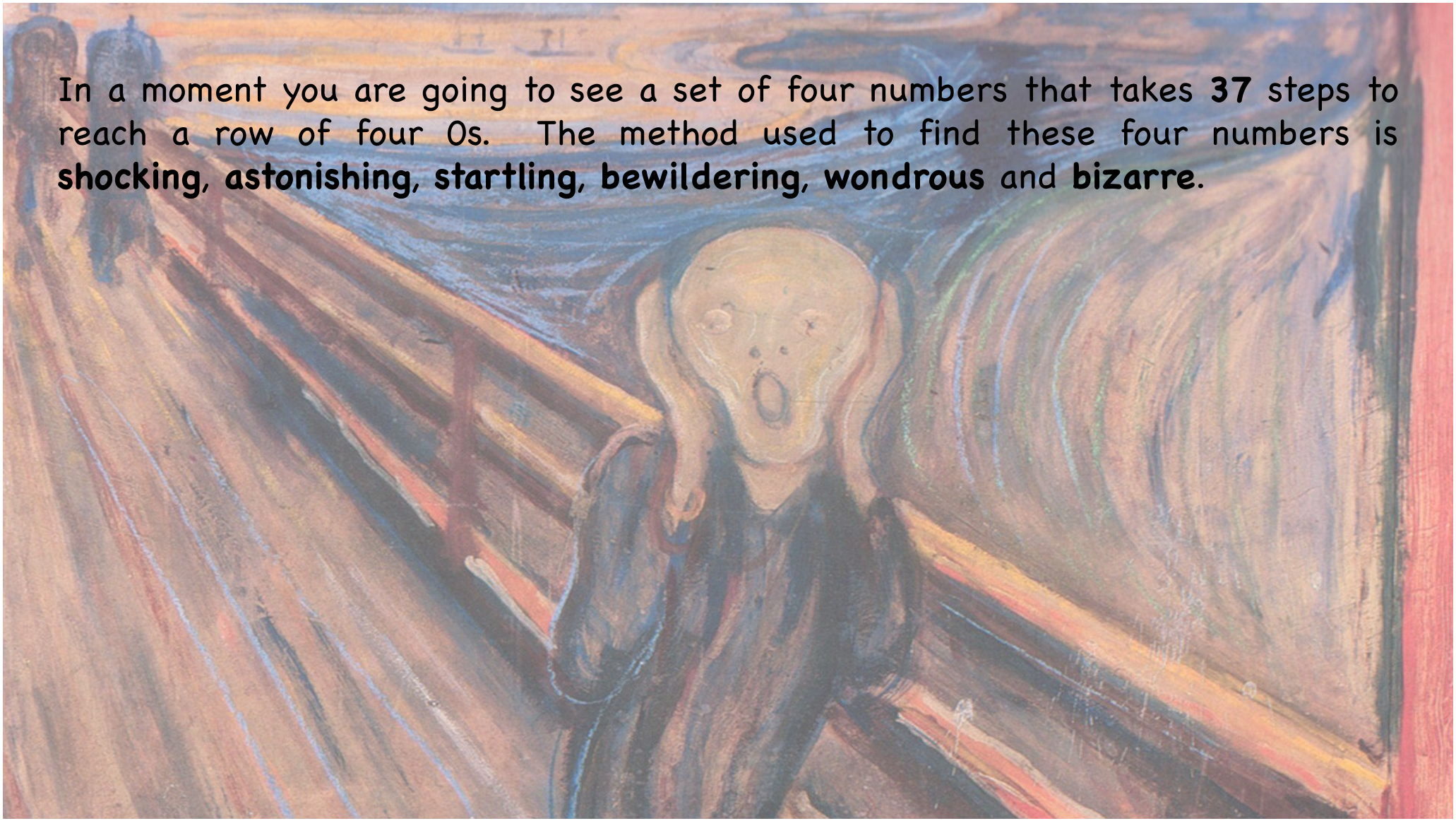
Starting with the four **matrices** given below, how many steps does it take to reach a row of four null matrices?

$$\begin{pmatrix} 3 & 1 \\ 4 & 1 \end{pmatrix} \quad \begin{pmatrix} 5 & 9 \\ 2 & 6 \end{pmatrix} \quad \begin{pmatrix} 5 & 3 \\ 5 & 8 \end{pmatrix} \quad \begin{pmatrix} 9 & 7 \\ 9 & 3 \end{pmatrix}$$

A challenge

Find a set of four numbers that takes a **long time** to reach a row of four 0s. You will find this is not easy to do.

174	326788	2	293828276
326614	326786	293828274	293828102
172	293501488	172	293501488
293501316	293501316	293501316	293501316
0	0	0	0



In a moment you are going to see a set of four numbers that takes **37** steps to reach a row of four 0s. The method used to find these four numbers is **shocking**, **astonishing**, **startling**, **bewildering**, **wondrous** and **bizarre**.

0, 543689013, 839286755, 1000000000

0	543689013	839286755	1000000000
543689013	295597742	160713245	1000000000
248091271	134884497	839286755	456310987
113206774	704402258	382975768	208219716
591195484	321426490	174756052	95012942
269768994	146670438	79743110	496182542
123098556	66927328	416439432	226413548
56171228	349512104	190025884	103314992
293340876	159486220	86710892	47143764
133854656	72775328	39567128	246197112
61079328	33208200	206629984	112342456
27871128	173421784	94287528	51263128
145550656	79134256	43024400	23392000
66416400	36109856	19632400	122158656
30306544	16477456	102526256	55742256
13829088	86048800	46784000	25435712
72219712	39264800	21348288	11606624
32954912	17916512	9741664	60613088
15038400	8174848	50871424	27658176
6863552	42696576	23213248	12619776
35833024	19483328	10593472	5756224
16349696	8889856	4837248	30076800
7459840	4052608	25239552	13727104
3407232	21186944	11512448	6267264
17779712	9674496	5245184	2860032
8105216	4429312	2385152	14919680
3675904	2044160	12534528	6814464
1631744	10490368	5720064	3138560
8858624	4770304	2581504	1506816
4088320	2188800	1074688	7351808
1899520	1114112	6277120	3263488
785408	5163008	3013632	1363968
4377600	2149376	1649664	578560
2228224	499712	1071104	3799040
1728512	571392	2727936	1570816
1157120	2156544	1157120	157696
999424	999424	999424	999424
0	0	0	0

$$x^3 - 5x^2 + 7x - 1 = 0$$

$$x^3 - 4x^2 + 6x - 2 = 0$$

$$A = 0.1607132447\ldots$$

$$B = 0.4563109873\ldots$$

$$A' = 1 - B = 0.5436890126\ldots$$

$$B' = 1 - A = 0.8392867552\ldots$$

$$(0, 543689013, 839286755, 1000000000)$$

The Four Numbers Game, Michael Dumont and Jean Meeus, Journal of Recreational Mathematics, Volume 13 (2), 1980-81

3

1
5
3

As shown below, the number 153 has an unusual property. Find other 3-digit numbers with this property. How many numbers are there in all? How do you know you?

$$1^3 + 5^3 + 3^3 = 153$$

As shown below, the number 153 has an unusual property. Find other numbers with this property. How many numbers are there in all? How do you know you?

$$1^3 + 5^3 + 3^3 = 153$$

There are four other numbers with this property.

1, 370, 371, 407

In the sequence given below, any given term after the first one is calculated by **cubing** the digits of the term before it and adding up the cubes. For example,

$$5^3 + 9^3 = 854 \qquad 8^3 + 5^3 + 4^3 = 701$$

59, 854, 701, 344, 155, 251, 134, 92, ...

Find additional terms of the sequence. What do you notice? What questions do you have? Use a different number for the first term and investigate what happens as more and more terms are calculated.

Note: A more formal way of describing how terms of this sequence are calculated is given below. S_3 represents the function that finds the sum of the cubes of the digits of a given number.

$$s_3\left(\sum_{i=0}^{n} a_i 10^i\right) = \sum_{i=0}^{n} a_i^3$$

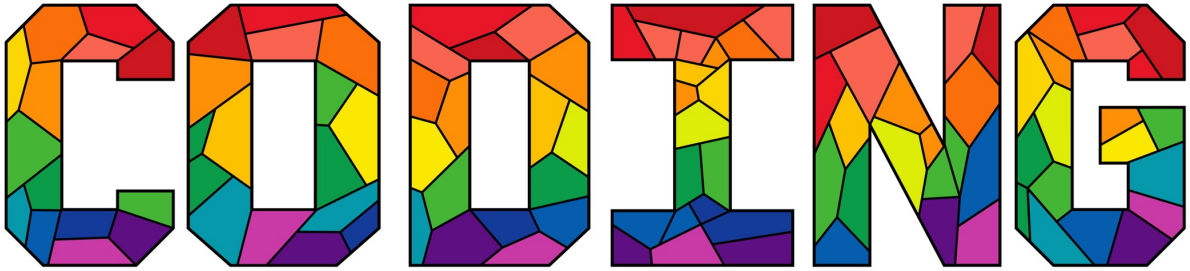
In the sequence given below, any given term after the first one is calculated by **cubing** the digits of the term before it and adding up the cubes. For example,

$$5^3 + 9^3 = 854 \qquad 8^3 + 5^3 + 4^3 = 701$$

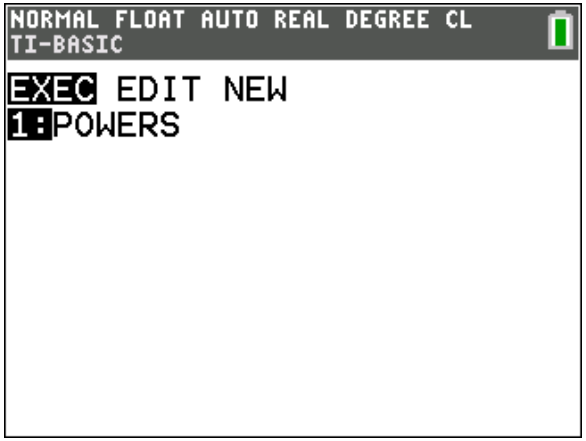
59, 854, 701, 344, 155, 251, 134, 92, 737, 713, **371**, 371, ...

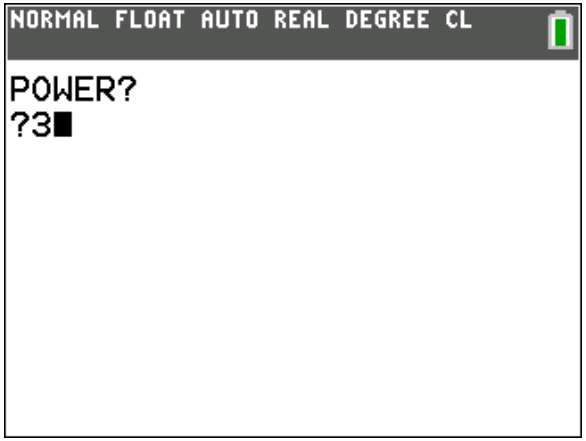
These are the different cycles that will appear, depending on what starting number is used. For any given starting number, is it possible to predict which cycle will come up without doing the calculations?

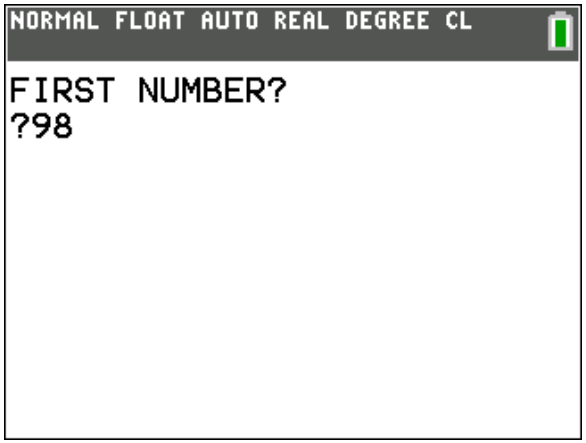
1, 1, ...
999, 954, 918, 1242, 81, 513, 153, **153**, ...
2971, 1081, 514, 190, 730, 370, **370**, ...
59, 854, 701, 344, 155, 251, 134, 92, 737, 713, **371**, 371, ...
98, 1241, 74, **407**, 407, ...
55, **250**, **133**, 55, ...
136, **244**, 136, ...
160, **217**, **352**, 160, ...
919, 1459, 919, ...

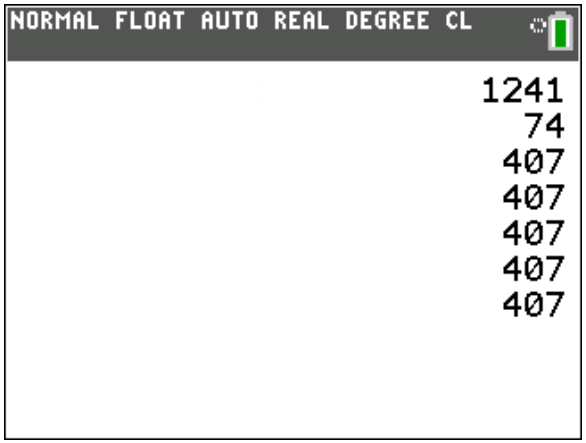


Program for the TI-83 graphing calculator









For any given starting number, is it possible to predict which cycle will come up without doing the calculations? **YES!**

If $a \equiv 0 \pmod 3$, then there exists an m such that $S_3^m(a) = 153$

If $a \equiv 1 \pmod 3$, then there exists an m such that $S_3^m(a) = 1, 55, 136, 160, 370,$ or 919

If $a \equiv 2 \pmod 3$, then there exists an m such that $S_3^m(a) = 371$ or 407

4

The Lyness Sequence

Let t_n be the sequence defined below. Let $t_1 = 8$ and $t_2 = 3$. Find the next 100 terms of the sequence.

$$t_1 = x, t_2 = y$$

$$t_n = \frac{1 + t_{n-1}}{t_{n-2}} \quad \text{for } n \geq 3$$

Let t_n be the sequence defined below. Let $t_1 = 8$ and $t_2 = 3$. Find the next 100 terms of the sequence.

8, **3**, 0.5, 0.5, 3, **8**, **3**, 0.5, 0,5, 3, **8**, **3**, 0.5, 0.5, 3, ...

$$t_1 = x, t_2 = y$$

$$t_n = \frac{1 + t_{n-1}}{t_{n-2}} \quad \text{for } n \geq 3$$

Some questions:

If $t_1 = x$ and $t_2 = y$, is it always true that $t_6 = t_1$ and $t_7 = t_2$? How do you know?

Are there values of x and y for which terms of the sequence are not defined?

From **algebra** to **geometry** and beyond to the **moon**...

$$7, 6, 1, \frac{1}{3}, \frac{4}{3}, 7, 6, 1, \ldots$$

From **algebra** to **geometry** and beyond to the **moon**...

$$7, 6, 1, \frac{1}{3}, \frac{4}{3}, 7, 6, 1, \ldots$$

From **algebra** to **geometry** and beyond to the **moon**…

$$7, 6, 1, \frac{1}{3}, \frac{4}{3}, 7, 6, 1, \ldots$$

$$A(7,6), B(6,1), C\left(1, \frac{1}{3}\right), D\left(\frac{1}{3}, \frac{4}{3}\right), E\left(\frac{4}{3}, 7\right), F(7,6)$$

$7, 6, 1, \frac{1}{3}, \frac{4}{3}, 7, 6, \ldots$

$A(7, 6)$

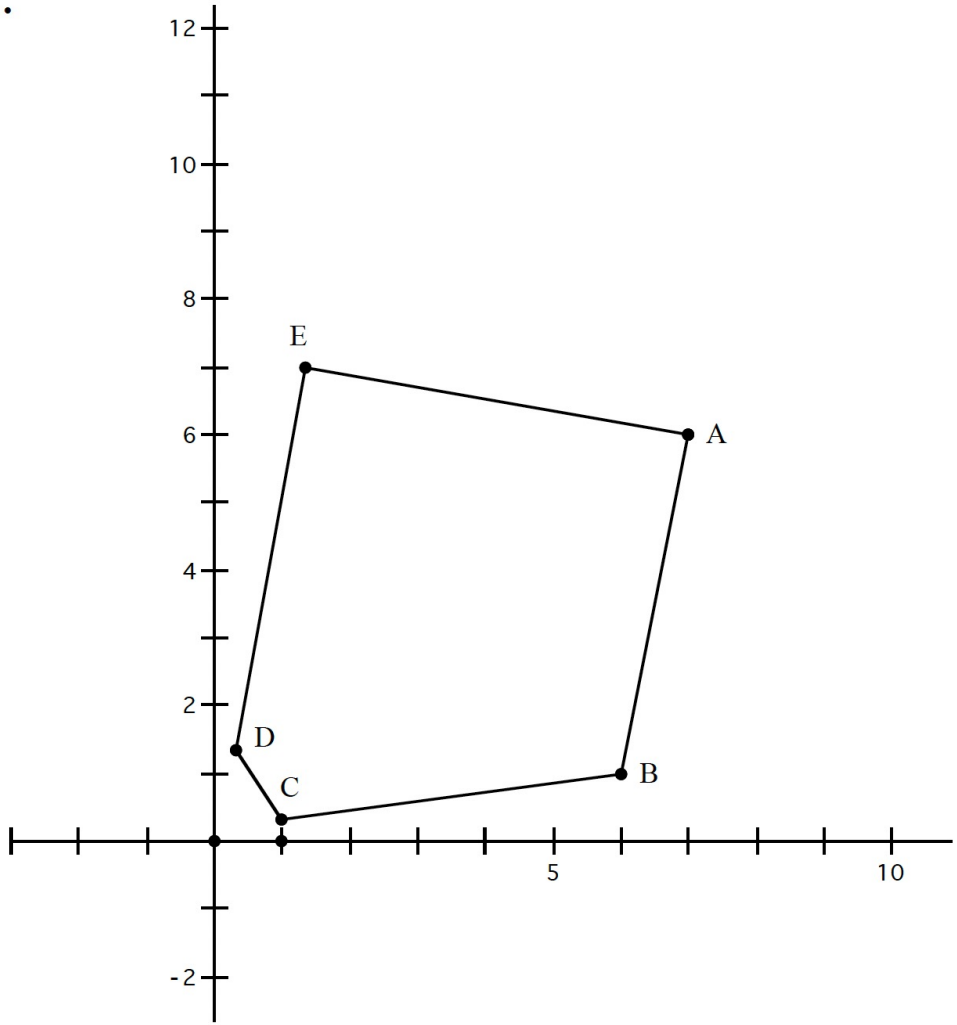
$B(6, 1)$

$C\left(1, \frac{1}{3}\right)$

$D\left(\frac{1}{3}, \frac{4}{3}\right)$

$E\left(\frac{4}{3}, 7\right)$

$F(7, 6)$



More questions:

How is the **area** of the pentagon related to the first two terms of the sequence?

Is it possible to adjust the first two terms so the figure is a **regular pentagon**? Why or why not?

How are the **angles** of the pentagon related to the first two terms of the sequence? When is angle DEA = **90°**?

$7, 6, 1, \frac{1}{3}, \frac{4}{3}, 7, 6, \ldots$

$A(7, 6)$

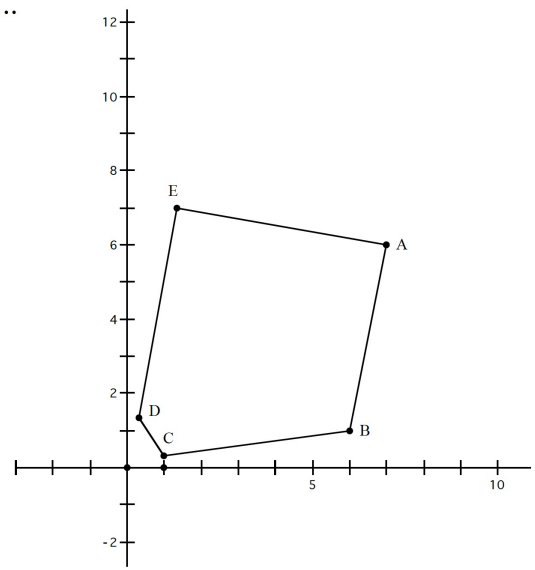
$B(6, 1)$

$C\left(1, \frac{1}{3}\right)$

$D\left(\frac{1}{3}, \frac{4}{3}\right)$

$E\left(\frac{4}{3}, 7\right)$

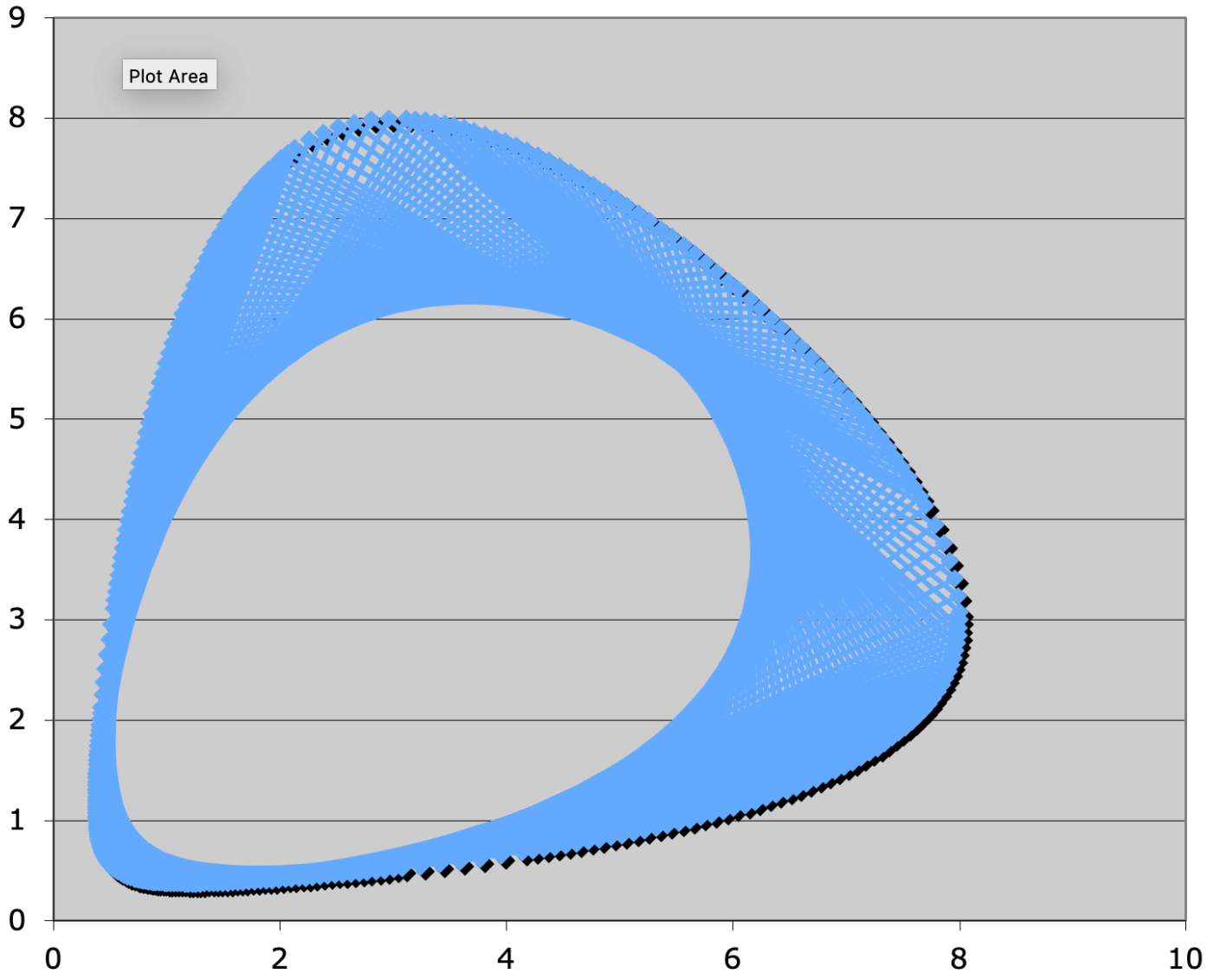
$F(7, 6)$



What happens if we **replace** the **1** with **1.1** in the formula for t_n?
Be prepared for a surprise.

$$t_1 = x, t_2 = y$$

$$t_n = \frac{1.1 + t_{n-1}}{t_{n-2}} \quad \text{for } n \geq 3$$


Plot Area
9
8
7
6
5
4
3
2
1
0
0
2
4
6
8
10

5, 6, 7, 8

The Hailstone Sequence, Collatz Conjecture, The $3n + 1$ conjecture, the Ulam conjecture (after Stanisław Ulam), Kakutani's problem (after Shizuo Kakutani), the Thwaites conjecture (after Sir Bryan Thwaites), Hasse's algorithm (after Helmut Hasse), the Syracuse problem

Look and Say Sequence (John Conway)

Kaprekar's constant (6174)

The Divisive Number 15

Resources:

The Book of Numbers by John H. Conway & Richard K. Guy

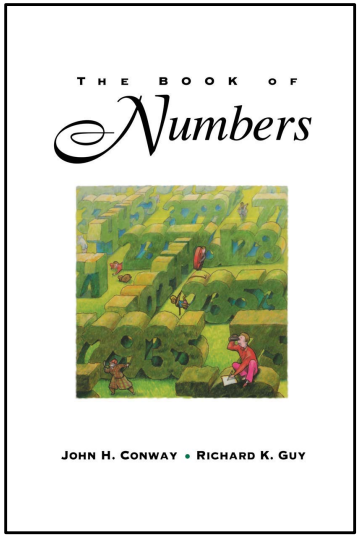
The Penguin Dictionary of Curious and Interesting Numbers by David Wells

Those Fascinating Numbers by Jean-Marie De Koninck

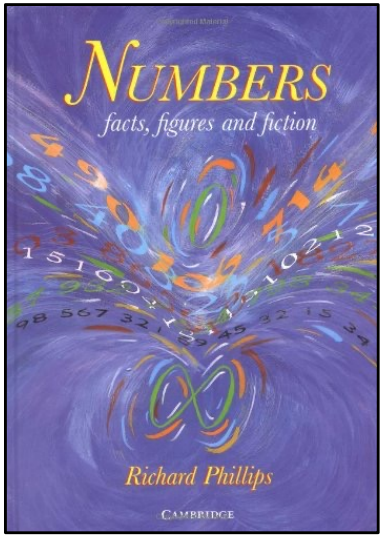
Numbers: facts, figures and fiction by Richard Phillips

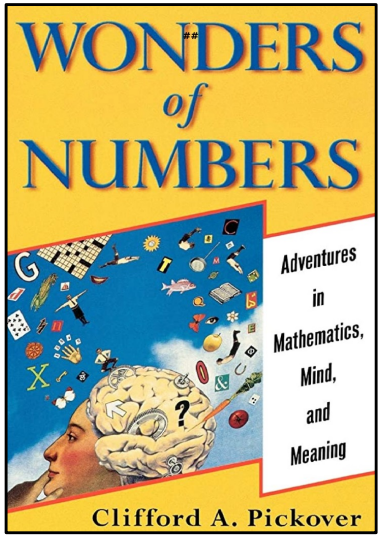
Wonders of Numbers: Adventures in Mathematics, Mind and Meaning by Clifford A. Pickover

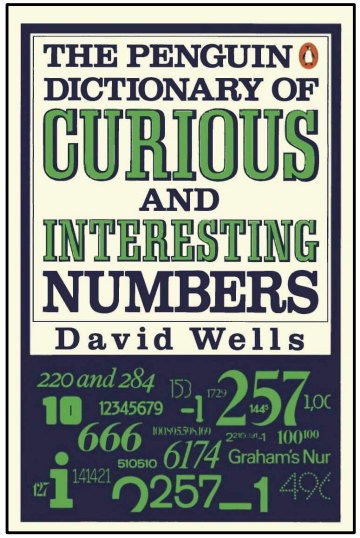
Any book by **Martin Gardner**


THE BOOK OF
Numbers
JOHN H. CONWAY • RICHARD K. GUY

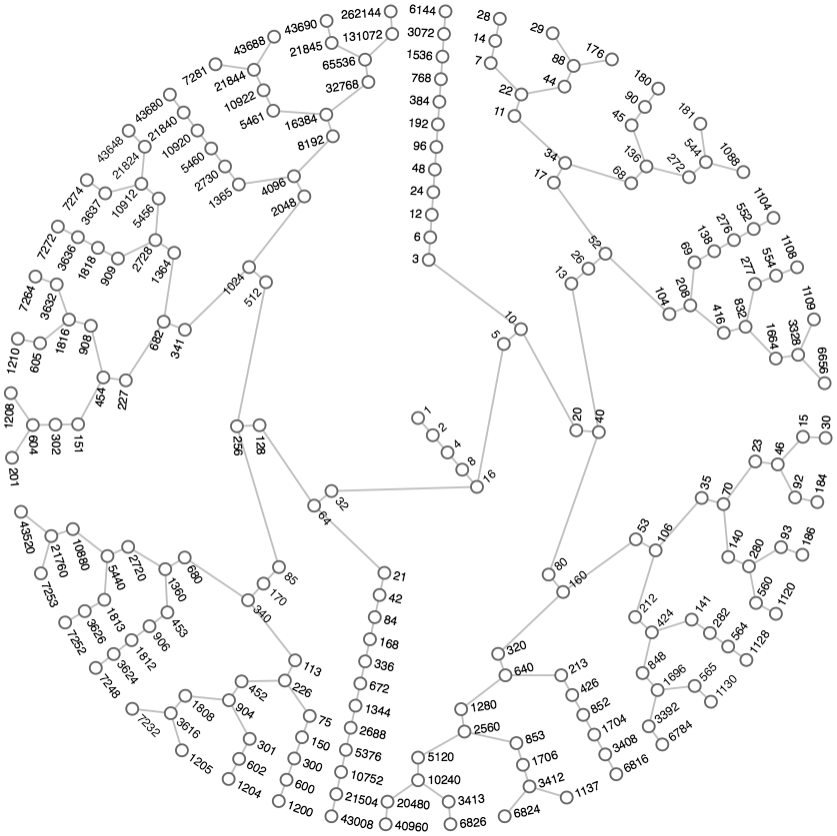
Those Fascinating Numbers
Jean-Marie De Koninck
AMS


NUMBERS
facts, figures and fiction
Richard Phillips
CAMBRIDGE

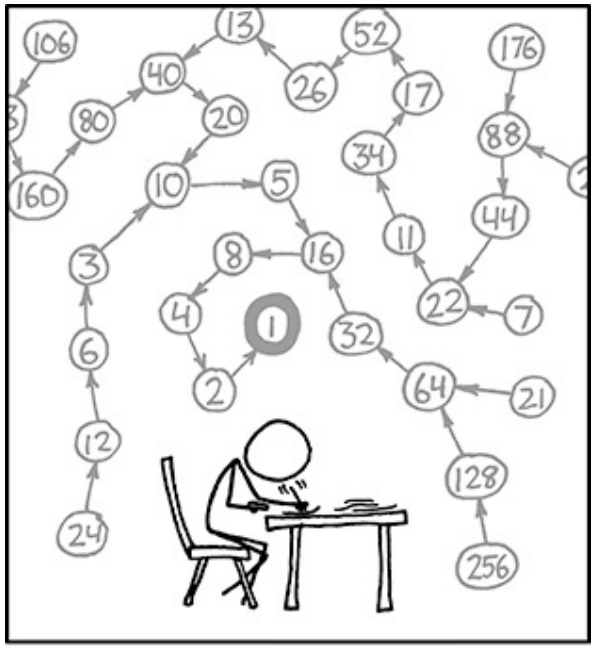

WONDERS of NUMBERS
Adventures in Mathematics, Mind, and Meaning
Clifford A. Pickover


THE PENGUIN DICTIONARY OF
CURIOUS
AND
INTERESTING
NUMBERS
David Wells
220 and 284
12345679
666
6174
Graham's Nur

The Haistone sequence



Collatz Graph
https://xkcd.com/710/



THE COLLATZ CONJECTURE STATES THAT IF YOU PICK A NUMBER, AND IF IT'S EVEN DIVIDE IT BY TWO AND IF IT'S ODD MULTIPLY IT BY THREE AND ADD ONE, AND YOU REPEAT THIS PROCEDURE LONG ENOUGH, EVENTUALLY YOUR FRIENDS WILL STOP CALLING TO SEE IF YOU WANT TO HANG OUT.

The Look and Say sequence

term number	term	number of digits
1	1	1
2	11	2
3	21	2
4	1211	4
5	111221	6
6	312211	6
7	1311221	7
8	1113213211	10
9	31131211131221	14
10	13211311123113100000	20
11	11131221133112132113212221	26
12	3113112221232112111312211312113211	34
13	1321132132111213122112311311222113111221131221	46
14	11131221131211131231121113112221121321132132211331222113112211	62

The Look and Say sequence

THE LOOK AND SAY SEQUENCE

About how many digits has the nth term in the sequence

1 11 21 1211 111221 312211 13112221 1113213211 31131211131221 . . . ?

The Look and Say sequence

Have you guessed the general rule yet? The first term consists of one "one," so the second is "one one." This consists of two "ones," so the third is "two one." This in turn is made from one "two," and one "one" and so gives "one two one one," and so on.

It can be shown that the number of digits in the nth term is roughly proportional to

$$(1.303577269034296391257099112152551890730702504659 4\ldots)^n,$$

The Look and Say sequence

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where the simplest algebraic equation defining the number in parentheses is

$$\begin{aligned}
&x^{71} - x^{69} - 2x^{68} - x^{67} + 2x^{66} + 2x^{65} + x^{64} - x^{63} - x^{62} - x^{61} - x^{60} - x^{59} \\
&+ 2x^{58} + 5x^{57} + 3x^{56} - 2x^{55} - 10x^{54} - 3x^{53} - 2x^{52} + 6x^{51} \\
&+ 6x^{50} + x^{49} + 9x^{48} - 3x^{47} - 7x^{46} - 8x^{45} - 8x^{44} + 10x^{43} \\
&+ 6x^{42} + 8x^{41} - 5x^{40} - 12x^{39} + 7x^{38} - 7x^{37} + 7x^{36} + x^{35} \\
&- 3x^{34} + 10x^{33} + x^{32} - 6x^{31} - 2x^{30} - 10x^{29} - 3x^{28} + 2x^{27} \\
&+ 9x^{26} - 3x^{25} + 14x^{24} - 8x^{23} - 7x^{21} + 9x^{20} + 3x^{19} - 4x^{18} \\
&- 10x^{17} - 7x^{16} + 12x^{15} + 7x^{14} + 2x^{13} - 12x^{12} - 4x^{11} - 2x^{10} \\
&+ 5x^{9} + x^{7} - 7x^{6} + 7x^{5} - 4x^{4} + 12x^{3} - 6x^{2} + 3x - 6 = 0.
\end{aligned}$$

Kaprekar's constant (6174)

$$t_1 = 3941$$
$$t_2 = 9431 - 1349 = 8082$$
$$t_3 = 8820 - 0288 = 8532$$
$$t_4 = 8532 - 2358 = 6174$$
$$t_5 = 7641 - 1467 = \mathbf{6174}$$

The Divisive Number 15

The Divisive Number 15

Take any natural number larger than 1 and write down its divisors, including 1 and the number itself. Now take the sum of the digits of these divisors. Iterate until a number repeats.

The black-hole number this time is 15. Its divisors are 1, 3, 5, and 15, and these digits sum to 15. This one is a bit more tedious, but it is also that much more strange at the same time. This one may defy not only your ability to explain it, but your very equilibrium.

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tinyurl.com/T3WEB2023RL

1	THE NAME GAME Shirley Ellis	6	WOOLY BULLY Sam The Sham & The Pharoahs
2	A LOVER'S CONCERTO The Toys	7	1-2-3 Len Barry
3	THIS DIAMOND RING Gary Lewis & The Playboys	8	TREAT HER RIGHT Roy Head & The Traits
4	HELP ME, RHONDA The Beach Boys	9	I GOT YOU BABE Sonny & Cher
5	YOU'VE LOST THAT LOVIN' FEELIN' The Righteous Brothers	10	EVE OF DESTRUCTION Barry McGuire