

“How-To”
With Your TI-84 Plus CE
Graphing Calculator
Part 4

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This handout and related accessories (programs, image files, graphical databases) can be downloaded at <http://users.pfw.edu/lamaster/technology> if you scroll to the presentation *What's On Your Table*, at the 2022 Virtual T³ International Conference, July 28, 2022. On this page you can also get links to Parts 1, 2, and 3 of this Webinar Series.

What's On Your (84) Table?

By John LaMaster, T³ National Instructor, Purdue University Fort Wayne, lamaster@pfw.edu

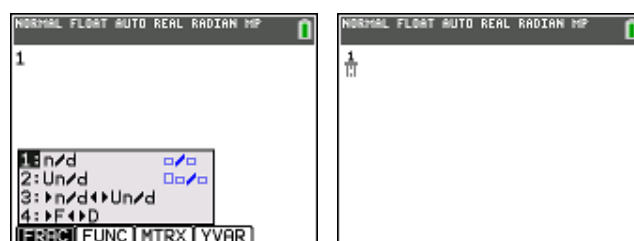
Explorations with tables can provide opportunities to support students with the following Math Practices:

- MP1 Make sense of problems and persevere in solving them.
- MP2 Reason abstractly and quantitatively.
- MP3 Construct viable arguments and critique the reasoning of others.
- MP4 Model with mathematics.
- MP5 Use appropriate tools strategically.
- MP6 Attend to precision.
- MP7 Look for and make use of structure.
- MP8 Look for and express regularity in repeated reasoning.

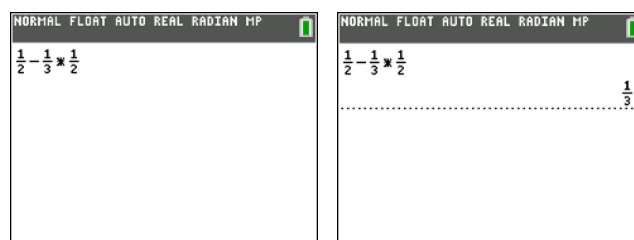
A. Using Stacked Fractions to Build a Table on the Home Screen (MP5, MP6, MP7)

Increase student self-efficacy and build the bridge from arithmetic to algebra with this investigation that will articulate across multiple content areas.

1. On the Home screen, press $\boxed{1}$.
2. Press $\boxed{\text{ALPHA}}$ $\boxed{[F1]}$ to get to the shortcut FRAC menu.
3. Press $\boxed{\text{ENTER}}$ or $\boxed{1}$ to select n/d .



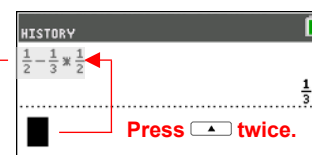
4. Press $\boxed{2}$ $\boxed{\rightarrow}$ $\boxed{-}$ $\boxed{1}$ $\boxed{\text{ALPHA}}$ $\boxed{[F1]}$ $\boxed{\text{ENTER}}$ $\boxed{3}$ $\boxed{\rightarrow}$ $\boxed{\times}$ $\boxed{1}$ $\boxed{\text{ALPHA}}$ $\boxed{[F1]}$ $\boxed{\text{ENTER}}$ $\boxed{2}$ $\boxed{\rightarrow}$.



5. Press $\boxed{\text{ENTER}}$ (to say "please").

6. Press the $\boxed{\uparrow}$ key twice to climb up the tree and highlight the expression. Press $\boxed{\text{ENTER}}$ to "pluck the fruit off the tree."

Once highlighted press $\boxed{\text{ENTER}}$.

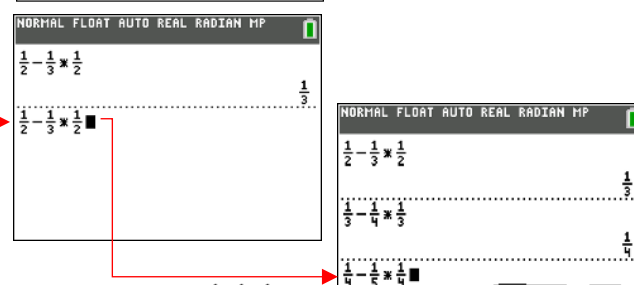


7. Press $\boxed{2\text{nd}}$ $\boxed{\leftarrow}$ to go to the beginning of the line ($\boxed{\text{H}}$). Change the expression to increase each denominator by 1. Press $\boxed{\text{ENTER}}$.

Repeat.

We have created a table on the home screen.

What do you notice? What do you wonder?



Edit $\frac{1}{2} - \frac{1}{3} \times \frac{1}{2}$ to make $\frac{1}{3} - \frac{1}{4} \times \frac{1}{3}$.

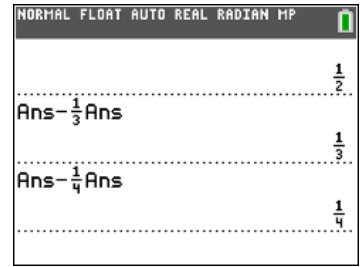
B. Using the Last Ans with Stacked Fractions to Build a Table on the Home Screen and Copy and Paste a Generalized Form into the Y= Editor (MP4, MP5, MP7, MP8)

Use the TI-84's Last Ans to represent the previous pattern as a fractional reduction. Such a table is not only easier to build, but it unveils the representation shown below.

$$\frac{1}{2} \quad \begin{array}{|c|c|c|c|c|c|} \hline \text{Yellow} & \text{Yellow} & \text{Yellow} & \text{White} & \text{White} & \text{White} \\ \hline \end{array}$$

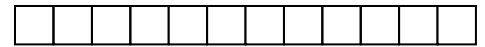
$$- \frac{1}{3} \text{ of } \frac{1}{2} \quad \begin{array}{|c|} \hline \text{Yellow} \\ \hline \end{array}$$

$$\frac{2}{3} \text{ of } \frac{1}{2} \quad \begin{array}{|c|c|} \hline \text{Yellow} & \text{Yellow} \\ \hline \end{array} = \frac{1}{3} \quad \begin{array}{|c|c|c|c|c|c|} \hline \text{Yellow} & \text{Yellow} & \text{White} & \text{White} & \text{White} & \text{White} \\ \hline \end{array}$$



Vertical Articulation: Later, when studying repeated exponential decay by $\frac{1}{3}$, they will represent $P - \frac{1}{3}P$ as $\frac{2}{3}P$.

1. Study the above diagram for $\frac{1}{2} - \frac{1}{3} \cdot \frac{1}{2}$.



Then create a diagram to model $\frac{1}{3} - \frac{1}{4} \cdot \frac{1}{3}$ in a similar way.

TIP: Unifix cubes with multiple colors can be helpful.

Why do we need 12 cubes to model $\frac{1}{3} - \frac{1}{4} \cdot \frac{1}{3}$ but

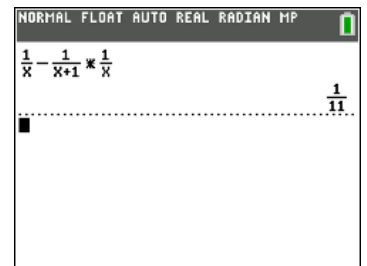
we need just 6 cubes to model $\frac{1}{2} - \frac{1}{3} \cdot \frac{1}{2}$?

Use algebra to simplify the expression $\text{Ans} - \frac{1}{4} \text{Ans}$ in terms of **Ans**. Then substitute $\frac{1}{3}$ for **Ans**.

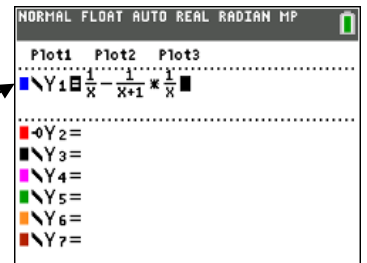
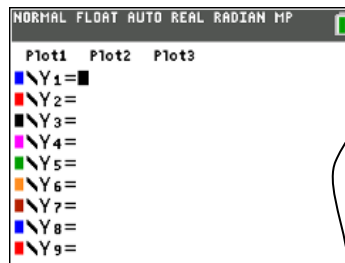
What do you notice?

3. Type $\frac{1}{x} - \frac{1}{x+1} \cdot \frac{1}{x}$ on the home screen and press **ENTER**.

Ask students to conjecture what the value of x is and explain their reasoning.



4. Press **Y=**, position your cursor in Y1,



and press **2nd** **ENTER**

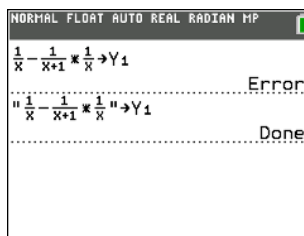
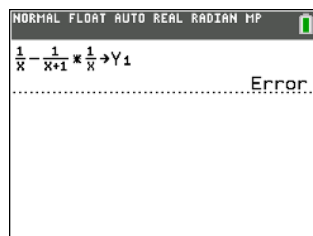
to "beam the expression up" into Y1.

Note: After the evaluation of an expression on the home screen, the input expression is stored in

2nd [entry] (above the **ENTER** key). **2nd** **ENTER** overwrites the entire line. It is a way to copy and paste.

Alternate Approach to Import Into Y=:

You can also do the above from the home screen but must use quotes. Use **[ALPHA]** **[F4]** **[ENTER]** to get Y1.



C. Using the Y= Editor to Explore the Table with Auto/Auto, Auto/Ask, Ask/Auto, and Ask/Ask (MP4, MP5, MP7, MP8)

With an algebraic function in the Y= Editor, students can explore patterns, make conjectures to answer the question which is the title of this handout, “What’s On Your Table?”, and then use algebraic reasoning to prove their conjecture. This shifts the ownership and motivation of using algebra from the teacher to the student.

1. Ask students to scroll the table and then conjecture a simplified formula for $\frac{1}{x} - \frac{1}{x+1} \cdot \frac{1}{x}$.

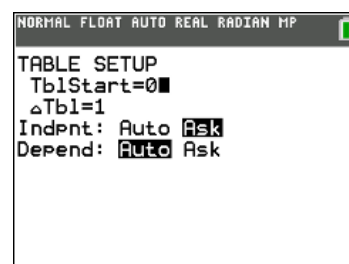
Students then can use algebra to verify their conjecture.

$$\begin{aligned} \frac{1}{x} - \frac{1}{x+1} \cdot \frac{1}{x} &= \frac{x+1}{x+1} \cdot \frac{1}{x} - \frac{1}{x+1} \cdot \frac{1}{x} \\ &= \frac{x+1-1}{(x+1)x} \\ &= \frac{x}{(x+1)x} \\ &= \frac{1}{(x+1)} \end{aligned}$$

2. Change the settings for the table using **[2nd]** **[WINDOW]** so you can **Ask** for any independent variable.



| X | Y1 |
|---|-----|
| 1 | 1/2 |
| 2 | 1/3 |
| 3 | 1/4 |
| 4 | 1/5 |



| X | Y1 |
|-----|-----|
| 1/2 | 2/3 |
| 1/3 | 3/4 |
| 1/4 | 4/5 |

Use the stacked fraction template to explore

the value of y for different inputs of the form $\frac{1}{a}$.

Use **[ALPHA]** **[F1]** **[ENTER]** to select **n/d** or **[ALPHA]** **[X, T, θ, n]**.

Ask students to conjecture what the numerical expression is in terms of a . What is the numerator? The denominator?

3. Change the settings for the table using **2nd** **WINDOW** so you can **Ask** to show the dependent variable.

Enter a value for $\frac{1}{a}$ such as $\frac{1}{99}$, then sit the cursor in the output column and press **ENTER** after students predict the value according to their conjecture from the previous step and test if it works.

| NORMAL FLOAT AUTO REAL RADIAN MP | | | | | |
|----------------------------------|-----------------|--|--|--|--|
| TABLE SETUP | | | | | |
| TblStart=1 | | | | | |
| ΔTbl=1 | | | | | |
| Indent: Auto | | | | | |
| Depend: Auto | | | | | |
| | | | | | |
| X | Y1 | | | | |
| $\frac{1}{2}$ | $\frac{2}{3}$ | | | | |
| $\frac{1}{3}$ | $\frac{3}{4}$ | | | | |
| $\frac{1}{10}$ | $\frac{10}{11}$ | | | | |
| $\frac{1}{99}$ | | | | | |
| Y1= | | | | | |

Ask students to substitute $x = \frac{1}{a}$ in the expression $\frac{1}{x+1}$ and simplify to prove their conjecture.

$$\begin{aligned}
 x = \frac{1}{a} &\Rightarrow \frac{1}{x+1} = \frac{1}{\frac{1}{a}+1} \\
 &= \frac{1}{\frac{1}{a}+1} \cdot \frac{a}{a} \\
 &= \frac{a}{1+a}
 \end{aligned}$$

Optional: For more fun this same idea can be extended to inputs of the form $\frac{a}{b}$

4. Similarly to the previous exploration,

enter values for $\frac{a}{b}$

such as those shown to the right,

then sit the cursor in the output column and press **ENTER**.

Ask students to conjecture what the numerical expression is in terms of a and b .

What is the numerator? The denominator?

Enter a value for $\frac{a}{b}$ such as $\frac{3}{4}$, then sit the cursor

in the output column and press **ENTER** after students predict the value to unveil the answer.

| NORMAL FLOAT AUTO REAL RADIAN MP | | | | | |
|----------------------------------|----------------|--|--|--|--|
| X | Y1 | | | | |
| $\frac{2}{3}$ | $\frac{3}{5}$ | | | | |
| $\frac{4}{7}$ | $\frac{7}{11}$ | | | | |
| $\frac{3}{8}$ | $\frac{3}{8}$ | | | | |
| | | | | | |
| | | | | | |
| Y1= | | | | | |

Once they construct a conjecture, ask students to substitute $x = \frac{a}{b}$

in the expression $\frac{1}{x+1}$ and simplify to justify their answer.

(See next page for the spoiler.)

$$\begin{aligned}
 x = \frac{a}{b} &\Rightarrow \frac{1}{x+1} = \frac{1}{\frac{a}{b}+1} \\
 &= \frac{1}{\frac{a}{b}+1} \cdot \frac{b}{b} \\
 &= \frac{b}{a+b}
 \end{aligned}$$

D. Changing the Expression and the ΔTbl Directly in the Table (MP1, MP3, MP5, MP6, MP7, MP8)

While the original expression $\frac{1}{x} - \frac{1}{x+1} \cdot \frac{1}{x}$ simplifies to a very recognizable pattern, $\frac{1}{x+1}$, we can modify the function slightly to $\frac{1}{x} - \frac{1}{x-1} \cdot \frac{1}{x}$ and ask similar questions before using algebra to simplify.

1. Ask students to sit their cursor on Y1, press **ENTER** to go to the entry line, and change the expression

$$\text{to } \frac{1}{x} - \frac{1}{x-1} \cdot \frac{1}{x}.$$

| NORMAL FLOAT AUTO REAL RADIAN MP | | | | | |
|----------------------------------|------|--|--|--|--|
| X | Y1 | | | | |
| 2/3 | 3/5 | | | | |
| 4/7 | 7/11 | | | | |
| 5/3 | 8/3 | | | | |
| 3/4 | 4/7 | | | | |
| Y1 = 1/X - 1/(X-1) * 1/X | | | | | |

| NORMAL FLOAT AUTO REAL RADIAN MP | | | | | |
|----------------------------------|------|--|--|--|--|
| X | Y1 | | | | |
| 2/3 | 3/5 | | | | |
| 4/7 | 7/11 | | | | |
| 5/3 | 8/3 | | | | |
| 3/4 | 4/7 | | | | |
| Y1 = 1/X - 1/(X-1) * 1/X | | | | | |

2. Change the settings for the table using **2nd** **WINDOW** so your table starts at 1 and climbs in steps of $\Delta Tbl = 1$.

| NORMAL FLOAT AUTO REAL RADIAN MP | | | | | |
|----------------------------------|--|--|--|--|--|
| TABLE SETUP | | | | | |
| TblStart=1 | | | | | |
| $\Delta Tbl=1$ | | | | | |
| Indent: Auto Ask | | | | | |
| Depend: Auto Ask | | | | | |

This pattern is not easy to see with what is shown. We will first explore only odd values of x .

3. Notice the context help "Press + ΔTbl ". Make sure your start value is 1 (or any odd number) and press the **+** key to change the step size to $\Delta Tbl = 2$.

| NORMAL FLOAT AUTO REAL RADIAN MP | | | | | |
|----------------------------------|-------|--|--|--|--|
| PRESS + FOR ΔTbl | | | | | |
| X | Y1 | | | | |
| 1 | ERROR | | | | |
| 2 | 0 | | | | |
| 3 | 1/6 | | | | |
| 4 | 1/6 | | | | |
| 5 | 3/20 | | | | |
| X=1 | | | | | |

| NORMAL FLOAT AUTO REAL RADIAN MP | | | | | |
|----------------------------------|-------|--|--|--|--|
| PRESS + FOR ΔTbl | | | | | |
| X | Y1 | | | | |
| 1 | ERROR | | | | |
| 2 | 0 | | | | |
| 3 | 1/6 | | | | |
| 4 | 1/6 | | | | |
| 5 | 3/20 | | | | |
| $\Delta Tbl=2$ | | | | | |

Press the **▼** key to scroll.

What is the next numerator? The next denominator?

Ask students to explain the pattern communicating precisely.

(The numerator is 2 less than x and the denominator is the product of x and $x-1$)

Ask for students to show this is true for any x using algebraic reasoning:

$$\frac{1}{x} - \frac{1}{x-1} \cdot \frac{1}{x} = \frac{1}{x} \cdot \frac{x-1}{x-1} - \frac{1}{x-1} \cdot \frac{1}{x} = \frac{x-1-1}{x(x-1)} = \frac{(x-2)}{x(x-1)}$$

| NORMAL FLOAT AUTO REAL RADIAN MP | | | | | |
|----------------------------------|-------|--|--|--|--|
| PRESS + FOR ΔTbl | | | | | |
| X | Y1 | | | | |
| 1 | ERROR | | | | |
| 3 | 1/6 | | | | |
| 5 | 3/20 | | | | |
| 7 | 5/42 | | | | |
| 9 | 7/72 | | | | |
| X=9 | | | | | |

E. Using a Slow Reveal by Displaying Selected Inputs and Outputs (MP2, MP3, MP5, MP6, MP7, MP8)

With the setting of Ask/Ask for both columns, you can create a dynamic table to heighten engagement.

1. Change the settings for the table using

2nd **WINDOW**

so your table has columns set to Ask.

NORMAL FLOAT AUTO REAL Radian MP

TABLE SETUP
TblStart=1
ΔTbl=2
Indent: Auto Ask
Depend: Auto Ask

NORMAL FLOAT AUTO REAL Radian MP

| X | Y1 | Y2 | Y3 | Y4 | Y5 | Y6 | Y7 | Y8 |
|----|----------------|----|----|----|----|----|----|----|
| 4 | $\frac{1}{6}$ | | | | | | | |
| 6 | $\frac{2}{15}$ | | | | | | | |
| 8 | $\frac{3}{28}$ | | | | | | | |
| 10 | | | | | | | | |

Y1=

2. Display inputs which are even integers and ask for a conjecture for $x = 10$.
What is the numerator? The denominator?
Press **ENTER** to unveil after conjectures are offered.

NORMAL FLOAT AUTO REAL Radian MP

Plot1 Plot2 Plot3

Y1 $\frac{1}{x} - \frac{1}{x-1} \times \frac{1}{x}$
Y2 $x(x-1)$
Y3=
Y4=
Y5=
Y6=
Y7=
Y8=

3. If more help is needed for the denominator, press Y=.
In Y2 enter the product of the denominators, $x(x-1)$, as shown.

4. You can use a slow reveal and display selected cells in any order you wish.
Sit your cursor on the selected cell and press **ENTER** to unveil its value. Here is one possible sequence.

NORMAL FLOAT AUTO REAL Radian MP

| X | Y1 | Y2 | Y3 | Y4 | Y5 | Y6 | Y7 | Y8 |
|----|----|----|----|----|----|----|----|----|
| 20 | | | | | | | | |
| 22 | | | | | | | | |
| 24 | | | | | | | | |
| 26 | | | | | | | | |

Y1=

NORMAL FLOAT AUTO REAL Radian MP

| X | Y1 | Y2 | Y3 | Y4 | Y5 | Y6 | Y7 | Y8 |
|----|------------------|----|----|----|----|----|----|----|
| 20 | | | | | | | | |
| 22 | $\frac{10}{231}$ | | | | | | | |
| 24 | | | | | | | | |
| 26 | | | | | | | | |

Y2=

NORMAL FLOAT AUTO REAL Radian MP

| X | Y1 | Y2 | Y3 | Y4 | Y5 | Y6 | Y7 | Y8 |
|----|------------------|-----|----|----|----|----|----|----|
| 20 | | | | | | | | |
| 22 | $\frac{10}{231}$ | 462 | | | | | | |
| 24 | | | | | | | | |
| 26 | | | | | | | | |

Y2=

NORMAL FLOAT AUTO REAL Radian MP

| X | Y1 | Y2 | Y3 | Y4 | Y5 | Y6 | Y7 | Y8 |
|----|------------------|-----|----|----|----|----|----|----|
| 20 | | | | | | | | |
| 22 | $\frac{10}{231}$ | 462 | | | | | | |
| 24 | | | | | | | | |
| 26 | | 650 | | | | | | |

Y1=

What is the numerator of the expression in Y1? The denominator? How might Y2 help?

NORMAL FLOAT AUTO REAL Radian MP

| X | Y1 | Y2 | Y3 | Y4 | Y5 | Y6 | Y7 | Y8 |
|----|------------------|-----|----|----|----|----|----|----|
| 20 | | | | | | | | |
| 22 | $\frac{10}{231}$ | 462 | | | | | | |
| 24 | | | | | | | | |
| 26 | $\frac{12}{325}$ | 650 | | | | | | |

Y2=

NORMAL FLOAT AUTO REAL Radian MP

| X | Y1 | Y2 | Y3 | Y4 | Y5 | Y6 | Y7 | Y8 |
|----|------------------|-----|-----|----|----|----|----|----|
| 20 | | | 380 | | | | | |
| 22 | $\frac{10}{231}$ | 462 | | | | | | |
| 24 | | | | | | | | |
| 26 | $\frac{12}{325}$ | 650 | | | | | | |

Y1=

Students may offer how the numerator 10 in the output for $x = 22$ is related to the numerator 11 in the output for $x = 24$ (climb in steps of 1).

Ask students to communicate how $x = 22$ is related to 10, how $x = 24$ is related to 11, how $x = 26$ is related to 12 (subtract 2, then take half).

Ask students to relate the denominator of the value in Y1 with the value in Y2. As more values are displayed they may notice Y2 is twice the value of the denominator of Y1.

This may be clearer if you return to the setting starting at 2 climbing in steps of 2.

If $x = 16$, what is Y1?
(Its numerator is half of 14 and the denominator is half the product of 16×15 .)

$$\frac{1}{x} - \frac{1}{x-1} \cdot \frac{1}{x} = \frac{(x-2)/2}{x(x-1)/2} = \frac{(x-2)}{x(x-1)}$$

NORMAL FLOAT AUTO REAL Radian MP

TABLE SETUP
TblStart=2
ΔTbl=2
Indent: Auto Ask
Depend: Auto Ask

NORMAL FLOAT AUTO REAL Radian MP

PRESS + FOR ΔTbl

| X | Y1 | Y2 | Y3 | Y4 | Y5 | Y6 | Y7 | Y8 |
|----|----------------|----|----|----|----|----|----|----|
| 2 | | 2 | | | | | | |
| 4 | $\frac{1}{6}$ | 12 | | | | | | |
| 6 | $\frac{2}{15}$ | 30 | | | | | | |
| 8 | $\frac{3}{28}$ | 56 | | | | | | |
| 10 | $\frac{4}{45}$ | 90 | | | | | | |

X=2

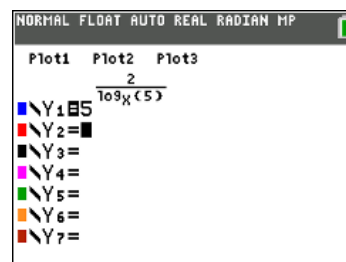
F. Using an Invariant (MP1, MP2, MP3, MP5, MP6, MP7, MP8)

In this investigation students are asked to conjecture what the expression in Y= is based on the table of values. Looks may be deceiving. Students find that the function remains unchanged when one of the

parameters of $y = a^{\frac{b}{\log_x a}}$ is modified.

- Before class, enter the expression $y = a^{\frac{b}{\log_x a}}$ in the Y= editor with $a = 5$ and $b = 2$. Use an Auto/Auto setting and show the table to the class. Ask them what is in your Y1.

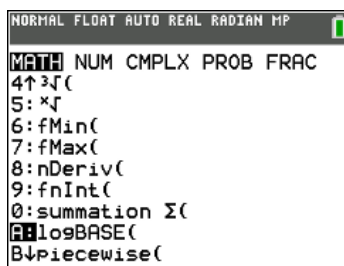
If using TI-SmartView™, you can also use a graphical database to put the expression in Y= without showing students. (See G.)



- Some students may conjecture Y1 contains $y = x^2$. Show your formula in Y1 and ask students to enter this in their own calculators.

| X | Y1 |
|----|-----|
| 2 | 4 |
| 3 | 9 |
| 4 | 16 |
| 5 | 25 |
| 6 | 36 |
| 7 | 49 |
| 8 | 64 |
| 9 | 81 |
| 10 | 100 |
| 11 | 121 |
| 12 | 144 |

Use the Math menu or the shortcut menu ALPHA [F2] for the logBASE function.



- Ask students to explore and discuss the following:
 - What happens when you change the parameter a to any positive number greater than 1?
 - What happens when you change the parameter b to 3? to 1? to 0? to -1?
 - Ask students to use properties of logarithms to explain.

Take the logarithms to the base x of both sides of the equation $y = a^{\frac{b}{\log_x a}}$

$$\log_x y = \log_x a^{\frac{b}{\log_x a}}$$

$$\log_x y = \frac{b}{\log_x a} \cdot \log_x a$$

$$\log_x y = b$$

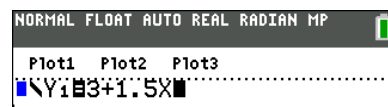
$$y = x^b$$

G. Walking the Table (MP2, MP3, MP7, MP8)

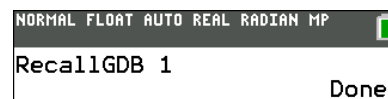
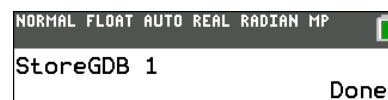
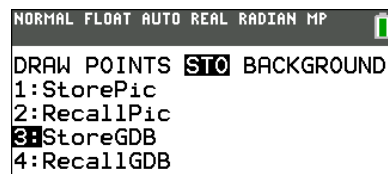
This exploration helps students recognize the tabular representation of a function if they were to see it waking down the street.

1. Enter an equation such as the following equation in Y1:
 $Y = 3 + 1.5X$ (or some other favorite).

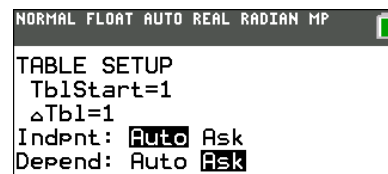
This should be done out of the sight of students.



- Usually computer projectors have a dark screen you can display while you modify the screen.
- Another option is to store the formula in a graph database (GDB) for later use.
 - i. After you have (secretly) entered the formula in Y1:
 - Press **2nd** **prgm** to access the DRAW menu. Press **▶** to reach the STO menu. Select **3: StoreGDB** and press **ENTER** to paste the command on the home screen, type the number (from 1 to 9, or 0) of the GDB variable to which you want to store the graph database and press **ENTER**.
 - ii. Now when you are ready for class, you can recall the graph database.
 - Return to the STO menu by following the above steps, only select **4: Recall GDB**, press **e** to paste the command to the home screen, type the number of your GDB, and press **ENTER**.



2. Select the Ask option for the dependent variable in TBLSET.



3. Reveal the table to the class and press **ENTER** to unveil one output at a time.

| NORMAL FLOAT AUTO REAL RADIAN MP | | | | |
|----------------------------------|-----|--|--|--|
| X | Y1 | | | |
| 1 | 4.5 | | | |
| 2 | 6 | | | |
| 3 | 7.5 | | | |
| 4 | | | | |
| 5 | | | | |
| 6 | | | | |
| 7 | | | | |
| 8 | | | | |
| 9 | | | | |
| 10 | | | | |
| 11 | | | | |

Y1=7.5

Possible prompts for discussion:

- What pattern do you see?
- Could it be linear? Why? (Yes, outputs increase by the same amount.)
- Do you think all of the even inputs will have integer outputs?
- What is the next output? How do you know?

4. Unveil a value anywhere in the table and challenge students to “walk the table” forward ...

| NORMAL FLOAT AUTO REAL RADIAN MP | | | | | |
|----------------------------------|-----|--|--|--|--|
| X | Y1 | | | | |
| 1 | 4.5 | | | | |
| 2 | 6 | | | | |
| 3 | 7.5 | | | | |
| 4 | | | | | |
| 5 | | | | | |
| 6 | | | | | |
| 7 | | | | | |
| 8 | 15 | | | | |
| 9 | | | | | |
| 10 | | | | | |
| 11 | | | | | |

Y1=

.... as well as backward.

| NORMAL FLOAT AUTO REAL RADIAN MP | | | | | |
|----------------------------------|------|--|--|--|--|
| X | Y1 | | | | |
| 1 | 4.5 | | | | |
| 2 | 6 | | | | |
| 3 | 7.5 | | | | |
| 4 | | | | | |
| 5 | | | | | |
| 6 | | | | | |
| 7 | | | | | |
| 8 | 15 | | | | |
| 9 | 16.5 | | | | |
| 10 | | | | | |
| 11 | | | | | |

Y1=

5. Challenge students to find the formula.
Walking backward, students can find the y-intercept is $4.5 - 1.5$.
Once they have the formula, check by substitution.

Highlighting Y1 _____ unveils the formula on the entry line without switching screens.

| NORMAL FLOAT AUTO REAL RADIAN MP PRESS ENTER TO EDIT | | | | | |
|---|------|--|--|--|--|
| X | Y1 | | | | |
| 1 | 4.5 | | | | |
| 2 | 6 | | | | |
| 3 | 7.5 | | | | |
| 4 | | | | | |
| 5 | | | | | |
| 6 | | | | | |
| 7 | 13.5 | | | | |
| 8 | 15 | | | | |
| 9 | 16.5 | | | | |
| 10 | | | | | |
| 11 | | | | | |

Y1=3+1.5X

6. Lead the group through several of these explorations.

- Engagement heightens when you have participants provide their own guess for the next input.
(Set Independent to **Ask** to change the x values.)
- Discuss choices for x that might make the function easier to guess.
- After you enter a new function, when Indpnt is Ask you will need to delete the x-values in the table before playing the game again.

| NORMAL FLOAT AUTO REAL RADIAN MP | | | | | |
|----------------------------------|--|--|--|-----|--|
| TABLE SETUP | | | | | |
| TblStart=1 | | | | | |
| ΔTbl=1 | | | | | |
| Indpt: Auto | | | | Ask | |
| Depend: Auto | | | | Ask | |

TIP: Load more than one equation into the same graph database prior to class.

Variation

How might this activity be used to construct and compare linear, quadratic, and exponential models and solve problems? For example, table investigations can build the ability to recognize the formula of a quadratic function from its numerical representation alone:

| NORMAL FLOAT AUTO REAL RADIAN MP PRESS ENTER TO EDIT | | | | | |
|---|----|--|--|--|--|
| X | Y1 | | | | |
| 1 | 35 | | | | |
| 2 | 26 | | | | |
| 3 | 19 | | | | |
| 4 | 14 | | | | |
| 5 | 11 | | | | |
| 6 | 10 | | | | |
| 7 | 11 | | | | |
| 8 | 14 | | | | |
| 9 | 19 | | | | |
| 10 | 26 | | | | |
| 11 | 35 | | | | |

This shows $a = 1$
 $y = (x - 6)^2 + 10$

Y1=(X-6)²+10

| NORMAL FLOAT AUTO REAL RADIAN MP PRESS ENTER TO EDIT | | | | | |
|---|-----|--|--|--|--|
| X | Y1 | | | | |
| -3 | -80 | | | | |
| -2 | -44 | | | | |
| -1 | -16 | | | | |
| 0 | 4 | | | | |
| 1 | 16 | | | | |
| 2 | 20 | | | | |
| 3 | 16 | | | | |
| 4 | 4 | | | | |
| 5 | -16 | | | | |
| 6 | -44 | | | | |
| 7 | -80 | | | | |

This shows $a = -4$
 $y = -4(x - 2)^2 + 20$

Y1=-4(X-2)²+20

H. Building Tables with Recursive Sequences (MP2, MP3, MP7, MP8)

Use the table in sequence mode to display the first few terms of a formula to the class. Students use recursive reasoning and problem solving strategies to find the next number in the sequence. For less difficult sequences, students also find the function defined explicitly and/or recursively.

1. Do the following before class or without displaying to students:

- Press **MODE**, highlight Sequence mode, then press **(ENTER)**.
- Press **(Y=)**. Your graphing variable is now n instead of x .

2. Change the settings for the table using **(2nd)** **(WINDOW)** so your table starts at 1 and climbs in steps of $\Delta Tbl = 1$.

3. You can get u , v , and w off the keypad or the shortcut menu by pressing **(ALPHA)** **[F4]**.

Enter the settings shown:

$$\begin{array}{ll} nMin = 1 & \text{the beginning input } u_n = u_{n-1} + n \\ u(n) = u(n-1) + n & \text{the recursive rule } u_1 = 1 \\ u(nMin) = 1 & \text{the beginning output} \end{array}$$

4. Store this into the graph database of your choice.

(See **G. Walking the Table** for steps.)

You may wish to enter more functions that are suggested in this activity at this time and store them all into the same GDB.

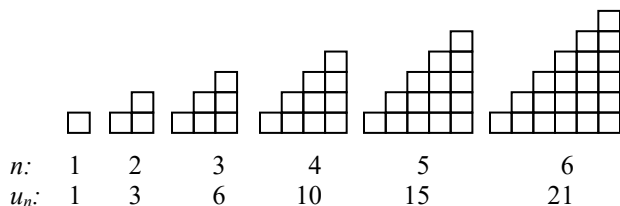
During class, recall the graph database.

Divide students in pairs. Ask students for the next number in the sequence as you place your cursor on the outputs and press **(ENTER)** to reveal each one. Ask for reasons for the answers. (Δu climbs by 2, 3, 4, etc.)

5. Once they have found the rule (The n th term is the PREVIOUS + n), you can show students the formula by walking the cursor up to the top of the second column.

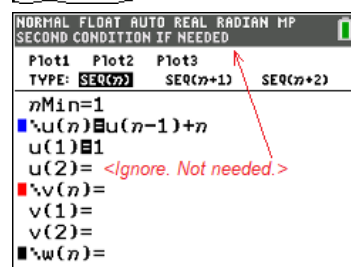
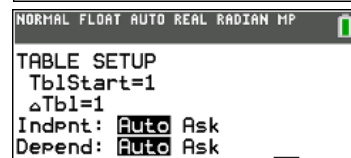
An equivalent “closed form” rule is $u_n = n(n-1)/2$ with $u_1 = 1$.

Note: These are called the **triangular numbers**.



The triangular numbers show up in several modeling scenarios.

- How many different handshakes are possible in a room with n people?
- Seven people are entered in a table-tennis tournament. If each person plays one game with each of the other persons in the tournament, how many games will be played together?



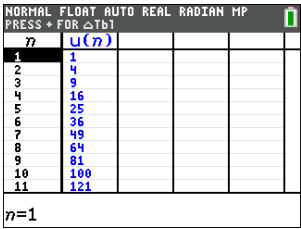
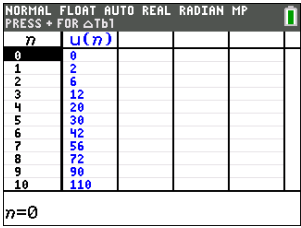
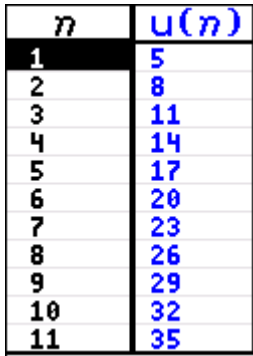
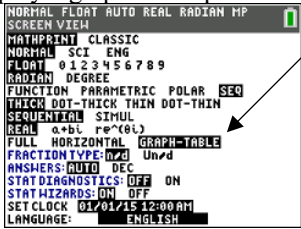
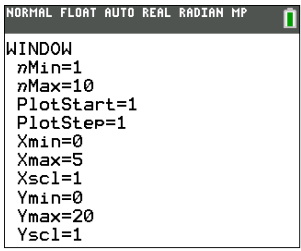
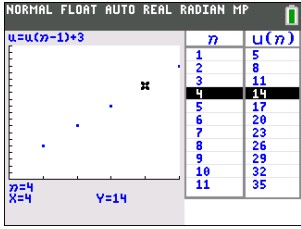
| n | $u(n)$ | | | |
|-----|--------|--|--|--|
| 1 | 1 | | | |
| 2 | 3 | | | |
| 3 | 6 | | | |
| 4 | 10 | | | |
| 5 | | | | |
| 6 | | | | |
| 7 | | | | |
| 8 | | | | |
| 9 | | | | |
| 10 | | | | |
| 11 | | | | |

$u(5)=$

| n | $u(n)$ | | | |
|-----|--------|--|--|--|
| 1 | 1 | | | |
| 2 | 3 | | | |
| 3 | 6 | | | |
| 4 | 10 | | | |
| 5 | 15 | | | |
| 6 | 21 | | | |
| 7 | | | | |
| 8 | | | | |
| 9 | | | | |
| 10 | | | | |
| 11 | | | | |

$u(n)=u(n-1)+n$

6. Give additional examples. Some ideas follow (or make up your own.)

| Create this rule behind the scenes | Display this to students | Ask them to find what is next. |
|---|---|---|
| <div> Plot1 Plot2 Plot3 TYPE: SEQ(n) SEQ(n+1) SEQ(n+2) nMin=1 <u>u(n)</u> u(n-1)+2n-1 u(1) 1 u(2)= $u_n = u_{n-1} + 2n - 1$ $u_1 = 1$ </div> |  | <p>Most may recognize the square numbers, but if this example comes immediately after the triangular numbers, some students may notice the difference between successive values is 3, 5, 7, 9, respectively.</p> |
| <div> Plot1 Plot2 Plot3 TYPE: SEQ(n) SEQ(n+1) SEQ(n+2) nMin=1 <u>u(n)</u> u(n-1)+2n u(1) 1 u(2)= $u_n = u_{n-1} + 2n$ $u_0 = 0$ <p>Explicitly, it is this quadratic function:</p> <div> Plot1 Plot2 Plot3 TYPE: SEQ(n) SEQ(n+1) SEQ(n+2) nMin=1 <u>u(n)</u> n(n+1) u(1) 2 u(2)= $u_n = n(n+1)$ $u_1 = 2$ </div> </div> |  | <p>These are called the rectangular numbers or oblong numbers. Some may notice the difference between successive values is 2, 4, 6, 8, 10, respectively.</p> <p>Others may see the products:</p> $0 \times 1 = 0$ $1 \times 2 = 2$ $2 \times 3 = 6$ $3 \times 4 = 12$ <p>Others may see they are twice the triangular numbers.</p> |
| <div> Plot1 Plot2 Plot3 TYPE: SEQ(n) SEQ(n+1) SEQ(n+2) nMin=1 <u>u(n)</u> u(n-1)+3 u(1) 5 u(2)= $u_n = u_{n-1} + 3$ $u_1 = 5$ <p>Explicitly, it is this linear function:</p> <div> Plot1 Plot2 Plot3 TYPE: SEQ(n) SEQ(n+1) SEQ(n+2) nMin=0 <u>u(n)</u> 3n+2 u(1) 2 u(2)= $nMin = 0$ $u_n = 3n + 2$ $u_1 = 2$ </div> </div> |  | <p>This is an arithmetic sequence with common difference 3 starting at 5. Display a graph in Graph-Table Mode</p>   <p>Press TRACE and left arrow.</p>  |

7. The following have rules that may be too challenging to guess, but you can ask for the **next** terms.

| Create this rule behind the scenes | Display this to students | Ask them to find what is next. |
|--|--------------------------|---|
| <div> Plot1 Plot2 Plot3 TYPE: SEQ(n) SEQ(n+1) SEQ(n+2) nMin=0 $u(n) = (-1)^n + 2n$ $u(0) = 1$ $u(1) = 1$ $u_n = (-1)^n + 2n$ $u_0 = 1$ $u_1 = 1$ </div> | | <p>Students may find it easy to predict the next value is 9.</p> <p>Some may notice the difference between successive pairs of values is 4.</p> |
| <div> Plot1 Plot2 Plot3 TYPE: SEQ(n) SEQ(n+1) SEQ(n+2) nMin=0 $u(n) = (-1)^n + u(n-1) - 1$ $u(0) = 1$ $u(1) = 1$ $u_n = (-1)^n + u_{n-1} - 1$ $u_0 = 1$ </div> | | <p>For increased drama, set Indpt to Auto and Depnt to Ask to unveil one output at a time.</p> <p>After finding the next term in the sequence, ask for 49th and 50th terms. Set Indpt to Ask and Depnt to Ask and check.</p> <p>Be patient as you wait. It calculates all of the previous terms behind the scenes before it displays the results.</p> |
| <div> Plot1 Plot2 Plot3 TYPE: SEQ(n) SEQ(n+1) SEQ(n+2) nMin=0 $u(n) = (-1)^n * u(n-1) - 1$ $u(0) = 1$ $u(1) = 1$ $u_n = (-1)^n * u_{n-1} - 1$ $u_0 = 1$ </div> | | <p>Use the strategy for the previous sequence to facilitate the class discussion.</p> |
| <div> NORMAL FLOAT AUTO REAL RADIAN MP SECOND CONDITION IF NEEDED Plot1 Plot2 Plot3 TYPE: SEQ(n) SEQ(n+1) SEQ(n+2) nMin=1 $u(n+2) = u(n+1) + u(n)$ $u(1) = 1$ $u(2) = 1$ $v(n+2) = \frac{u(n+1)}{u(n)}$ $v(1) = 1$ $v(2) = 1$ </div> | | <p>Ask students to compare how the columns are related.</p> <p>This table delights in the sweet aroma of Fibonacci.</p> |
| <p>Add the following to the above if desired:</p> <div> $w(n+2) = \frac{u(n+1)}{u(n)}$ $w(1) = 1$ $w(2) = 1$ $w(n+2) = u(n+1) / u(n)$ $w(1) = 1$ $w(2) = 1$ </div> | | <p>To see the golden ratio, include the fractional expression in w using \div and (patiently) scroll the table.</p> <p>See the presentation Embark on the Voyage of Discovery with the TI-84 Plus C Silver Edition and the CCSS at the site users.pfw.edu/lamaster/technology/ for a way to build this sequence on the home screen with the Last Answer feature similar to parts A and B of this handout.</p> |

8. When finished with the activity, press MODE and return to function mode.

Warning: It is **not** a good idea to use Reset Defaults (2nd MEM 7 2 2) between classes with this activity because it will deselect all of your functions in the Y= Editor.

| |
|--|
| NORMAL FLOAT AUTO REAL RADIAN MP FUNCTION TYPES MATHPRINT CLASSIC NORMAL SCI ENG FLOAT 0 1 2 3 4 5 6 7 8 9 RADIAN DEGREE FUNCTION PARAMETRIC POLAR SEQ THICK DOT-THICK THIN DOT-THIN SEQUENTIAL SIMUL REAL a+bi re^(tL) FULL HORIZONTAL GRAPH-TABLE FRACTIONTYPE: 0/0 Unrd ANSWERS: AUTO DEC STAT DIAGNOSTICS: OFF ON STAT WIZARDS: ON OFF SET CLOCK 01/01/15 12:00 AM LANGUAGE: ENGLISH |
|--|