

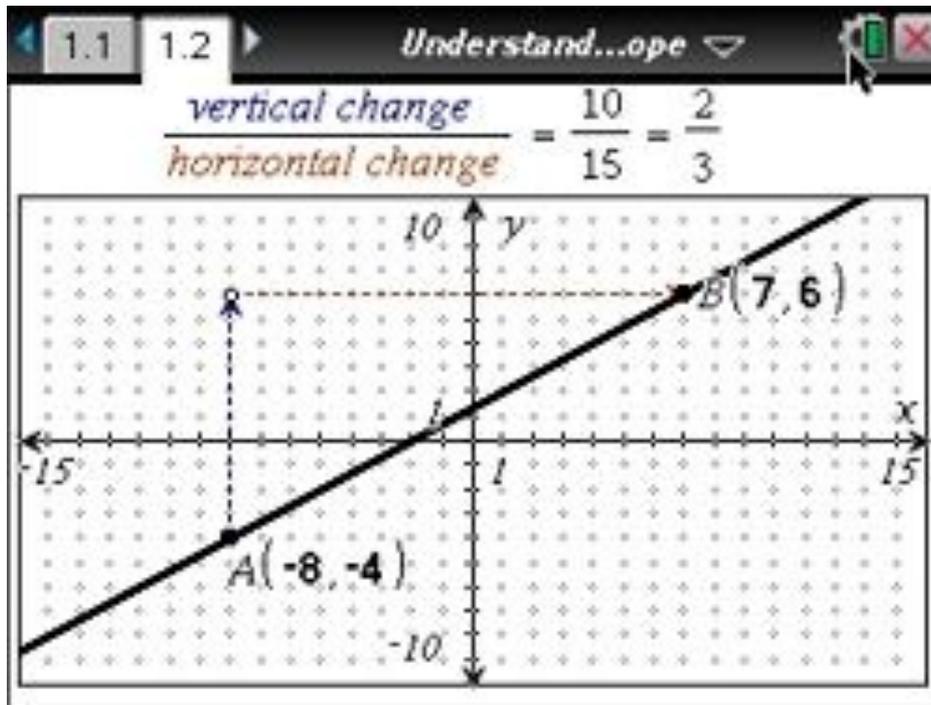
Rates from Algebra to Calculus



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Understanding Slope

- Students need to “experience” slope
- What do we reference when comparing slopes?
- What is the “Parent Function”?



Connecting Rates with Multiple Representations

Problem adapted from Connected Mathematics Project, Moving Straight Ahead with the assistance of Dr. James Fetterly from University of Central Arkansas.

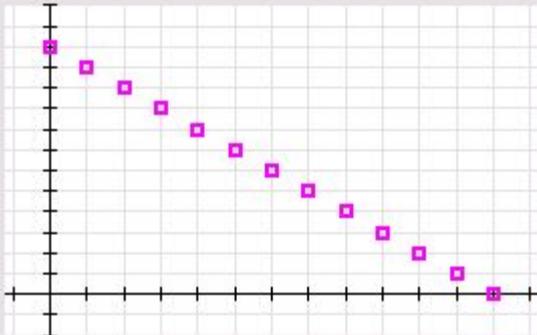
The First Situation: Ms. Chang's class collected \$144 from their walkathon fundraiser to provide books for the children's ward at the hospital. They put the money in the school safe and withdraw a fixed amount of \$12 each week to buy new books. To keep track of the money, Isabella makes a table of the amount of money in the account at the end of each week.

Take a few moments to recreate the table Isabella made. What would the graph of her table look like? Could we make an equation (do we even need to)?

The First Situation

NORMAL FLOAT AUTO REAL DEGREE MP					
L1	L2	L3	L4	L5	2
0	144	-----	-----	-----	
1	132				
2	120				
3	108				
4	96				
5	84				
6	72				
7	60				
8	48				
9	36				
10	24				

NORMAL FLOAT AUTO REAL DEGREE MP					
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Week	Value	Subtraction	Rewrite	Rewrite 2
0	144	144	144 - 0	144 - 12(0)
1	132	144-12	144 - 12	144 - 12(1)
2	120	144-12-12	144 - 24	144 - 12(2)
3	108	144-12-12-12	144 - 36	144 - 12(3)
4	96	144-12-12-12-12	144 - 48	144 - 12(4)
5	84	144-12-12-12-12-12	144 - 60	144 - 12(5)
6	72	144-12-12-12-12-12-12	144 - 72	144 - 12(6)
N	A			$A = 144 - 12N$

Connecting Rates with Multiple Representations

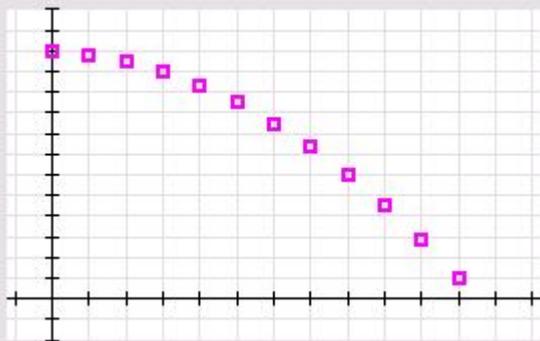
The Second Situation: Ms. Chang's class collected \$144 from their walkathon fundraiser to provide books for the children's ward at the hospital. They put the money in the school safe and withdraw a \$2 for the first week, then \$4 the second week, then \$6 for the third week, and so on (\$2 more than the previous week) to buy new books. To keep track of the money, Isabella makes a table of the amount of money in the account at the end of each week.

Take a few moments to recreate the table Isabella made. What would the graph of her table look like? Could we make an equation (do we even need to)?

The Second Situation

L1	L2	L3	L4	L5	3
0	144	-2	-----	-----	
1	142	-4			
2	138	-6			
3	132	-8			
4	124	-10			
5	114	-12			
6	102	-14			
7	88	-16			
8	72	-18			
9	54	-20			
10	34	-22			

L1	L2	L3	L4	L5	3
0	144	-2	-----	-----	
1	142	-4			
2	138	-6			
3	132	-8			
4	124	-10			
5	114	-12			
6	102	-14			
7	88	-16			
8	72	-18			
9	54	-20			
10	34	-22			



Week	Value	Subtraction	Rewrite	Rewrite 2
0	144	144	144 - 0	144 - 1(0)
1	142	144-2	144 - 2	144 - 2(1)
2	138	144-2-4	144 - 6	144 - 3(2)
3	132	144-2-4-6	144 - 12	144 - 4(3)
4	124	144-2-4-6-8	144 - 20	144 - 5(4)
5	114	144-2-4-6-8-10	144 - 30	144 - 6(5)
6	102	144-2-4-6-8-10-12	144 - 42	144 - 7(6)
N	A			$A = 144 - (N+1)N$ $A = 144 - N - N^2$

Connecting Rates with Multiple Representations

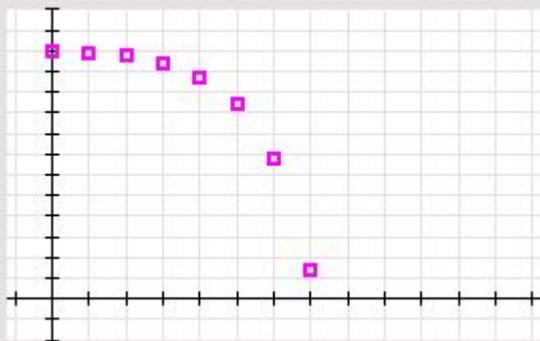
The Third Situation: Ms. Chang's class collected \$144 from their walkathon fundraiser to provide books for the children's ward at the hospital. They put the money in the school safe and withdraw \$1 the first week, then \$2 the second week, and then \$4 the third week and so on (doubling the amount withdrawn each week) to buy new books. To keep track of the money, Isabella makes a table of the amount of money in the account at the end of each week.

Take a few moments to recreate the table Isabella made. What would the graph of her table look like? Could we make an equation (do we even need to)?

The Third Situation

L1	L2	L3	L4	L5	3
0	144	-1	-----	-----	
1	143	-2			
2	141	-4			
3	137	-8			
4	129	-16			
5	113	-32			
6	81	-64			
7	17	-----			
-----	-----				

L1	L2	L3	L4	L5	3
0	144	-1	-----	-----	
1	143	-2			
2	141	-4			
3	137	-8			
4	129	-16			
5	113	-32			
6	81	-64			
7	17	-----			
-----	-----				



Week	Value	Subtraction	Rewrite	Rewrite 2
0	144	144	144 - 0	144 - (2 ⁰ - 1)
1	132	144-1	144 - 1	144 - (2 ¹ - 1)
2	120	144-1-2	144 - 3	144 - (2 ² - 1)
3	108	144-1-2-4	144 - 7	144 - (2 ³ - 1)
4	96	144-1-2-4-8	144 - 15	144 - (2 ⁴ - 1)
5	84	144-1-2-4-8-16	144 - 31	144 - (2 ⁵ - 1)
6	72	144-1-2-4-8-16-32	144 - 63	144 - (2 ⁶ - 1)
N	A			A = 144 - (2 ^N - 1)

Rate of change can change before calculus!!!

Linear

L1	L2	L3	L4	L5	3
0	144	-12	-----	-----	
1	132	-12			
2	120	-12			
3	108	-12			
4	96	-12			
5	84	-12			
6	72	-12			
7	60	-12			
8	48	-12			
9	36	-12			
10	24	-----			

$L_3(1) = -12$

Quadratic

L1	L2	L3	L4	L5	3
0	144	-2	-----	-----	
1	142	-4			
2	138	-6			
3	132	-8			
4	124	-10			
5	114	-12			
6	102	-14			
7	88	-16			
8	72	-18			
9	54	-20			
10	34	-22			

$L_3 = \{-2, -4, -6, -8, -10, -12, -14\}$

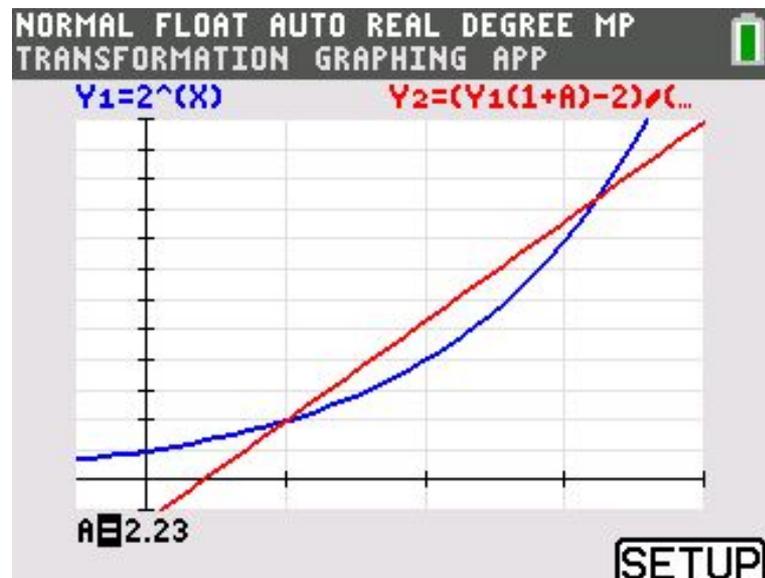
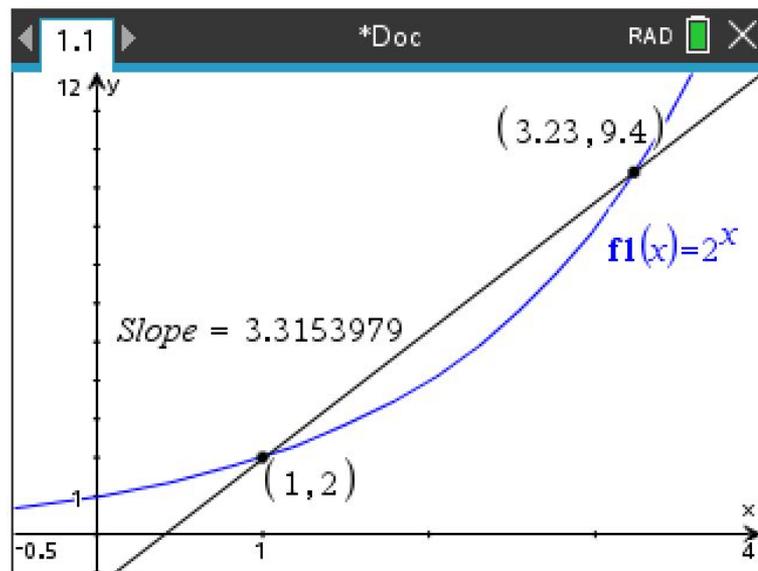
Exponential

L1	L2	L3	L4	L5	3
0	144	-1	-----	-----	
1	143	-2			
2	141	-4			
3	137	-8			
4	129	-16			
5	113	-32			
6	81	-64			
7	17	-----			
-----	-----				

$L_3(1) = -1$

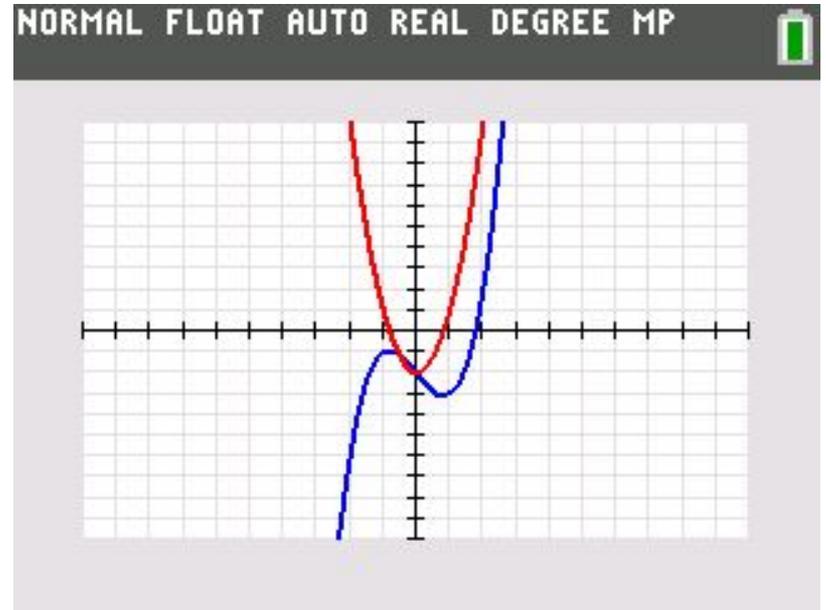
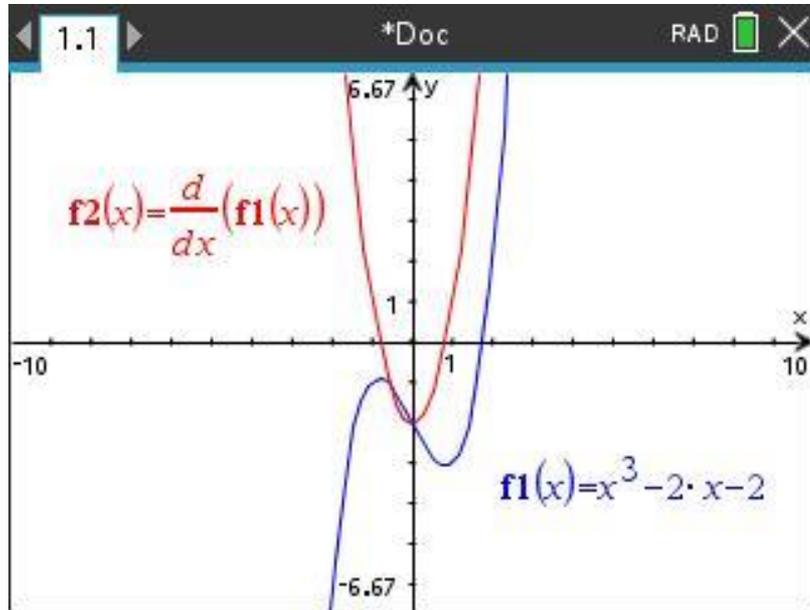
Moving onto Calculus

Secant Lines approaching a tangent



Moving onto Calculus

Graphing Derivatives



AP Problem - AB/BC #1 2021, Part a

r (centimeters)	0	1	2	2.5	4
$f(r)$ (milligrams per square centimeter)	1	2	6	10	18

- The density of a bacteria population in a circular petri dish at a distance r centimeters from the center of the dish is given by an increasing, differentiable function f , where $f(r)$ is measured in milligrams per square centimeter. Values of $f(r)$ for selected values of r are given in the table above.
 - Use the data in the table to estimate $f'(2.25)$. Using correct units, interpret the meaning of your answer in the context of this problem.

AB/BC #1 2021, Part a - Scoring

Worth two points

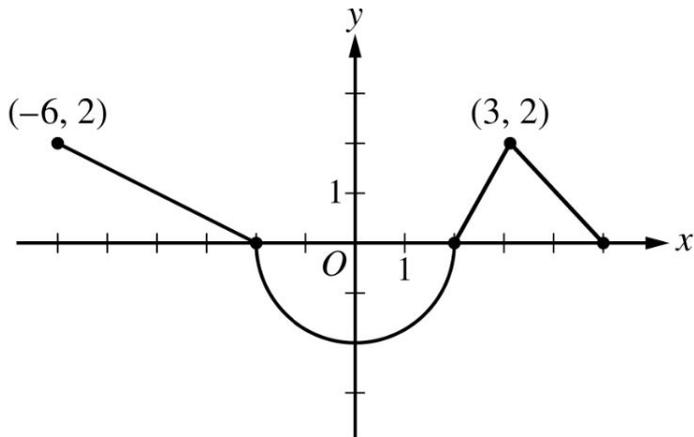
1st point is earned for the estimate: $(10-6)/(2.5-2) = 8$

- Must provide a difference and quotient and use table values
- Simplification not required, errors in simplification cost the point

2nd point is earned for interpretation: At 2.25 cm from the center of the petri dish the density of the population is increasing at a rate of 8 mg/cm^3

- Must have 3 things: the distance, density increasing/decreasing (changing) at a rate consistent with 1st point, and correct units.

AB4 b&d, 2017



Graph of f'

3. The function f is differentiable on the closed interval $[-6, 5]$ and satisfies $f(-2) = 7$. The graph of f' , the derivative of f , consists of a semicircle and three line segments, as shown in the figure above.
- Find the values of $f(-6)$ and $f(5)$.
 - On what intervals is f increasing? Justify your answer.
 - Find the absolute minimum value of f on the closed interval $[-6, 5]$. Justify your answer.
 - For each of $f''(-5)$ and $f''(3)$, find the value or explain why it does not exist.

AB4, 2017 , Scoring, Parts b&d

(b) $f'(x) > 0$ on the intervals $[-6, -2)$ and $(2, 5)$.

Therefore, f is increasing on the intervals $[-6, -2]$ and $[2, 5]$.

$$(d) f''(-5) = \frac{2 - 0}{-6 - (-2)} = -\frac{1}{2}$$

$$\lim_{x \rightarrow 3^-} \frac{f'(x) - f'(3)}{x - 3} = 2 \text{ and } \lim_{x \rightarrow 3^+} \frac{f'(x) - f'(3)}{x - 3} = -1$$

$f''(3)$ does not exist because

$$\lim_{x \rightarrow 3^-} \frac{f'(x) - f'(3)}{x - 3} \neq \lim_{x \rightarrow 3^+} \frac{f'(x) - f'(3)}{x - 3}.$$

2 : answer with justification

2 : $\left\{ \begin{array}{l} 1 : f''(-5) \\ 1 : f''(3) \text{ does not exist,} \\ \text{with explanation} \end{array} \right.$