Avon High School
AP Calculus AB

SOLUTIONS

Period _____ Score ______ / 10

Skill Builder: Rate In/Rate Out Problems (Circuit Within a Circuit)

Begin in the first cell marked #1 and find the requested information. To advance in the circuit, search for your answer and mark that cell #2. Continue in this manner until you complete the circuit. Then verify your results by picking one of the remaining cells, calling it A, and continuing with another circuit until it loops back to A. The numbered circuit and lettered circuit will eventually use every cell. You will need your calculator.

 # 1 **Ans:** 13,738

A cylindrical tank is having water pumped in at a rate of 45 liters/minute and is releasing water from a valve at its bottom at a rate of 25 liters/minute. After five minutes, what is the net change in the volume of the water in the tank in liters?

$$\text{Change in Volume} = (45)(5) - (25)(5)$$
$$= 225 - 125$$
$$= 100$$

$14 + \int_1^{2.5} 9\,dt - \int_0^{2.5} 7\,dt$ 10.000

 # _9_ **Ans:** 5.188

A stormwater tank contains 13,000 gallons at the start of a rain storm (t = 0). Over the next $0 \le t \le 24$ hours, rain adds water to the tank at a rate modeled by $r(t) = 110 + 15\sin\left(0.5\sqrt{t}\right)$ in gallons per hour and usage by buildings attached to the tank is modeled by $u(t) = 95 - 10\cos\left(\frac{t}{24}\right)$ in gallons per hour. How much water (to the nearest gallon) is in the tank 20 hours after the storm begins?

$$W(20) = 13000 + \int_0^{20} \left(110 + 15\sin\left(0.5\sqrt{t}\right)\right)dt - \int_0^{20} \left(95 - 10\cos\left(\frac{t}{24}\right)\right)dt$$
$$= 13737.684$$

 # _7_ **Ans:** 148.851

A grocery store has their new cereal Calculos® on sale. For the first six hours the store is open, Calculos® are removed from the shelf at a rate modeled for $0 \le t \le 6$ by $f(t) = 14 + 3t\sin\left(\frac{t^2}{3}\right)$.

Two hours after opening, employees add boxes to the shelf at a rate modeled by $g(t) = 5 + 3\ln\left(t^2 + 2\right)$ for $2 \le t \le 6$. Is the number of boxes of Calculos® increasing or decreasing at t = 4? Find the value at which the number of boxes is decreasing or increasing per hour to advance in the circuit.

$\text{Removal Rate} = 14 + 3(4)\cdot\sin\left(\frac{4^2}{3}\right) \approx 4.240$

$\text{Added Rate} = \left(5 + 3\ln\left(4^3 + 2\right)\right) \approx 17.569$

Added Rate > Removal Rate (The number of boxes is increasing)

$17.569 - 4.240 = 13.329$

$14+3\cdot 4\cdot \sin\left(\frac{4^2}{3}\right)$	4.240
$5+3\cdot \ln\left(4^3+2\right)$	17.569
17.569−4.24	13.329

 # _4_ **Ans:** 149.302

Water is entering a pond from the north inlet. Water is exiting from the south inlet. Rates sampled from both inlets at specific times are shown in the table below. Use left-hand Riemann sums to approximate the net change in the pond's water volume (in liters) over the 20 minutes samples.

t (minutes)	0	8	14	20
North inlet (l/min)	4200	4800	3800	4400
South inlet (l/min)	5000	4800	4600	4900

$V_N = (8)(4200) + (6)(4800) + (6)(3800) = 85{,}200$

$V_S = (8)(5000) + (6)(4800) + (6)(4600) = 96{,}400$

$V_N - V_S = -11{,}200$

8· 4200+6· 4800+6· 3800	85200
8· 5000+6· 4800+6· 4600	96400
85200−96400	-11200

# _A_	**Ans:** -17.318

The rate of a snowfall on a driveway is modeled in inches/hour by $s(t)=\frac{1}{4}\ln(4+2\cos t)$ where t is hours since 7:00 am. At 9:00 am, Mike begins removing snow at a rate modeled by $m(t)$ = 1 inch/hour. If there was no snow on the driveway at 7:00 am and snow stops falling at 10:00 am, how many inches of snow remain for Mike to remove?

$$\int_0^3\left(\frac{1}{4}\ln(4+2\cos t)\right)dt-\int_2^3 1\,dt\approx 0.010$$

$$\int_0^3\left(\frac{1}{4}\cdot\ln(4+2\cdot\cos(t))\right)dt-\int_2^3 1\,dt \qquad 0.010$$

# _2_	**Ans:** 100

From the time a facility processing sand for sandboxes opens, sand is brought in at a constant rate of 500 lbs/hour. Sand is filtered, sanitized, and packaged. The play sand ships out at a rate modeled by $s(t)=395-2t\cos\left(\frac{1}{2}t\right)$ where t is measured in hours and s in pounds/hour. How much sand has been shipped 4 hours after the facility has opened?

$$\int_0^4 395-2t\cos\left(\frac{1}{2}t\right)dt\approx 1576.780$$

$$\int_0^4\left(395-2\cdot t\cdot\cos\left(\frac{1}{2}\cdot t\right)\right)dt \qquad 1576.780$$

# _B_	**Ans:** 0.010

A segment of pipe contains a blockage and 14 gallons of water at time t = 0. Water is draining from one end of the pipe at 7 gallons per hour. After 1 hour, water begins entering the pipe at the opposite end at a rate of 9 gallons per hour. How much water is in the pipe (in gallons) at time t = 2.5?

$$W(2.5)=14+\int_1^{2.5}9dt-\int_0^{2.5}7\,dt \quad \text{or} \quad 14+(1.5)(9)+(2.5)(7)$$

$$=10$$

# _5_	**Ans:** $-11,200$

Sweet tea is being brewed by workers at a restaurant at a constant rate of 1,920 oz per hour during peak business hours. At the start of peak hours, there are 960 oz of tea already made. Customers are known to be served tea at the rates given in the table below. Use a right-hand Riemann sum to approximate how much tea is available in ounces after the 4-hour peak period.

t (hours)	0	1	2	4
Serving (oz/hour)	2000	1800	2100	2000

$$\text{Amount of Tea Served}=(1)(1800)+(1)(2100)+(2)(2000)=7900$$

$$\text{Amount of Tea Brewed}=(4)(1920)=7680$$

$$\text{Tea Available (at time } t=4)=960+7680-7900=740$$

$1\cdot 1800+1\cdot 2100+2\cdot 2000$	7900
$4\cdot 1920$	7680
$960+7680-7900$	740

_3_

Ans: 1,576.780

A tank is having a liquid pumped in at a rate modeled by $p(t)=t+1-\dfrac{2}{t+2}$ gallons per hour (t is in hours) and is releasing the liquid at a rate modeled by $d(t)=1.5\ln(t+1)$ gallons/hour. After 20 hours what is the net change in the volume of the liquid in the tank in gallons?

$$\int_0^{20}\left(t+1-\frac{2}{t+2}\right)dt-\int_0^{20}\left(1.5\ln(t+1)\right)dt\approx 149.302$$

$$\int_0^{20}\left(t+1-\frac{2}{t+2}\right)dt-\int_0^{20}\left(1.5\cdot\ln(t+1)\right)dt$$

149.302

_C_

Ans: 10

At the start of the workday (t = 0), a factory has a surplus of 140 spinning toys. During a seven-hour shift, they manufacture spinning toys at a rate modeled by $s(t)=15-e^{\sin(2t)}$ in toys/hr. They ship out toys at a rate modeled by the function $m(t)=30+\ln\left(5+\cos(3t)\right)$. What is the rate of change in toys/hr of the inventory of toys in the factory at t = 6?

$$s(6)-m(6)=15-e^{\sin(2(6))}-\left(30+\ln\left(5+\cos(3(6))\right)\right)$$
$$=-17.318$$

$$15-e^{\sin(2\cdot 6)}-\left(30+\ln\left(5+\cos(3\cdot 6)\right)\right)$$

-17.318

_8_

Ans: 13.329

At a school pofessional development workshop, a generous administrator provided coffee at a rate modeled by $c(t)=30$ for $0\le t\le 5$ (in cups per hour). Teachers are drinking the coffee at a rate modeled by $d(t)=25-\dfrac{1}{15}t^2+10\cos(2t)$ for $0\le t\le 5$. Find how much the rate of change of the amount of coffee available is decreasing at t = 3. Identify units and find the numerical value to proceed.

$$A(t)=\int_0^t 30\,dx-\int_0^t\left(25-\frac{1}{15}x^2+10\cos(2x)\right)dt$$

$$A'(t)=30-\left(25-\frac{1}{15}t^2+10\cos(2t)\right)$$

$$A''(3)=\frac{d}{dt}\left[\left(30-\left(25-\frac{1}{15}t^2+10\cos(2t)\right)\right)\right]_{t=3}\approx-5.188$$

The rate of change of the amount of coffee is decreasing by 5.188 cups/hour^2

$$\frac{d}{dt}\left(30-\left(25-\frac{1}{15}\cdot t^2+10\cdot\cos(2\cdot t)\right)\right)\Big|t=3.$$

-5.188

_6_

Ans: 740

A spherical tank on benzine is being filled at a rate modeled by $f(t)=\dfrac{1}{5}t^2-6\sin\left(\dfrac{1}{2}t-3\right)+4$ for $0\le t\le 10$.

At t = 2, the tank starts leaking benzine at a rate modeled by $g(t)=e^{\cos t}+7$ for $2\le t\le 10$. Both $f(t)$ and $g(t)$ are measured in liters/hr and t is in hours. What is the volume of the tank at t = 10 (in liters) if the initial amount of benzine in the tank was 100 liters?

$$A_{Filled}=\int_0^{10}\left(\frac{1}{5}t^2-6\sin\left(\frac{1}{2}t-3\right)+4\right)dt\approx 113.553$$

$$A_{Leaked}=\int_2^{10}\left(e^{\cos t}+7\right)dt\approx 64.702$$

$$A(10)=100+A_{Filled}-A_{Leaked}=100+113.553-64.702=148.581$$

$$\int_0^{10}\left(\frac{1}{5}\cdot t^2-6\cdot\sin\left(\frac{1}{2}\cdot t-3\right)+4\right)dt \quad 113.553$$

$$\int_2^{10}\left(e^{\cos(t)}+7\right)dt \quad 64.702$$

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