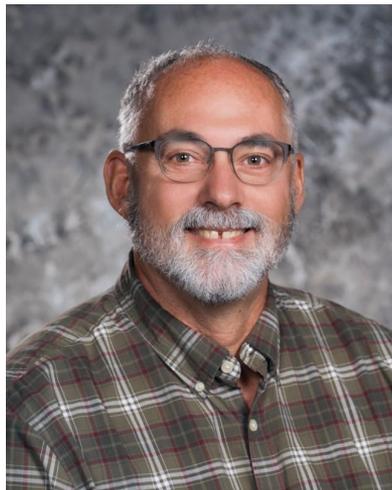


Puzzle Problems

A Path to Student Justification

Tim Collier



Tracy Matthews



Math teachers from McAlester High School, McAlester Oklahoma

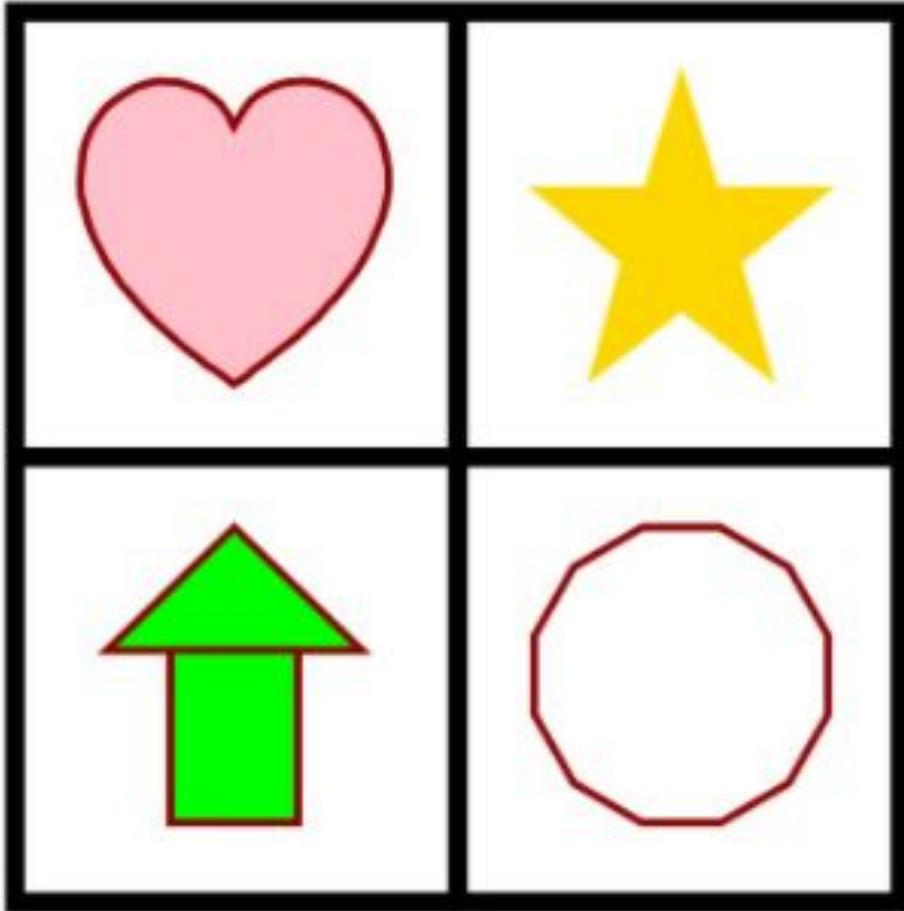
Tim taught math for 34 years.
Secondary Academic Design Coordinator.

Tracy has been teaching for 20 years
Math department chair.

Waterfall Response Question

There is a question on the next slide. We will give you 45 seconds to type your response in the chat. When we say go.....everyone submit your answers at the same time.

Which One Does Not Belong? Justify your answer.



Our Journey Thus Far

Today's presentation reflects an ongoing project to understand this idea. The presentation is organized following our own growth.

- Seven years of GEAR/UP- TI training with Doug Roberts and Sandra Hocutt
- T³™ International Conference Attendees and Presenters
- T³™ Teacher Leader Cadre 2017
- T³™ Regional Instructor
- Oklahoma Excel Community Members

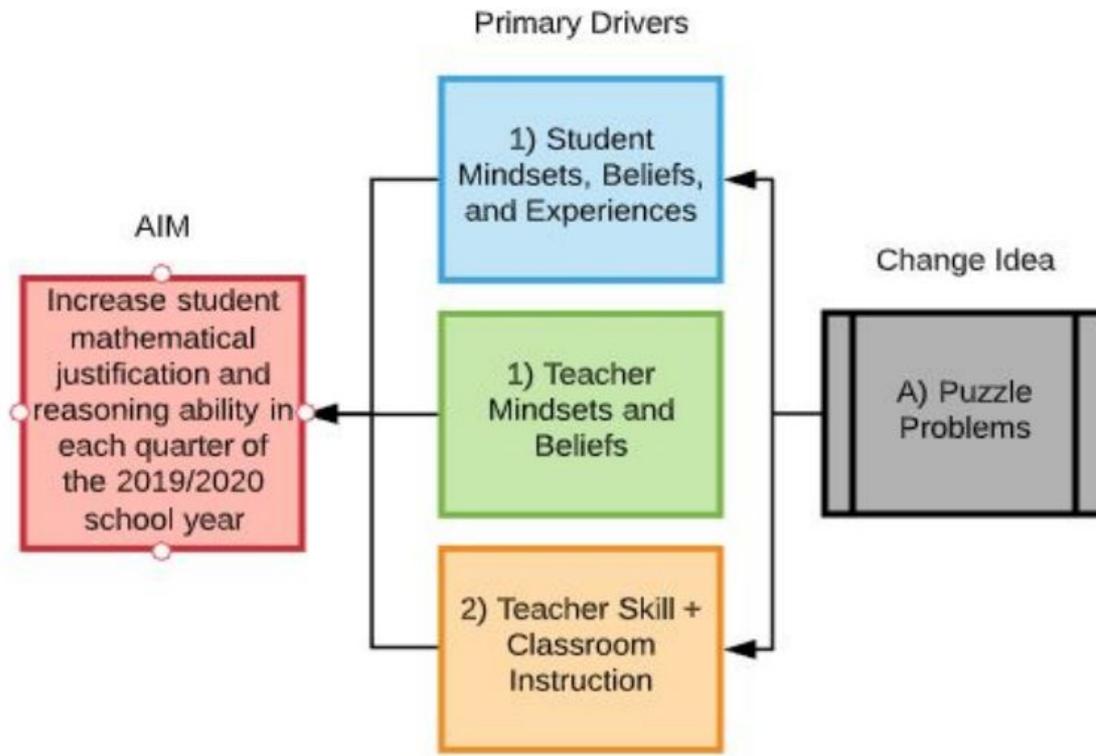
Oklahoma Excel Networked Improvement Community

- Statewide network of teachers that studied the value of student justification.
- Implemented the change idea of puzzle problems.
- Collected and analyzed data in 6 week PDSA cycles to refine our practice.
- We are now in year 3.

Why Justification?

- Promotes analysis of the mathematics and revision
- Deeper student understanding of the mathematics, not just mimicking the teacher.
- Democratization of knowledge
- Student ownership of the mathematics increases.
- Teacher can see and hear student thinking.

Driver Diagram



Puzzle Problems

Puzzle Problems can be an on-ramp for students to think deeply and justify their reasoning. These structures can provide first steps and a process for continuous improvement in justification and reasoning.

The focus is the process not the problem.

Introduction to Puzzle Problems

- **WHAT?** - Short problems that allow students to practice flexible math thinking and justifying their answers collaboratively.
- **WHEN?** - These problems are 5-15 minutes in length, and can be added whenever it makes sense in your lesson. They can be...
 - at the beginning as a launch
 - embedded into the math lesson
 - as part of the lesson closing

Student Growth

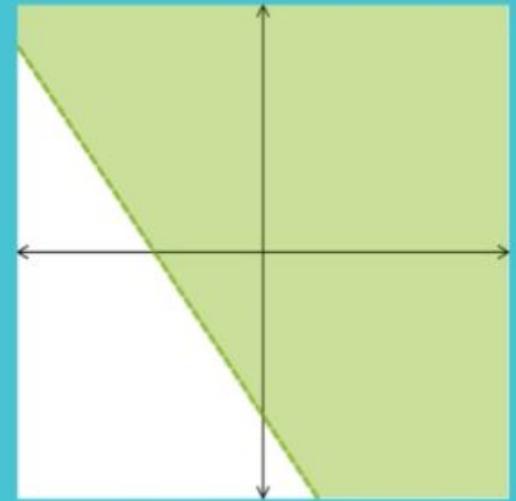
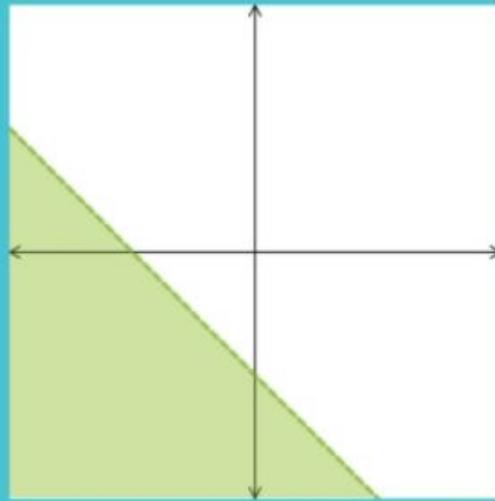
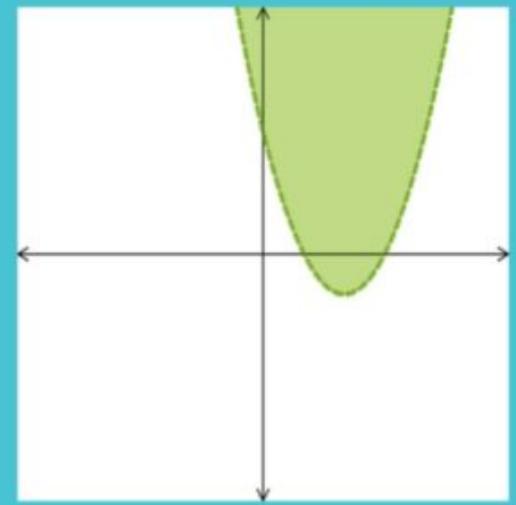
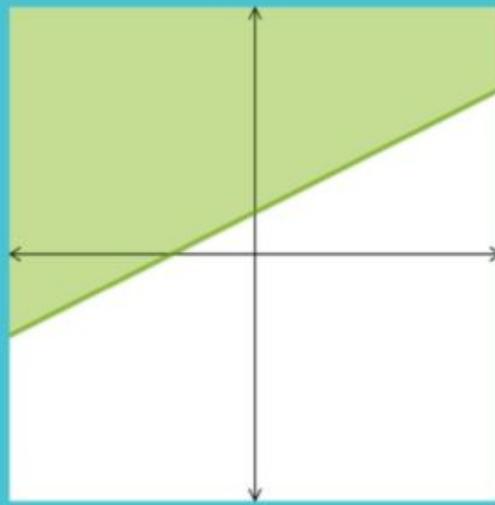
Students start with accessible questions where they can give reason for their answers.

Oklahoma Excel Puzzle Problems

- Which One Does Not Belong?
- Always, Sometimes, Never
- Would You Rather
- Number Talks

Which one does not belong?

Put your answer in the chat.



Work as a server at Restaurant A
OR
Work as a server at Restaurant B?

Restaurant A

\$18 per hour
No tipping allowed

(meals range from \$8
to \$25 each)



Restaurant B

\$10.50 per hour
Tipping encouraged

(meals range from \$8
to \$25 each)

Justifying Anchor Chart from Jennifer Findley

How to Justify your Answer...

- Label your work.
- Show all of your work.
 - * models * equations * all steps
- Explain how you got your answer.
 - * First, I... Then...
- Explain why you chose an operation.
 - * I ___ because...
- Explain how you know your answer is correct.
- Show that your answer is correct with multiple representations.

Math Justification and Reasoning Rubric

This rubric allows for a formative assessment of how students are able to explain their mathematical thinking and provide a justification. Justifications evaluated using the rubric can be written or verbal.

0	1	2	3	4
<p>No attempt was made to justify</p> <p>OR</p> <p>Justification is missing all critical components</p>	<p>Justification is too difficult to understand or is <i>missing</i> 3 of the critical components</p> <p>AND</p> <p>The solution may be correct or incorrect</p>	<p>Justification is difficult to understand and/or is <i>missing</i> 2 of the critical components</p> <p>AND</p> <p>The solution may be correct or incorrect</p>	<p>Justification is clear and <i>includes</i> at least 3 critical components</p> <p>AND</p> <p>The solution may be correct, but may also include minor calculation errors</p>	<p>Justification is detailed and clear and <i>includes</i> all 4 critical components</p> <p>AND</p> <p>The solution must be correct</p>

Critical Components of Mathematical Justifications

- WHAT they got as the answer
- HOW they got to that answer
- WHY they chose the strategies or operations they used
- WHY their answer is correct

Determine whether the statement is always, sometimes, or never true.

Two triangles with the same perimeter are congruent to each other.

Add a notes page or geometry page to justify your answer.

Always

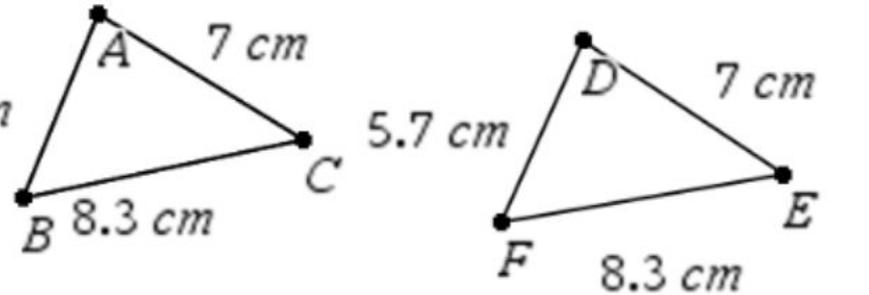
Sometimes

Never

Sample Student Response

1.1 1.2 1.3 *Doc DEG 1 cm

Always true because congruent triangles must have the same perimeter.



The image shows two triangles, ABC and DEF, drawn on a white background. Triangle ABC has vertices A, B, and C. Side AB is labeled 5.7 cm, side AC is labeled 7 cm, and side BC is labeled 8.3 cm. Triangle DEF has vertices D, E, and F. Side DF is labeled 5.7 cm, side DE is labeled 7 cm, and side FE is labeled 8.3 cm. The triangles are identical in shape and size, representing congruent triangles. The text above the triangles states: "Always true because congruent triangles must have the same perimeter." The text is written in a cursive font. The entire diagram is enclosed in a window-like border with a title bar containing "1.1 1.2 1.3 *Doc DEG 1 cm" and a mouse cursor is visible at the bottom right.

Score from Rubric:
What, How, and Why are included. Argued the converse of the statement. Not a minor calculation error. Score: 2 or 3 depending on how minor you see logical error.

Sample Student Response

Determine whether the statement is always, sometimes, or never true.

Two triangles with the same perimeter are congruent to each other.

Add a notes page or geometry page to justify your answer.

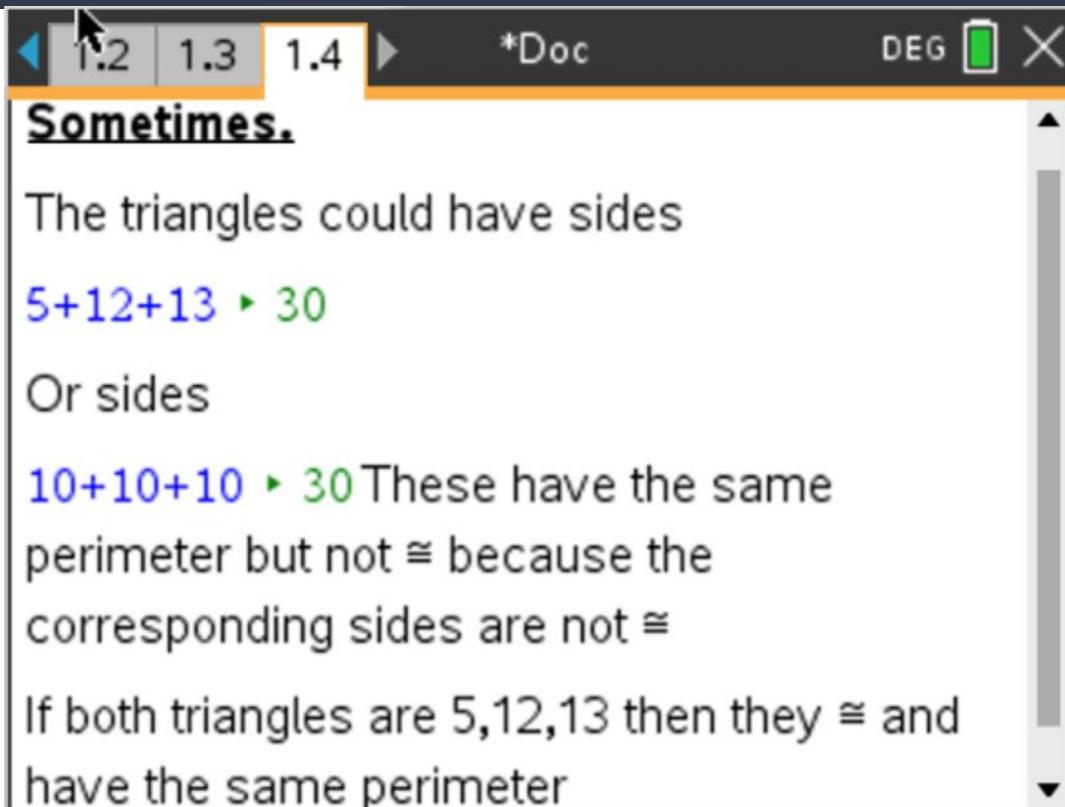
Always

Sometimes

Never

Score From
Rubric:
Answer but no
justification
Score: 0

Sample Student Response



A screenshot of a document editor window. The title bar shows a navigation pane with tabs for 1.2, 1.3, and 1.4, with 1.4 selected. The document title is '*Doc'. The editor content includes the word 'Sometimes.' followed by three paragraphs of text. The first paragraph says 'The triangles could have sides' followed by the equation $5+12+13 \blacktriangleright 30$. The second paragraph says 'Or sides' followed by the equation $10+10+10 \blacktriangleright 30$ and the text 'These have the same perimeter but not \cong because the corresponding sides are not \cong '. The third paragraph says 'If both triangles are 5,12,13 then they \cong and have the same perimeter'. The editor has a scroll bar on the right and a mouse cursor is visible at the top left.

Sometimes.

The triangles could have sides

$$5+12+13 \blacktriangleright 30$$

Or sides

$$10+10+10 \blacktriangleright 30$$

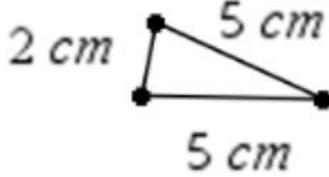
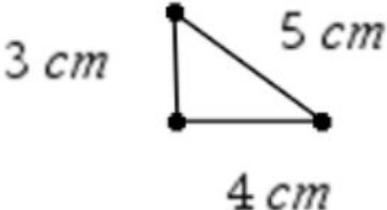
These have the same perimeter but not \cong because the corresponding sides are not \cong

If both triangles are 5,12,13 then they \cong and have the same perimeter

Score from
Rubric: What,
How, Why, and
Why included.
Score: 4

Sample Student Response

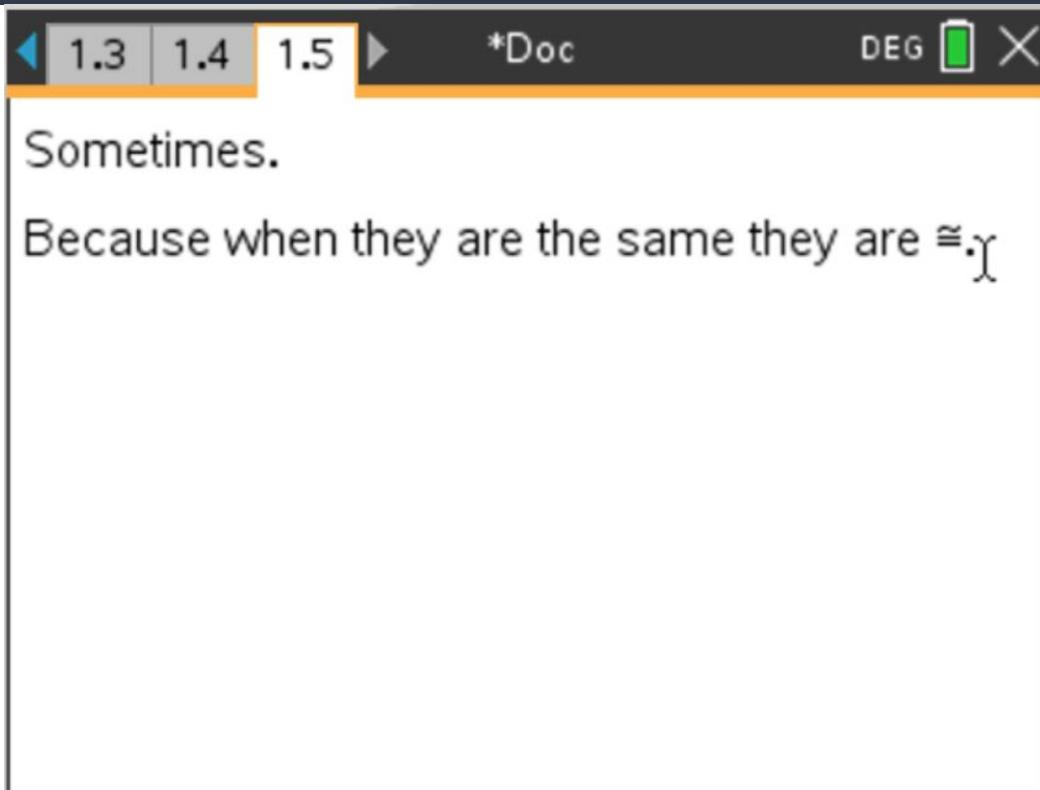
1.1 ASN response DEG  



Sometimes. The sides may add up to the same perimeter, but not be the same length.

Score from
Rubric: What,
How, Why, and
Why included but
no congruent
example.
Score: 3

Sample Student Response



A screenshot of a digital document editor interface. The top bar shows navigation tabs for sections 1.3, 1.4, and 1.5, with 1.5 selected. To the right of the tabs are icons for a document (*Doc), a calculator (DEG), a battery level indicator, and a close button (X). The main text area contains the following response:

Sometimes.
Because when they are the same they are \cong .

Score from Rubric:
Answer correct with
a what and why but
the justification is
hard to understand
Score: 2

Teacher Growth

- Start with simple puzzle problems with the intention of building classroom culture
- Problems do not need to come directly from your curriculum
- Students talking (writing) about their thinking is what is most important

T³ Chicago for Tracy, Baltimore for Tim

This is the point in our growth where we made the connection between puzzle problems and T³TM pedagogies.

Cognitively Challenging Tasks

Doug Roberts, in a local PD, defined cognitively challenging tasks. Suddenly they made sense in light of puzzle problems and justification.

Change a textbook question into a more cognitively challenging task.

John walks 2 miles east from his house on Gore Blvd to get a drink at the convenience store then turns south on 52nd street for 4 miles to visit his friend Trisha. He is running late, so when he leaves, he walks by the train tracks that go diagonally from Trisha's house to his own. How far did he walk?

Rewritten Question:

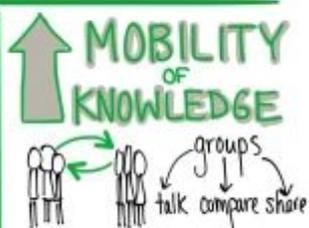
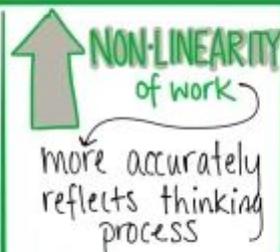
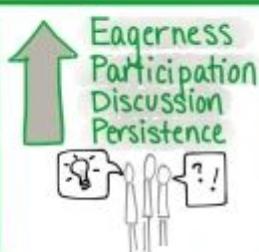
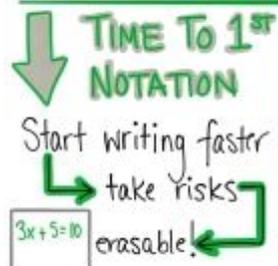
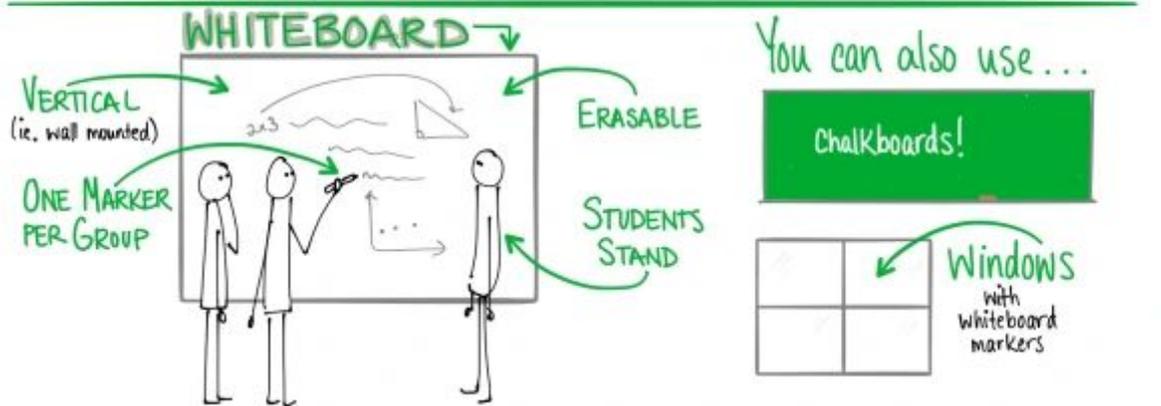
John walks due east from his house on Gore Blvd to get a drink at the convenience store then walks due south on 52nd street to visit his friend Trisha. He is running late so, when he leaves, he walks 5 miles along the train tracks that go diagonally from Trisha's house to his own. How far could it be from his house to the store? How far does Trisha live from the store?

Non-Permanent Vertical Surfaces with Random Grouping.

Having learned from Dr. Peter Liljedahl the value and method of making student thinking visible we suddenly saw a path to make puzzle problems, and thus justification, even more powerful.

VERTICAL NON-PERMANENT SURFACES

in math class



Research: @pgiljedahl

Sketchnot: @wheeler_laura

TI Tech is a wonderful tool for discovery. But why stop there? Students own the math when they justify their thinking!

TI Tech as a tool for students to justify their thinking

- Calculations
- Notes Pages
- Tables
- Graphs
- Probes
- CBR
- Rovers
- Innovators

Dr. Annie Fetter

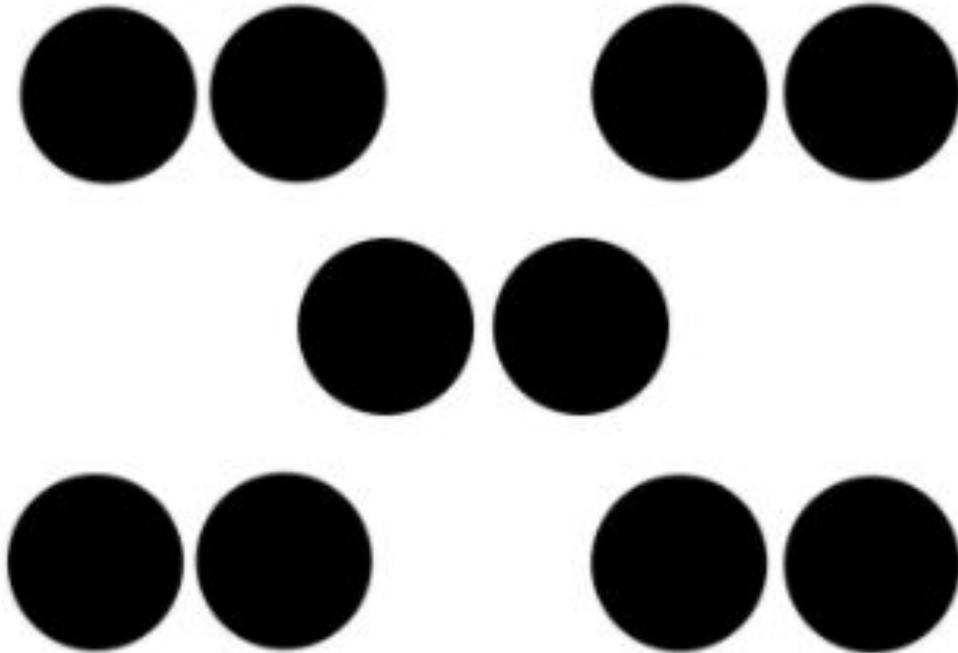
What Do Notice,
What Do Wonder

Notice

Wonder

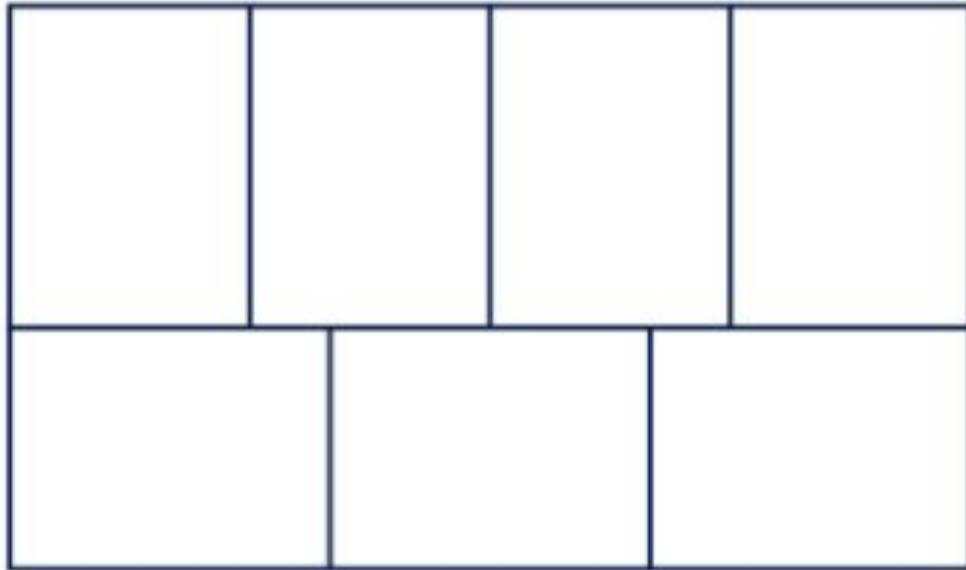
Notice	Wonder

What do Notice? What do you Wonder?

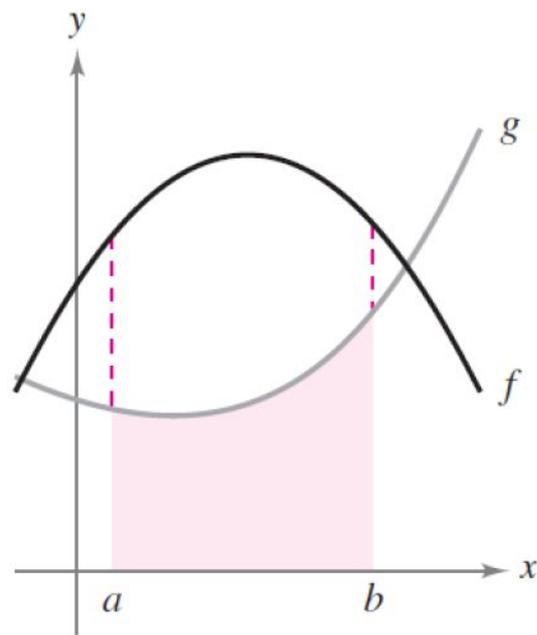
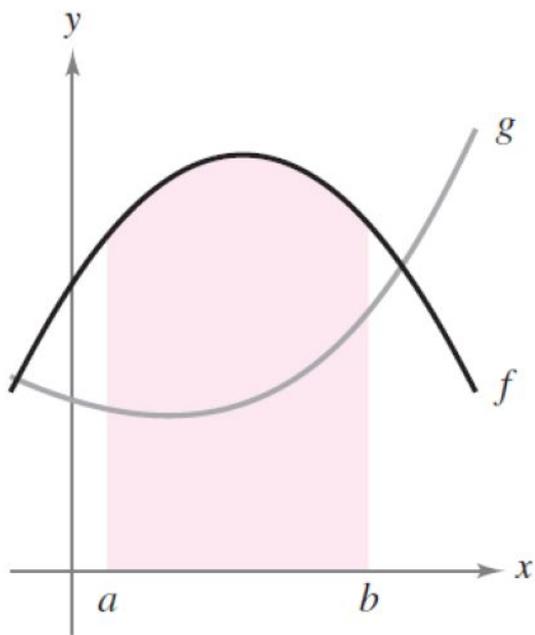
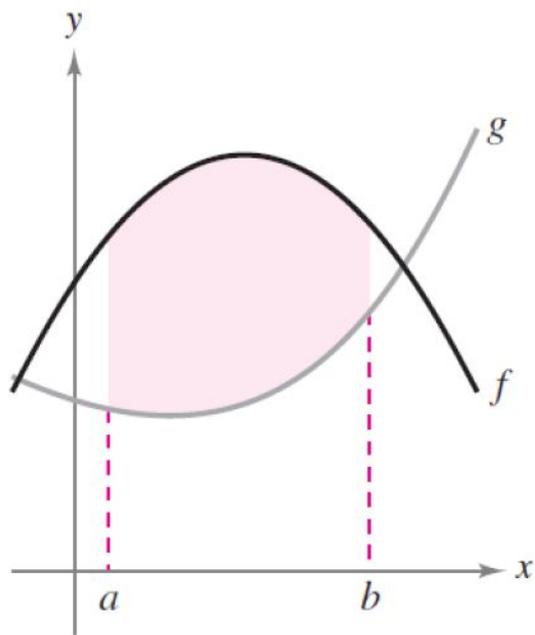


What do Notice? What do you Wonder?

The seven small rectangles in this picture are congruent.



What do you notice? What do you wonder?



Next Natural Step?

Mathematical Modeling

A dark blue diagonal gradient bar that starts from the bottom left corner and extends towards the top right corner, covering the lower half of the slide.

Mathematics Devoid of Meaning is Empty

Nancy Butler Wolf

In the chat:

What sense do you make of this quote?

In light of your teaching,
How does this quote
make you feel?

GAIMME 2016

Mathematical modeling is a process that uses mathematics to represent, analyze, make predictions or otherwise provide insight into real-world phenomena.

NAEP 2026

The practice of modeling requires students to make sense of the scenario, identify a problem to be solved, mathematize it, and apply the mathematics to reach a solution and check the viability of the solution.

Aim

Students will improve their mathematical reasoning and justification skills (as measured by rubric) each quarter.

Drivers

Student Attitudes and Beliefs

Student Knowledge and Skills

Teacher Mindsets and Beliefs

Teacher Knowledge and Skills

Change Ideas

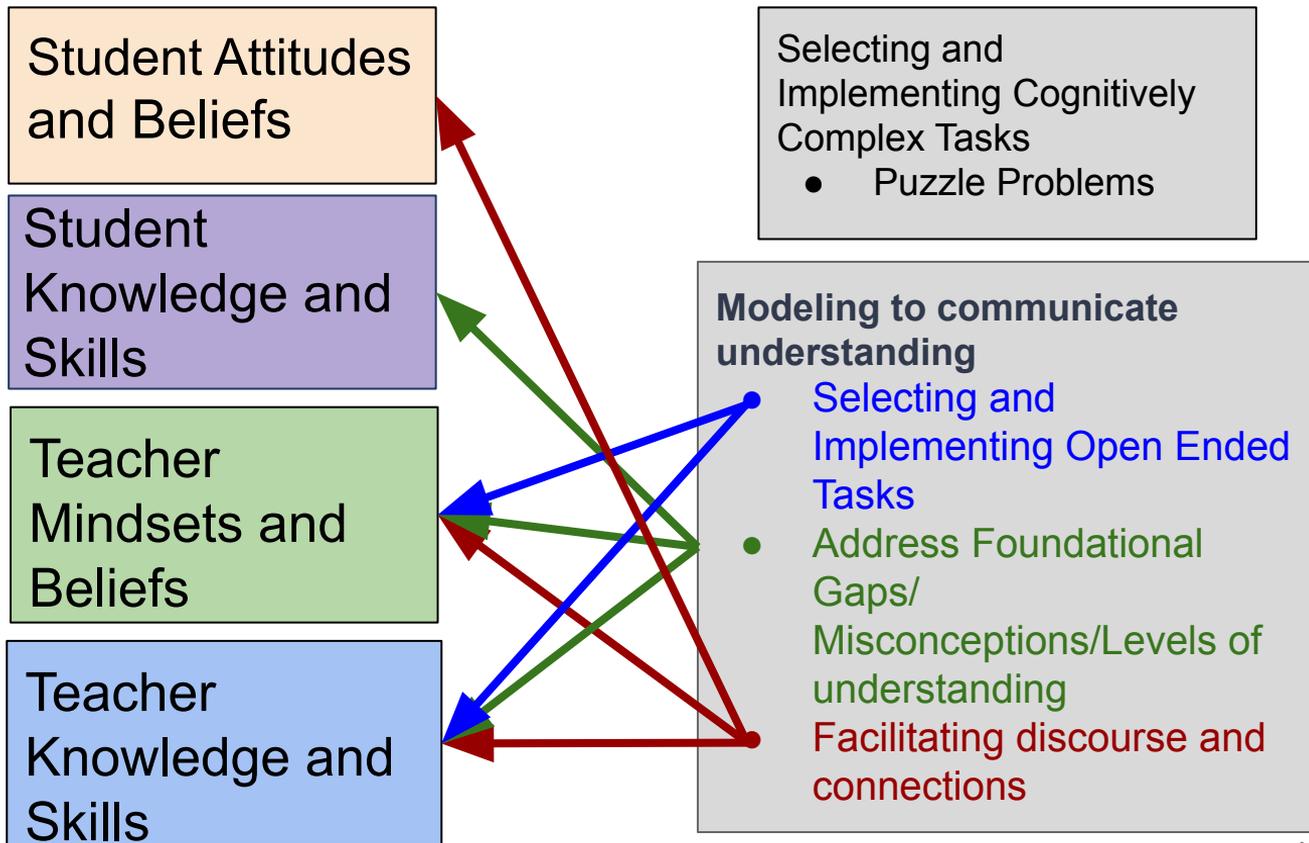
Selecting and Implementing Cognitively Complex Tasks

- Puzzle Problems

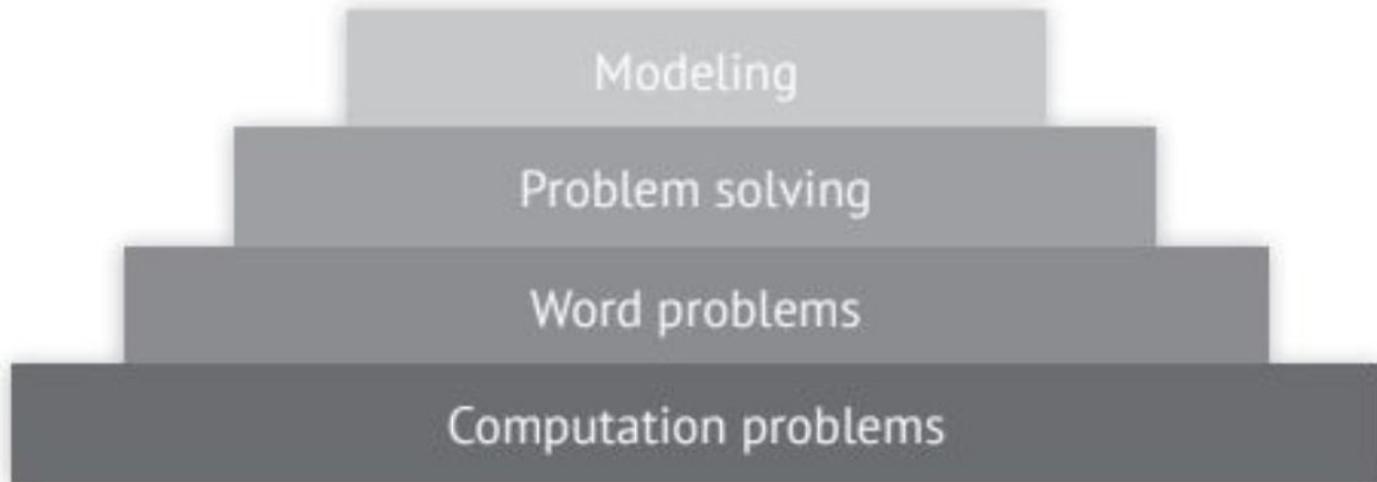
Modeling to communicate understanding

Selecting and Implementing Open Ended Tasks

- Address Foundational Gaps/
Misconceptions/Levels of understanding
Facilitating discourse and connections



Problem Pyramid





Act 1

1. How many sugar packets do you think are inside a 20-oz bottle of soda?
2. Guess as close as you can.
3. Give an answer you know is too high.
4. Give an answer you know is too low.

Act 2

5. What information will you need to know to solve this problem? ***How many packets of sugar are in a 20 oz. bottle of soda?***

Act 2 (continued)



similar products



Nutrition Facts

Serving Size 1 bottle

Servings Per Container 1

Amount Per Serving

Calories 240

% Daily Value*

Total Fat 0g **0%**

Sodium 75mg **3%**

Total Carbohydrate 65g **22%**

Sugars 65g

Protein 0g

Not a significant source of fat calories, saturated fat, trans fat, cholesterol, fiber, vitamin A, vitamin C, calcium and iron.

*Percent Daily Values (DV) are based on a 2,000 calorie diet.

Act 2 (continued)

http://www.dominosugar.com/Product.aspx?id=32, retrieved 02 February 2011

Domino SUGAR
We'll Always Be Your Sugar!

ABOUT US | PARTY CENTRAL | BAKING MADE EASY | FUN CORNER | RECIPES | OUR PRODUCTS | NEWSLETTER

SEARCH our site
GO
Search Recipes
Search Entire Site

MY recipes
Create a virtual cookbook by collecting all your favorite recipes from this site. Returning users please login below. New users please [click here to register](#).
USERNAME: _____
PASSWORD: _____ LOGIN

15 CALORIES
per teaspoon

PERSONALIZE your baking
Add a special touch to your homemade baked goods with these handy accessories you can personalize and print right from your home computer. What better way to say -- "Baked with love!"
Gift Tags for your Baked Items:
[Baking Gift Tags](#)
[Holiday Gift Tags 1](#)

SHARE | EMAIL | PRINT

Sugar Packets
Each packet contains one portion of 100% pure Superfine Granulated Sugar. At home, at work, or on the go, Domino® Sugar Packets are always neat and convenient.
Available Sizes:
50 count box
100 count box

JOIN BAKER'S CIRCLE
Sign up to receive recipes, our newsletter and special offers. Join Today!

BAKING made easy
Find answers to many of your baking questions. Our experts share tips, videos and handy charts to help you bake like a pro!
[Learn More](#)

SHARE OUR STRENGTHS GREAT AMERICAN BAKESALE
NO KID HUNGRY

Domino® Sugar - Nutrition Facts -- Sugar Packets

Nutrition Facts	
Serving size 1 Teaspoon (4g)	
Amount Per Serving	
Calories	15
% Daily Value*	
Total Fat	0g 0%
Sodium	0mg 0%
Total Carbohydrate	4g 1%
Sugars 4g	
Protein	0g
*Percent Daily Values are based on a 2,000 calorie diet.	

Example:

coke Sugars = 65g

Domino Sugar paket = 4g

$\boxed{4}$ $\boxed{4}$ $\boxed{4}$ $\boxed{4}$ $\boxed{4}$ = 20g

$\boxed{4}$ \square \square \square \square = 20g

$\boxed{4}$ \square \square \square \square ~~40g~~
+ 20g

$\boxed{4}$ $\overset{x}{\square}$ not enough for a full pak

60g

+ 4g

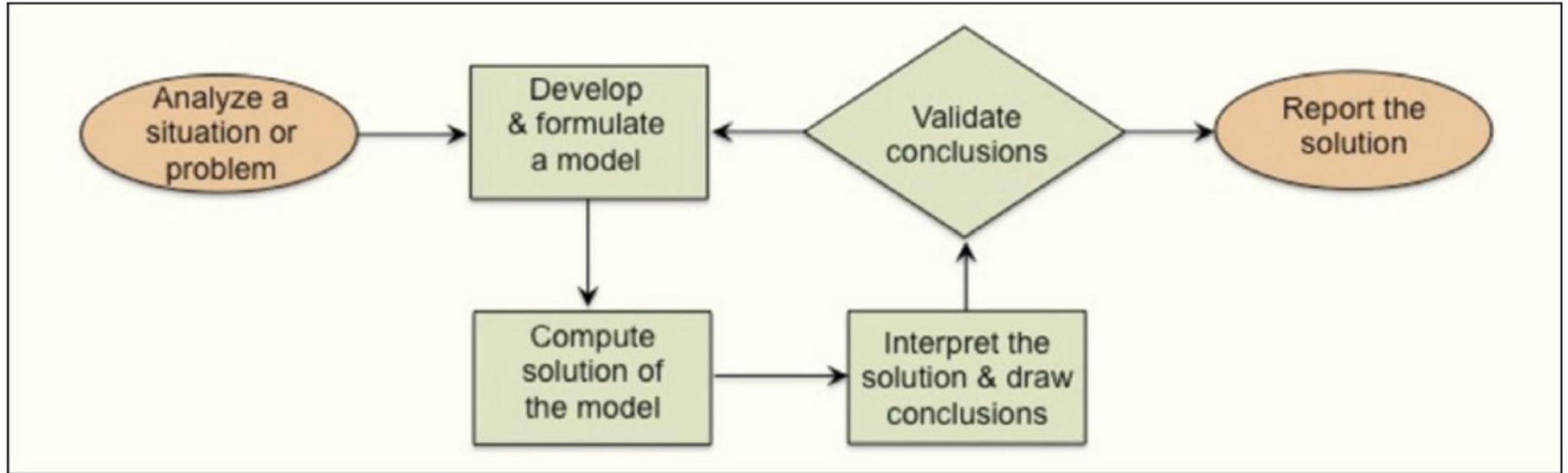
16 full packs

64g
+ $\frac{1}{4}$ of another paket



You'd never **EAT 16** packs of sugar.

Mathematical Modeling Cycle



I used to think my job was to teach students to see what I see. I no longer believe this. My job is to teach students to see; and to recognize that no matter what the problem is, we don't all see things the same way. But when we examine our different ways of seeing, and look for the relationships involved, everyone sees more clearly; everyone understands more deeply.

—*Ruth Parker*

Never Say Anything
A Student Could Say



How many toes on the bus?

Which One Does Not Belong? Justify your answer.

9	16
25	43



You'd never **EAT 16 packs** of sugar.