

Investigate real data from real contexts

The screenshot shows the Texas Instruments Education Technology website. The main navigation bar includes: Products, Downloads, Activities, Professional Development, Resources, Customer Support, Where to Buy, About Us, and Site US and Canada. A search bar is located in the top right. The left sidebar contains a menu with categories like 'Welcome to Math Nspired', 'About Math Nspired', 'Middle Grades Math', 'Algebra I', 'Geometry', 'Algebra II', 'Precalculus', 'Calculus', 'Statistics', 'Mathematical Modeling', 'Ratings', 'Prediction', 'Simulation', 'Standards Search', and 'Textbook Search'. The main content area is titled 'Mathematical Modeling' and features a sub-header 'Find activities that support your lesson plans'. Below this, there is a paragraph describing how TI-Nspire graphing calculators support mathematical modeling. To the right of the text is an image of a TI-Nspire calculator, a laptop, and a tablet. Below the main text is a section titled 'Featured Activities' with two sub-sections: 'Modeling CO2 Levels' and 'Modeling QB Passing Rates'. Each sub-section includes a small image of a TI-Nspire calculator screen displaying a scatter plot and a line of best fit. The 'Modeling CO2 Levels' activity shows a scatter plot of CO2 ppm data over 12 months. The 'Modeling QB Passing Rates' activity shows a scatter plot of interceptions versus touchdowns for football quarterbacks, with a line of best fit and a point labeled (51, 69).

Mathematical Modeling

Find activities that support your lesson plans

Technology tools such as the TI-Nspire™ CX II and other TI graphing calculators can be a powerful support for mathematical modeling activities. It opens up opportunities for student to compare models graphically, to use regression for making and testing predictions, and to simulate processes that would be difficult or impossible to do without technology.

This set of activities seeks to illustrate how the TI-Nspire™ CX family of graphing calculators could be used to effectively support the vision of the [GAIMME](#) document. The document "Guidelines for Assessment & Instruction in Mathematical Modeling Education" (GAIMME) was published in 2016 by the Consortium for Mathematics and its Applications (COMAP) and the Society for Industrial and Applied Mathematics (SIAM). GAIMME is a rich resource for teachers at all levels K-16, and it provides valuable advice for making modeling an integral part of mathematics instruction.

Featured Activities

Modeling CO2 Levels

This lesson provides an opportunity to engage students with data that is relevant to contemporary discussions of climate change. Students will compare multiple box plots of the data across years, analyze scatter plots, and fit models

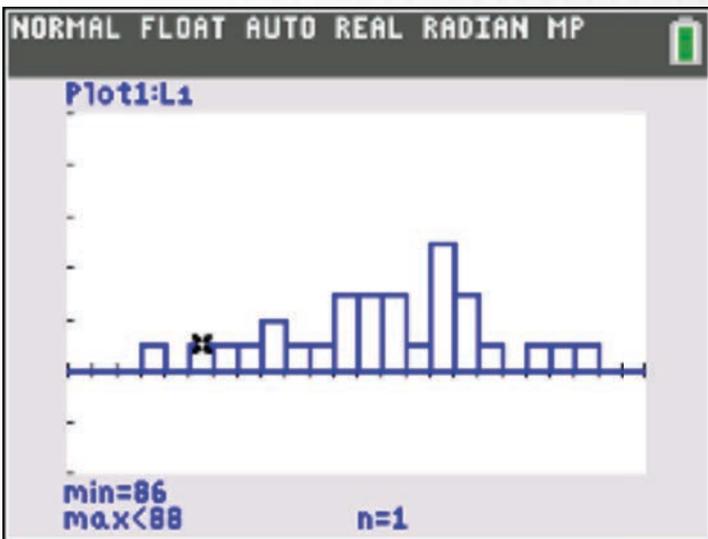
Modeling QB Passing Rates

This lesson involves using the measures commonly collected for football quarterbacks and provides an opportunity for students to create their own quarterback rating models. As a result, students will model a contextual

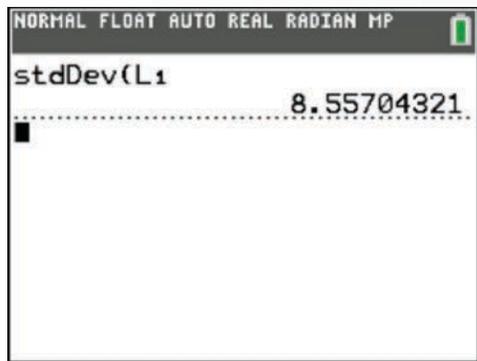
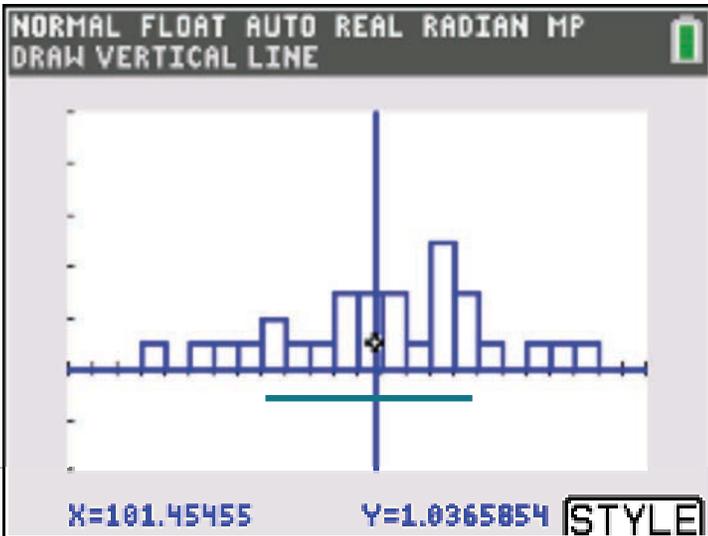
- Is women's income catching up to men's?
- Has the CO2 level in the air changed over time?
- What do we know about natural disasters?
- How does herd immunity work?
- Is Tom Brady really the all time "best" NFL passing quarterback?
- What is a false positive?
- How do you compile judge's votes?

Interpreting the news....

- “ELISA is rated 90%-97% sensitivity”
- “Coronavirus transmission rate climbing in L.A. County as economy reopens.”
- “... estimate of influenza incidence among all ages is about 8.3%”
- “To get herd immunity against measles ... 93% to 95% of people in a community have to be vaccinated.”
- “Abbott Lab test had a 40% false negative rate.”
- “Pfizer’s early data shows vaccine is more than 90% effective.”



Variability in number of positive in sample of 500 with 0.2 positivity rate



False positive/negative

prevalence 0.2, P(accuracy)=0.75

	Flu- infected	No flu- uninfected	Total
Test positive			
Test negative			
Total			500

• True positive: $\frac{\text{Number with the flu and positive}}{\text{Total number with flu}}$

$$\frac{\quad}{\quad} = 0.$$

• **False positive:** $\frac{\text{Number without the flu and positive}}{\text{Total number without flu}}$

$$\frac{\quad}{\quad} = 0.$$

• True negative: $\frac{\text{Number without the flu and negative}}{\text{Total number without flu}}$

$$\frac{\quad}{\quad} = 0.$$

• **False negative:** $\frac{\text{Number with the flu and negative}}{\text{Total number with flu}}$

$$\frac{\quad}{112} = 0.$$

Consequences of false negatives/false positives

- In evaluating risk, important to consider the consequences of the errors – false negatives and false positives?
- What is a potential consequence of a false positive?
- What is a potential consequence of a false negative?
- These two errors are inversely related – which would you consider more important to minimize?

False positive/negative

prevalence 0.2, P(accuracy)=0.75

	Flu- infected	No flu- uninfected	Total
Test positive			
Test negative			
Total			500

- Probability I **have** the virus given I test positive $\frac{\text{---}}{\text{---}} = 0.$
- Probability I **don't have** the virus given I test negative $\frac{\text{---}}{\text{---}} = 0.$

Conditional probabilities!



Screening Test Results

Flu prevalence: 0.20
 Prob (accurate positive) 0.75
 Prob (accurate negative) 0.75

	A pop1	B pop2	C pop3	D preval.
=				=randsa
1	flu	negflu	negnoflu	noflu
2	noflu	pflu	negnoflu	noflu
3	noflu	pflu	negnoflu	noflu
4	noflu	pflu	pnoflu	noflu
5	noflu			flu

	pop2	C pop3	D preval...	E test_flu
=			=randsam	=randsam
1	egflu	negnoflu	noflu	pflu
2	flu	negnoflu	noflu	pflu
3	flu	negnoflu	noflu	negflu
4	flu	pnoflu	noflu	pflu
5			flu	pflu

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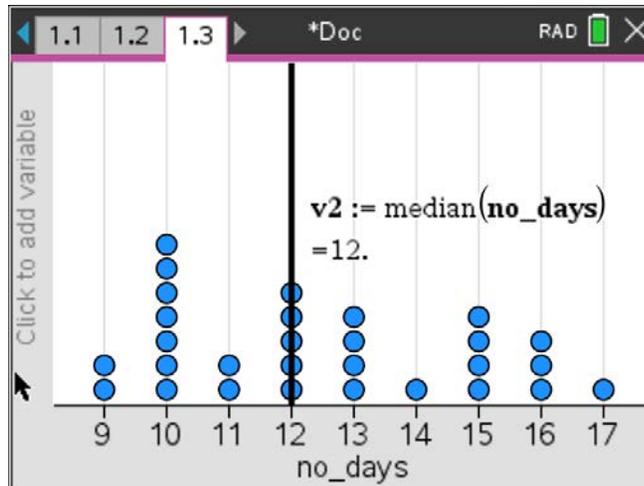
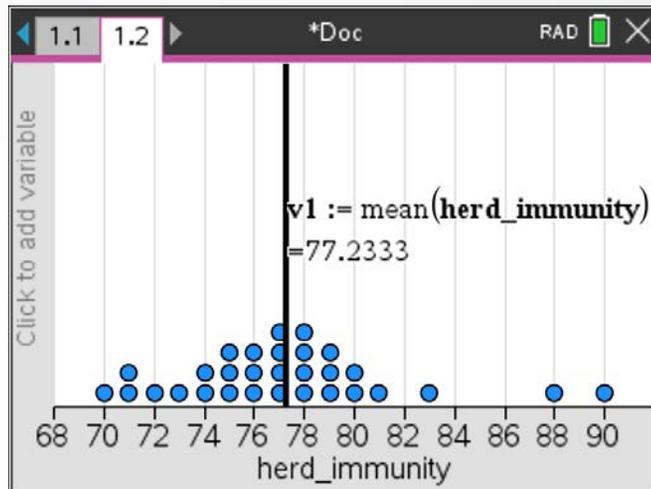
countif(prevalence,?="flu") ▶ 93.
countif(test_flu,?="pflu") ▶ 69.
countif(test_noflu,?="pnoflu") ▶ 107.
countif(prevalence,?="noflu") ▶ 407.
countif(test_flu,?="negflu") ▶ 24.
countif(test_noflu,?="negnoflu") ▶ 300.
    
```

	Flu-infected	No flu-uninfected	Total
Test positive	69	107	176
Test negative	24	300	324
Total	93	407	500

- Probability I have the virus given I test positive
 $69/176 = 0.39$
- Probability I don't have the virus given I test negative
 $300/324 = 0.93$

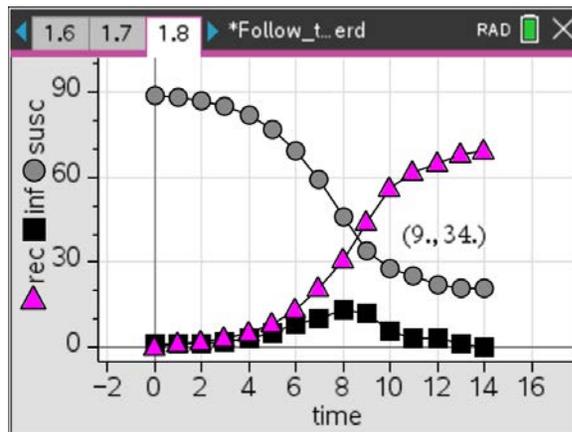
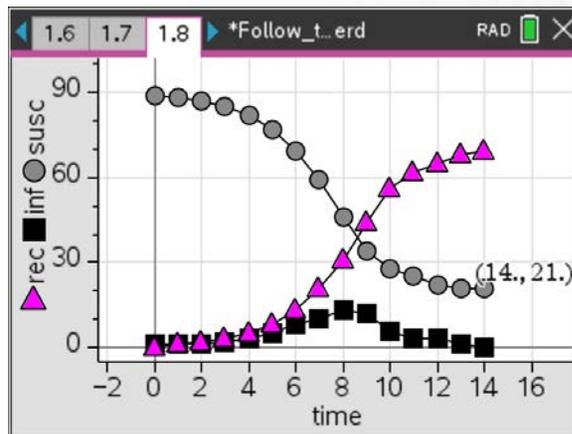
An automated simulation

Herd immunity and statistics



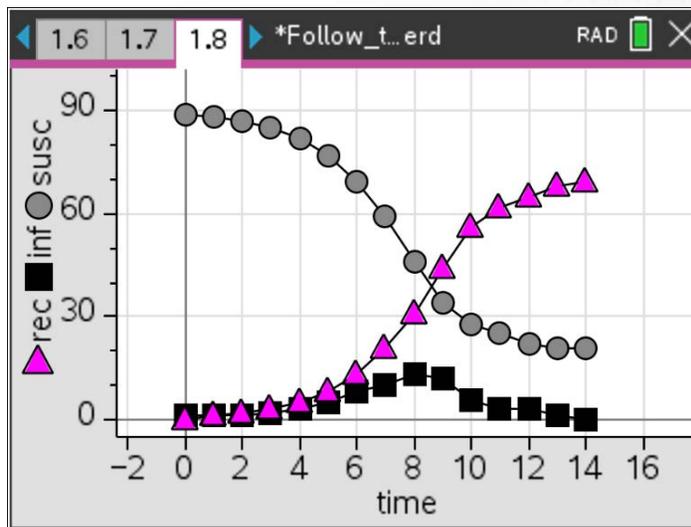
- What is a likely herd immunity threshold (HIT) for the given conditions?
- Estimate a confidence interval for HIT
- Are 88% and 90% outliers? Why or why not?
- Describe the number of days until herd immunity is reached.

SIR model and functions



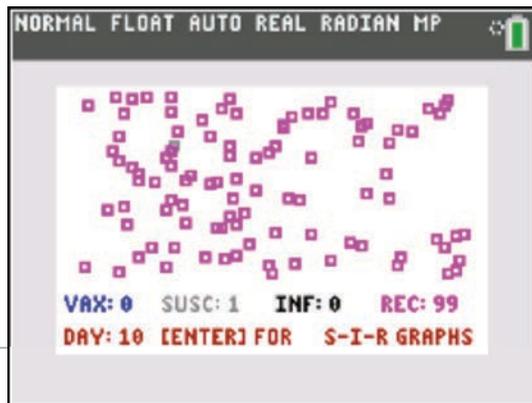
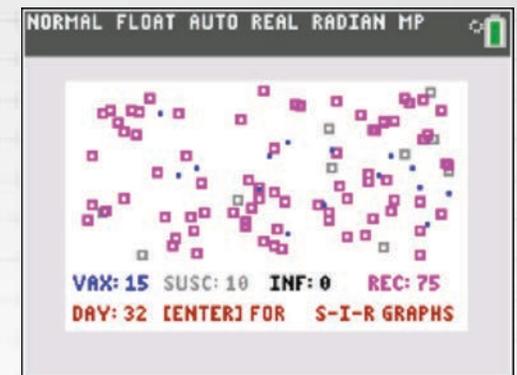
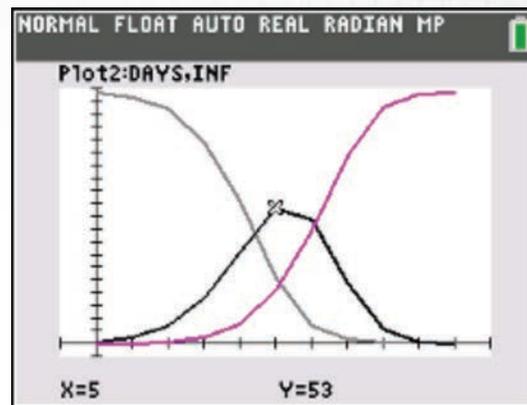
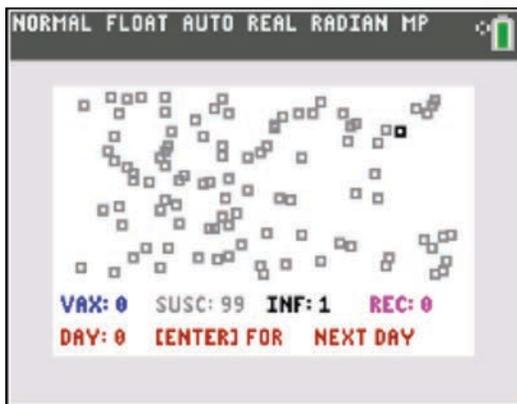
- Describe the graphs of the three functions (include the domain and range for each, increasing/decreasing intervals).
- How are the graphs related?
- How can you find the herd immunity threshold from the graphs?
- What do the points of intersection indicate?

The SIR model and calculus



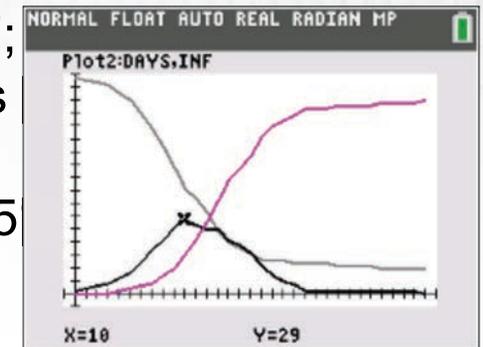
- Where is the point of inflection for susc and what does it mean?
- How are the graphs of the recovered and the infected related (pink and black)?
- Estimate the absolute maximum and minimum for each graph and the day they occurred. Interpret these in the context of the spread.
- Estimate the average value for the infected. Explain your thinking.

How will changing conditions affect the herd immunity threshold graphs and their interpretations?



Transmission 2,
Days contagious 2;
Population 100,
0 vaccinated
Herd Immunity 99%

Transmission 0.7;
Days contagious
Vaccinated 15
Herd Immunity 85



Why modeling?

- When tasks require reasoning, problem solving, and modeling (i.e., tasks with high cognitive demand) students have a positive orientation toward mathematics and themselves as doers of mathematics (Boaler and Staples 2008).
- “When students engage in mathematical modeling, they often have the opportunity to leverage mathematics to understand and critique the world.”
- “Modeling mathematics and statistics should be key components throughout any high school mathematics program.” (NCTM, 2018)

- The tns file Follow_the_Herd was created by Tom Dick from Oregon State University using a method based on Euler's formula. The theoretical background comes from differential calculus – one source you might want to check out is:
 - Collins, J., & Abdelal, N. (2018). Spread of disease. <https://calculate.org.au/wp-content/uploads/sites/15/2018/10/spread-of-disease.pdf>

COVID-19 Data sources

- Average number of days contagious for COVID-19 is 3 to 4 (Carlsen, 2021)
- Transmission by state (median of the states is 0.94, range from .84 in Georgia to 1.11 in Vermont) <https://www.statista.com/statistics/1119412/covid-19-transmission-rate-us-by-state/>
- Number of people who have had COVID per state (median is about 37% estimate, 8% reported ranging from ND with 12% reported and 62% estimate to Hawaii with 2% reported and 3% estimate) <https://www.npr.org/sections/health-shots/2021/02/06/964527835/why-the-pandemic-is-10-times-worse-than-you-think>

References

- Boaler, Jo, and Megan E. Staples. “Creating Mathematical Futures through an Equitable Teaching Approach: The Case of Railside School.” *Teachers College Record* 110, no. 3 (2008): 608–45.
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<http://www.siam.org/reports/gaimme.php>
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<https://education.ti.com/en/timathnspired/us/mathematical-modeling>
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- National Council of Teachers of Mathematics. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations*. Reston VA: The Council.

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<https://education.ti.com/en/84activitycentral/us/mathematical-modeling>