TI in Focus: AP® Calculus

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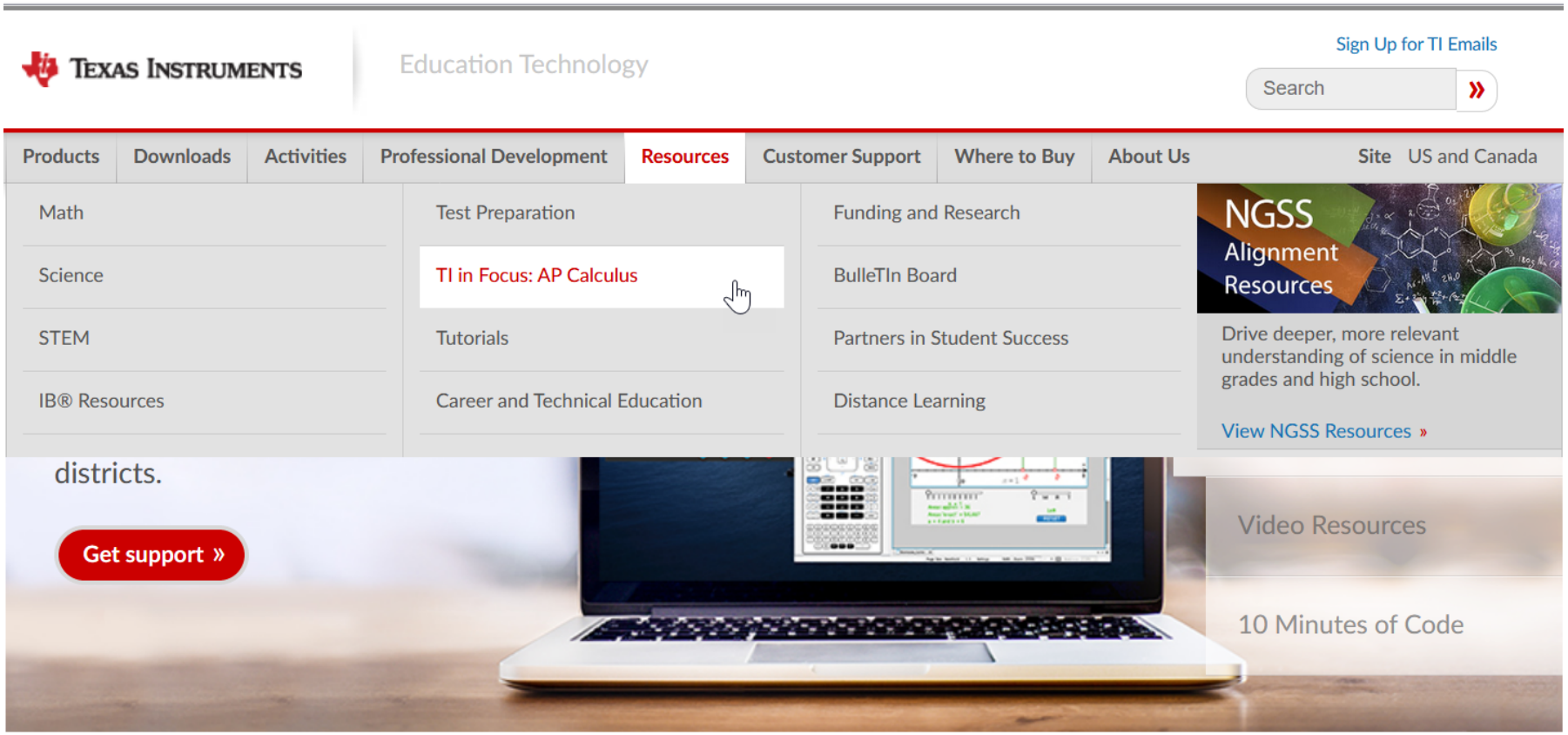


TI in Focus

AP® Calculus

Here you'll find AP® Calculus questions from the last few years

A collection of test-prep resources

Help students score on the AP® Calculus exam with solutions from Texas Instruments. The TI in Focus program supports teachers in preparing students for the AP® Calculus AB and BC test. Our library of files covers free-response questions (FRQ) from past exams through the lens of graphing technology.

Below you'll find released AP® Calculus questions from the last few years. Each section (or module) leads to a page with videos, downloads and learning objectives related to each free-response question.

Organized by Topic and Year

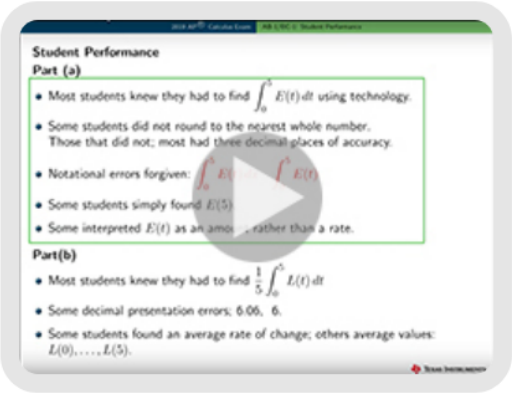
Topic

- Topic
- Area / Volume
- Applications / Approximations of Definite Integrals
- Differential Equations
- Function / Graphical Analysis
- Fundamental Theorem of Calculus (FTC)
- Polar and Parametric (2D)
- Position / Velocity / Acceleration
- Series and Improper Integrals
- Taylor Polynomial and Power Series

Year

- Year
- 2017
- 2018
- 2019

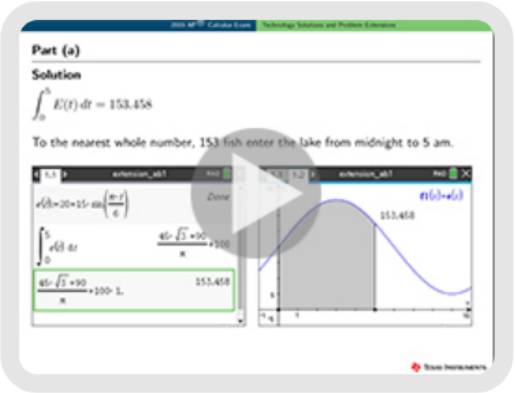
Resource Videos



Scoring and common errors

This video focuses on the scoring of the Free Response Question. The presentation is similar to a briefing at the AP® Calculus Reading, but condensed. The allotment of points, common student errors and present specific scoring examples are discussed.

Watch Video »



Technology solutions and extensions

This video focuses on technology solutions to the Free Response Question, even if the question is not calculator-active. Additional examples are presented based on the information given in the Free Response Question for instructional use and in preparing for the AP® Calculus Exam.

Watch Video »

Topic summary

This video presents a summary of a specific topic related to the Free Response Question. The examples included emphasize the use of technology, AP® Calculus-type questions, and some are left open for exploration and discussion.

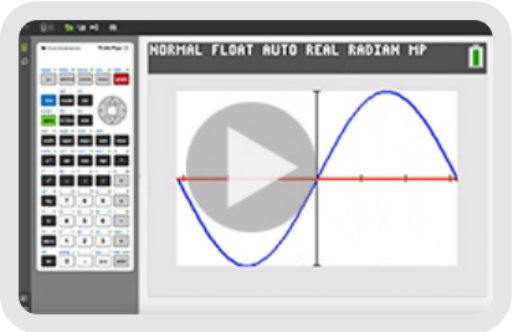
Watch Video »

Technology Videos

TI-Nspire™ CX CAS graphing calculator

This video focuses on how to perform specific operations and calculations related to the topic on TI-Nspire™ CX CAS graphing calculator.

Watch Video »



TI-84 Plus CE graphing calculator

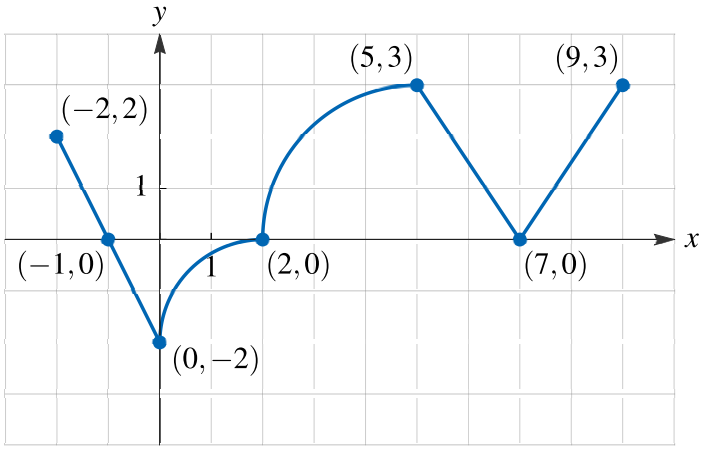
This video focuses on how to perform specific operations and calculations related to the topic on the TI-84 Plus CE graphing calculator.

Watch Video »

AP Calculus Mock Exam

AB 1

The continuous function f has domain $-2 \le x \le 9$. The graph of f, consisting of three line segments and two quarter circles, is shown in the figure.



Graph of f

Let g be the function defined by $g(x) = \int_0^x f(t)\,dt$ for $-2 \le x \le 9$.

(a) Find the x-coordinate of each critical point of g on the interval $-2 \leq x \leq 9$.

(b) Classify each critical point from part (a) as the location of a relative minimum, a relative maximum, or neither for g. Justify your answers.

(c) For $-2 \leq x \leq 9$, on what open intervals is g increasing and concave down? Give a reason for your answer.

(d) Find the value of $g(-1)$. Show the computations that lead to your answer.

(e) Find the value of $g(2)$. Show the computations that lead to your answer.

(f) Find the absolute maximum value of g over the interval $-2 \leq x \leq 5$.

(g) Find the value of $g''(6)$, or explain why it does not exist.

(h) Must there exist a value of d, for $0 < d < 2$, such that $g'(d)$ is equal to the average rate of change of g over the interval $0 \leq x \leq 2$? Justify your answer.

(i) Find $\lim\limits_{x \to 0} \dfrac{3x + g(x)}{\sin x}$. Show the computations that lead to your answer.

(j) The function h is defined by $h(x) = x \cdot g(x^2)$. Find $h'(\sqrt{2})$. Show the computations that lead to your answer.

(c) For $-2 \leq x \leq 9$, on what open intervals is g increasing and concave down? Give a reason for your answer.

Key Concepts

Definition

A function f is **increasing** on an interval I if

$$f(x_1) < f(x_2) \quad \text{whenever} \quad x_1 < x_1 \text{ in } I$$

The function is **decreasing** on I if

$$f(x_1) < f(x_2) \quad \text{whenever} \quad x_1 < x_2 \text{ in } I$$

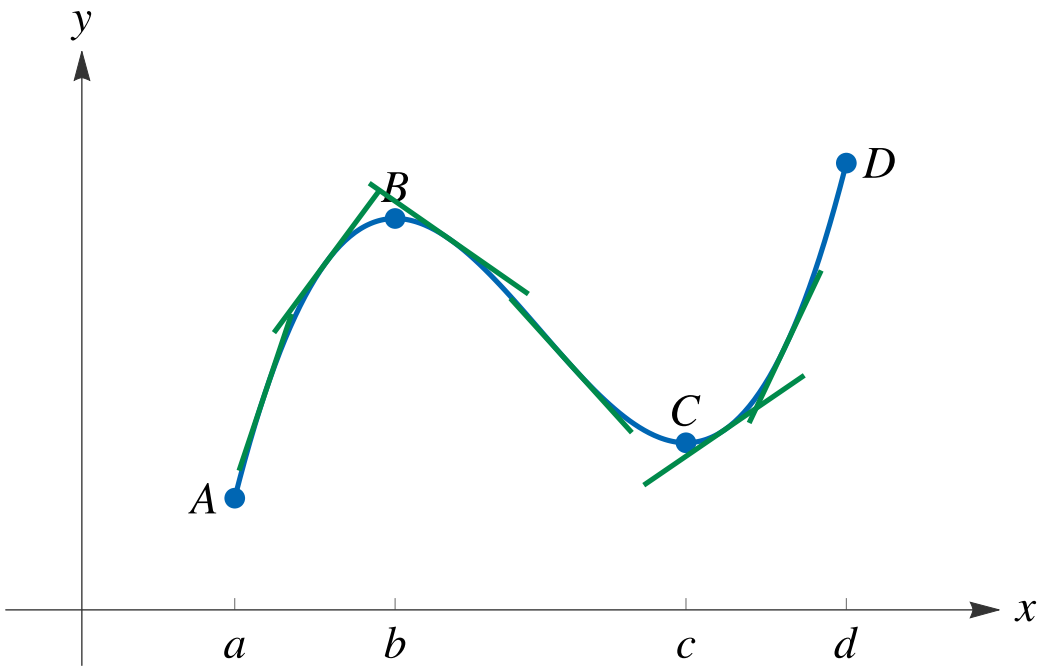
Increasing/Decreasing Test (I/D Test)

(a) If $f'(x) > 0$ on an interval, then f is increasing on that interval.

(b) If $f'(x) < 0$ on an interval, then f is decreasing on that interval.

A Closer Look

1. Consider the figure.



Between a and b, and between c and d:
tangent lines have positive slopes, and $f'(x) > 0$.

Between b and c:
tangent lines have negative slopes, and $f'(x) < 0$.

2. Proof: Mean Value Theorem

Definition

Let f be a differentiable function.

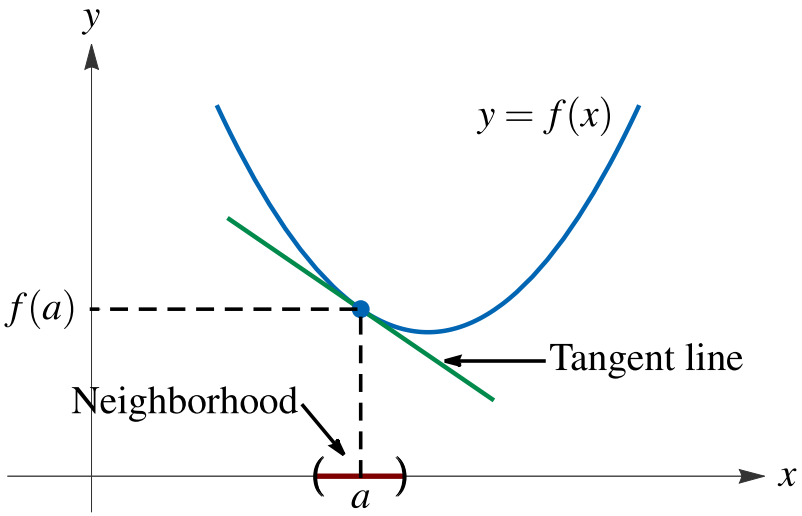
f is **concave up** at a if the graph of f is above the tangent line to f at a for all x in a neighborhood containing a (but not equal to a).

f is **concave down** at a if the graph of f is below the tangent line to f at a for all x in a neighborhood containing a (but not equal to a).

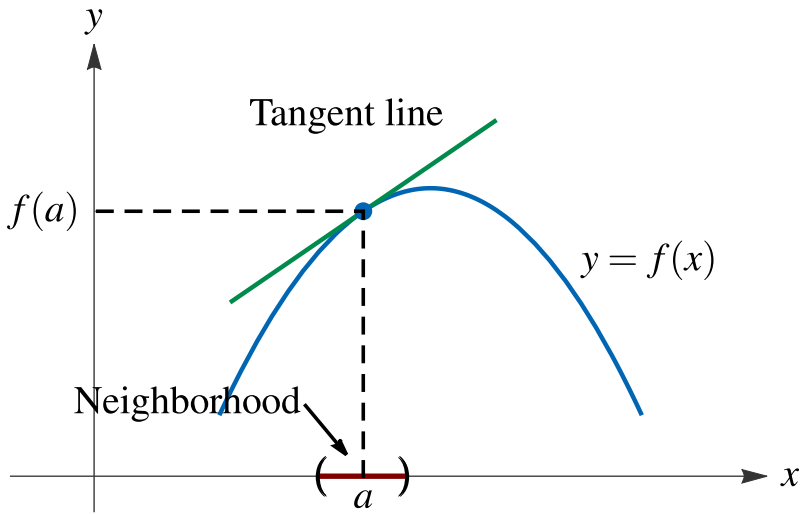
A Closer Look

1. Neighborhood: an open interval containing a.
2. Concavity is defined in terms of a single number, not an interval.

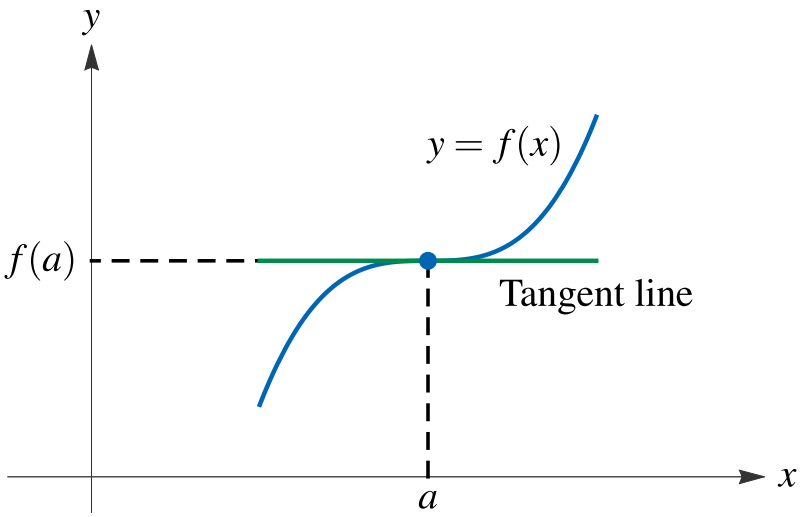
   The graph of a function can be concave up, concave down, or have no concavity at a number.



The graph of f is concave up at a. The graph of f lies above the tangent line in a neighborhood of a.



The graph of f is concave down at a. The graph of f lies below the tangent line in a neighborhood of a.



The graph of f has no concavity at a.

Concavity Test

(a) If $f''(x) > 0$ for all x in I, then the graph of f is concave up on I.

(b) If $f''(x) < 0$ for all x in I, then the graph of f is concave down on I.

A Closer Look

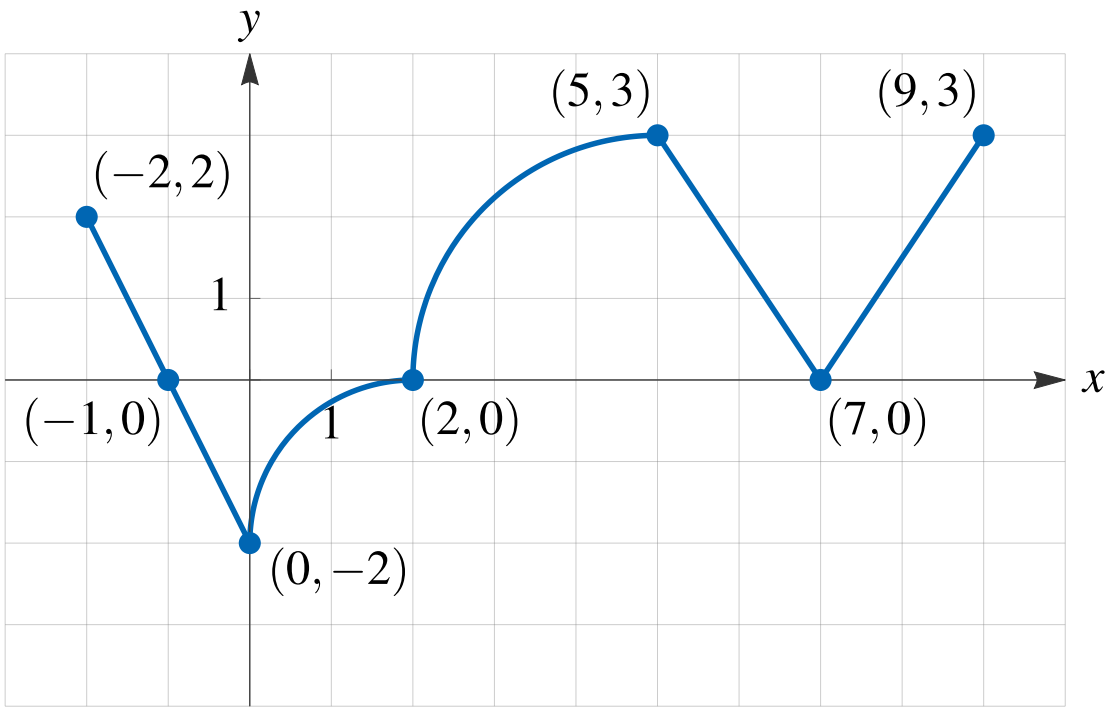
1. The Concavity Test allows us to draw a conclusion about the behavior of the graph of f if $f''(x) > 0$ or $f''(x) < 0$.

   Need to examine numbers a where $f''(a) = 0$ or $f''(a)$ DNE.

2. The graph of f is either concave up, concave down, or has no concavity at a number.

3. The point on the graph of f where it changes concavity is called an inflection point.

Solution



g is increasing where $g' = f$ is positive: $(-2,-1)$, $(2,7)$, $(7,9)$

g is concave down where $g' = f$ is decreasing: $(-2,0)$, $(5,7)$

g is increasing and concave down on the intervals: $(-2,-1)$ and $(5,7)$.

Scoring Guidelines

(c) g is increasing where $g' = f$ is positive.

g is concave down where $g' = f$ is decreasing.

g is increasing and concave down on the intervals $(-2, -1)$ and $(5, 7)$.

$$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$$

Scoring Notes

- The first point is earned for the correct intervals: open, closed, half-open.
- To be eligible for the second point: must have earned the first point.
- A reason referencing only g' is eligible for the second point.
- A reason referencing only f is not eligible for the second point unless the student has made the connection $g' = f$.
- The reason g' is decreasing can be expressed as: g'' is negative, or the slope of g' is negative.
- A response referencing *the function*, *the slope*, or *it* are too vague.

(d) Find the value of $g(-1)$. Show the computations that lead to your answer.

Key Concepts

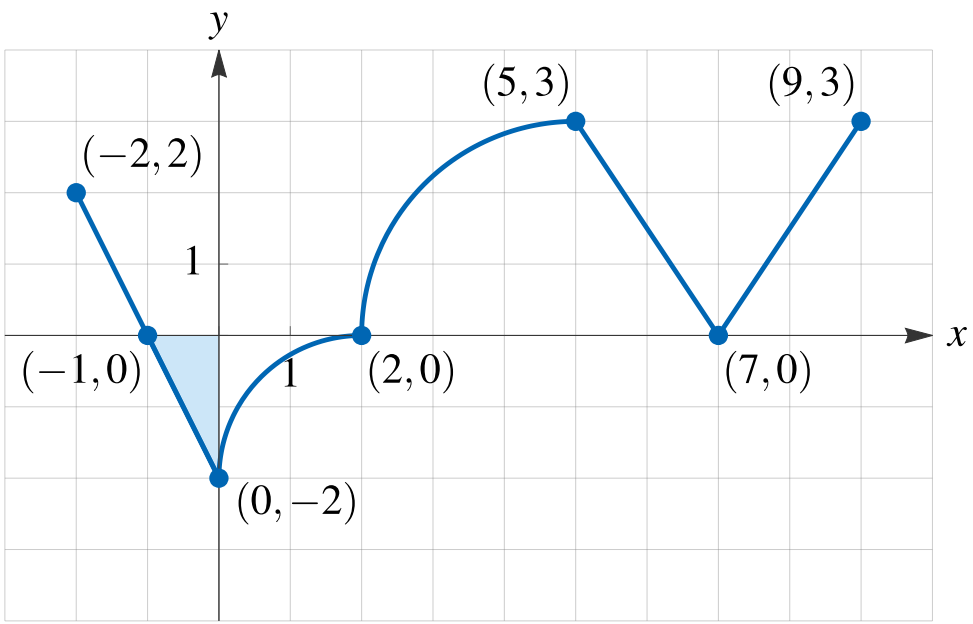
1. The definite integral can be interpreted as net area.

$$\int_a^b f(x)\,dx = A_1 - A_2$$

where A_1 is the area of the region above the x-axis and below the graph of f, and A_2 is the area of the region below the x-axis and above the graph of f.

2. $\displaystyle\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

Solution



$$g(-1) = \int_0^{-1} f(t)\,dt$$ Definition of the function g.

$$= -\int_{-1}^{0} f(t)\,dt$$ Property of definite integrals.

$$= -\left(-\frac{1}{2}(1)(2)\right) = 1$$ Area of a triangle.

Scoring Guidelines

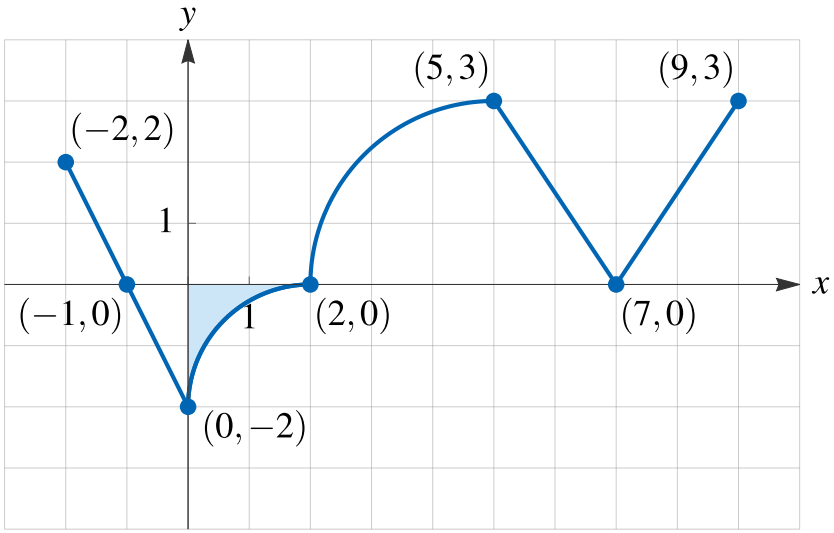
(d) $g(-1) = \int_0^{-1} f(t)\,dt$ $= -\int_{-1}^{0} f(t)\,dt = -\left(-\frac{1}{2}(1)(2)\right) = 1$	1 : answer

Scoring Notes

- Correct supporting work is needed to earn this point.
- A bald 1 does not earn this point.

(e) Find the value of $g(2)$. Show the computations that lead to your answer.

Solution



$$g(2) = \int_0^2 f(t)\,dt$$ Definition of the funciton g.

$$= -\left(2 \cdot 2 - \frac{1}{4} \cdot \pi \cdot 2^2\right)$$ Area of a square minus area of a quarter circle.

$$= -(4 - \pi) = \pi - 4$$ Simplify.

Scoring Guidelines

(e) $g(2) = \int_0^2 f(t)\,dt$

$$= -\left(2 \cdot 2 - \frac{1}{4} \cdot \pi \cdot 2^2\right) = -(4 - \pi)$$

$$2 : \begin{cases} 1 : \text{area of quarter circle} \\ 1 : \text{answer} \end{cases}$$

Scoring Notes

- The response does not need to include a definite integral.
- $\pi - 4$ with no supporting work earns one point.
- Evidence of area of quarter circle in calculation earns one point.

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