MUSIC IS … MATH WITH FEELINGS

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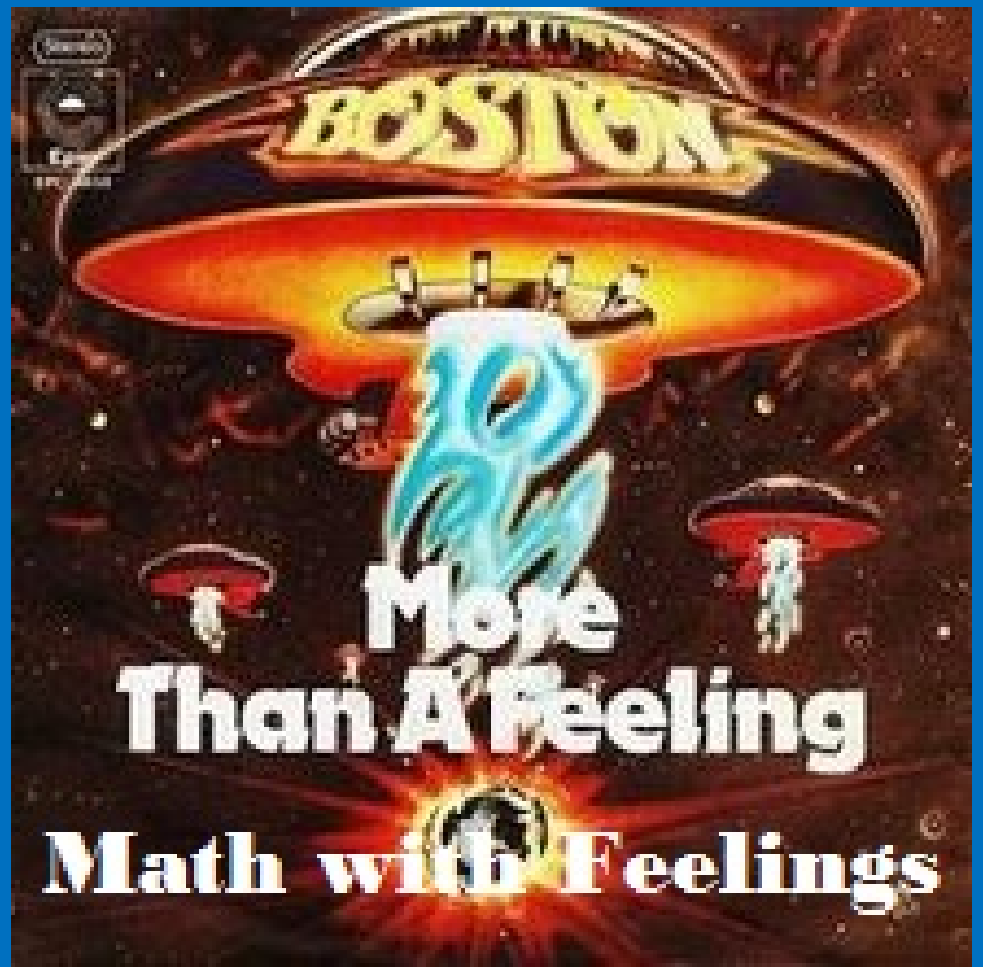
T^3 Webinar: April 21, 2020

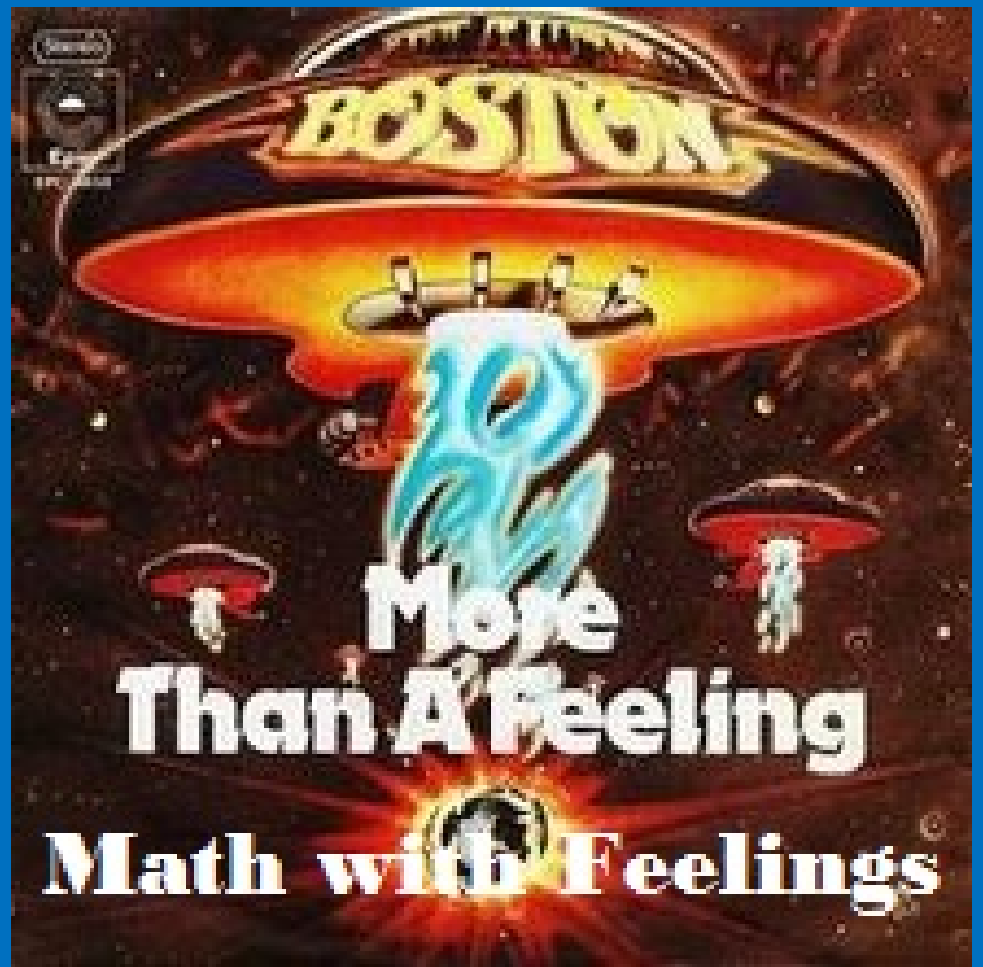
FOUR DISCLAIMERS …

- While many of these ideas were originally intended for “regular” classroom instruction, we will talk about how to adapt them as much as possible to a virtual learning environment.
- In this session we will be using both the TI-84 Plus CE, and the TI-nspire CX II CAS, but everything we will do can be done on any TI-83/84 or TI-nspire.
- Having said that, this session will not be 60 minutes of using the calculator to explore connections between music and math.
- Having said that, we are going to do some really cool stuff with the calculators!

2019 T^3 International Conf: Music is Math with Feelings Presentation

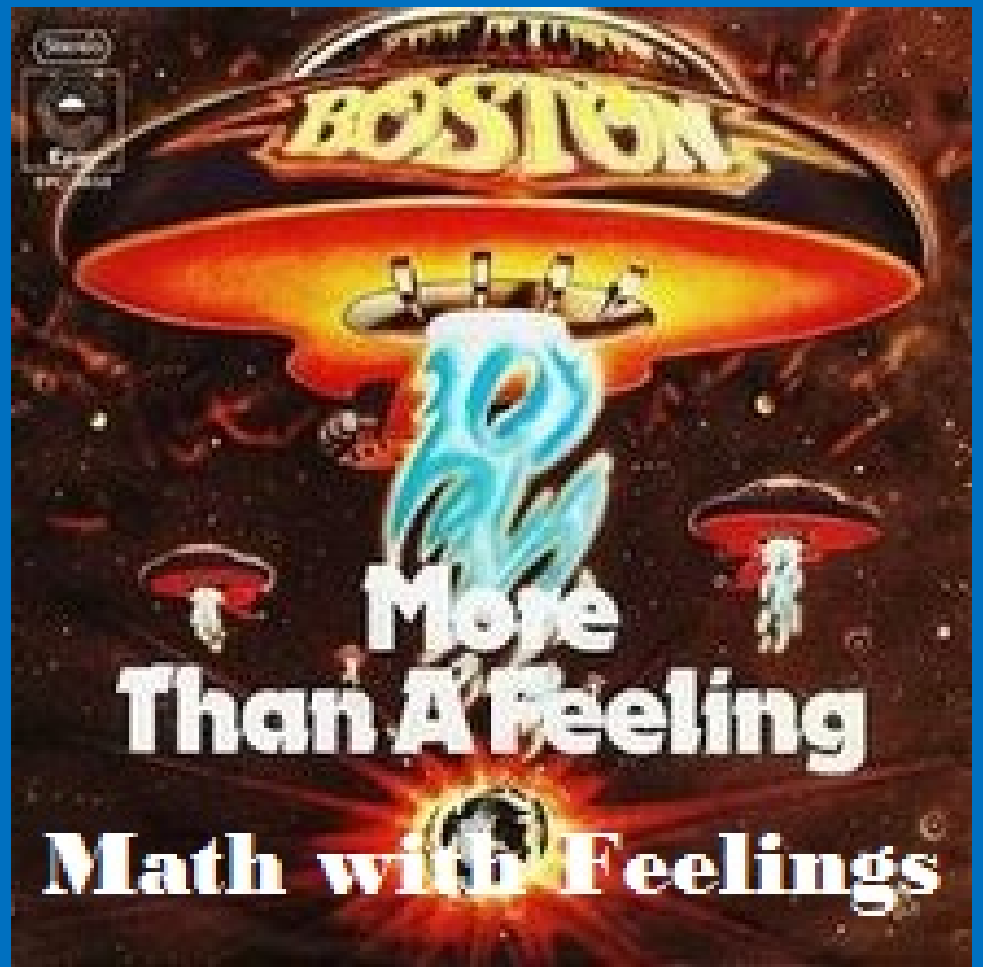

BOSTON
More
Than A Feeling
Math with Feelings

MUSIC AND MATH ARE DEEPLY INTERTWINED



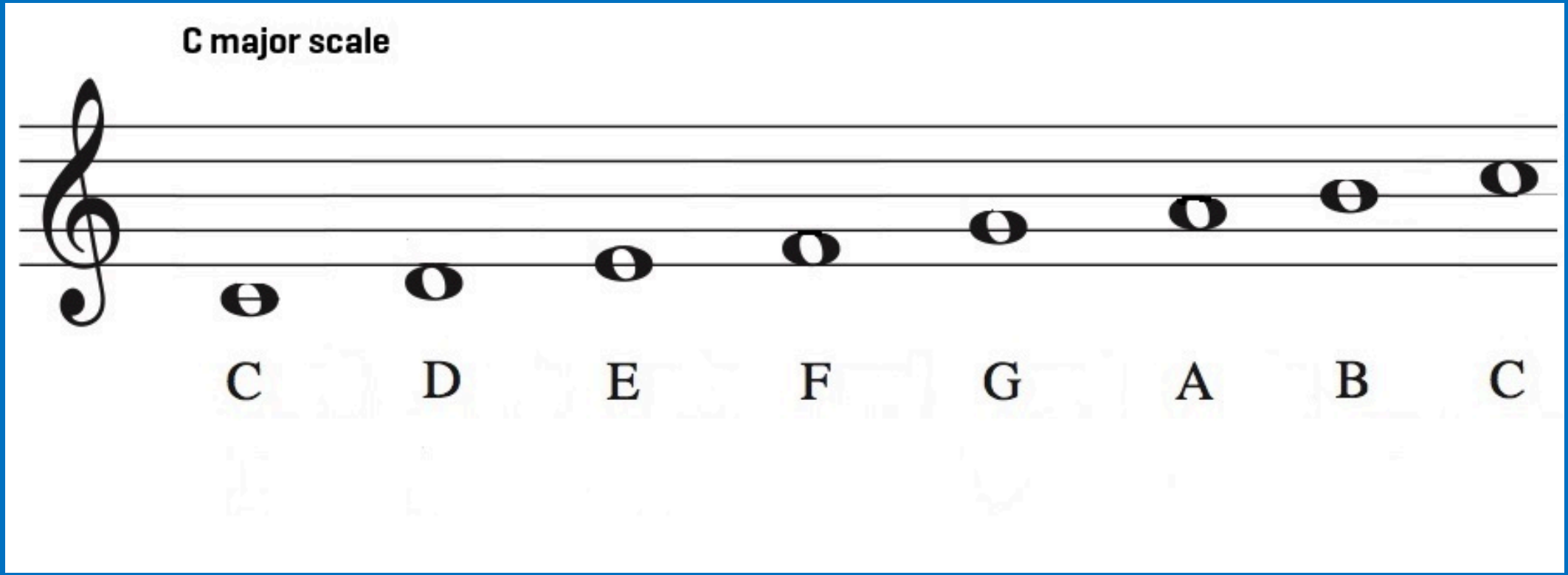
CHORDS LEADING INTO THE CHORUS

Duhm-da-da-uhm-
Duhm-duhm
Duhm-da-da-duhm
Da-duhm-da

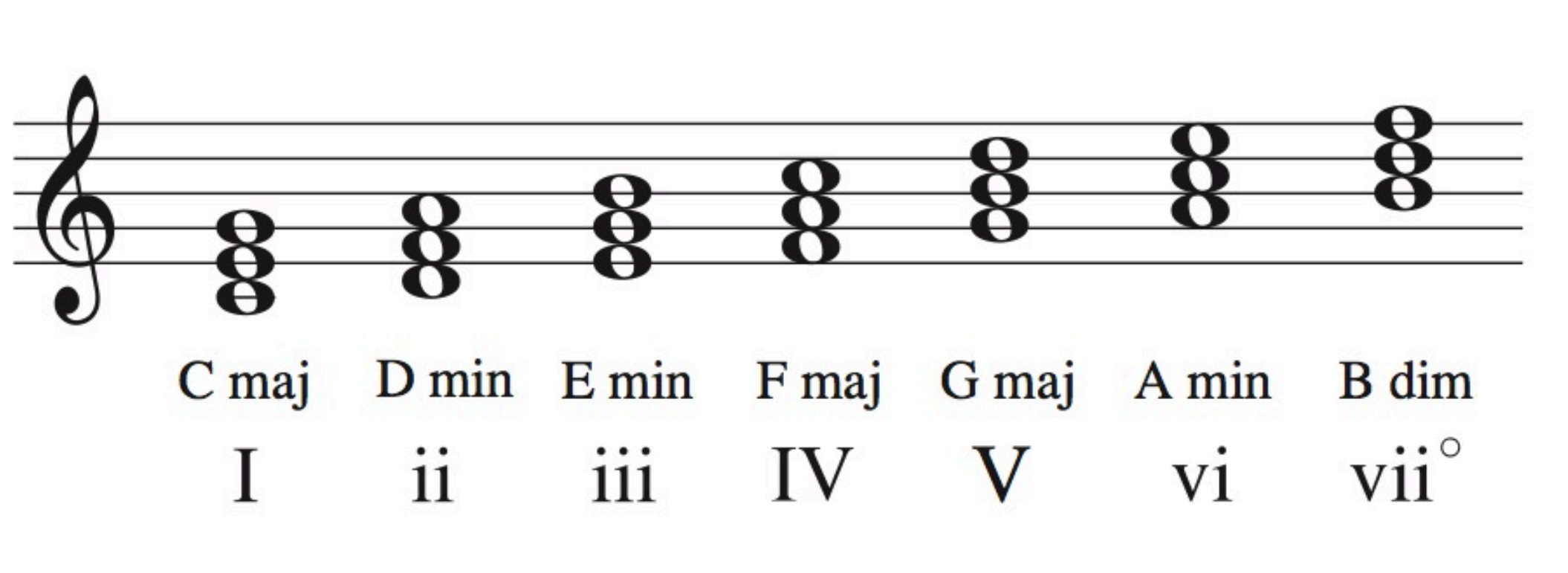


1-1-slide-up-to-4
6-mi-minor
5-5-4

C-MAJOR SCALE



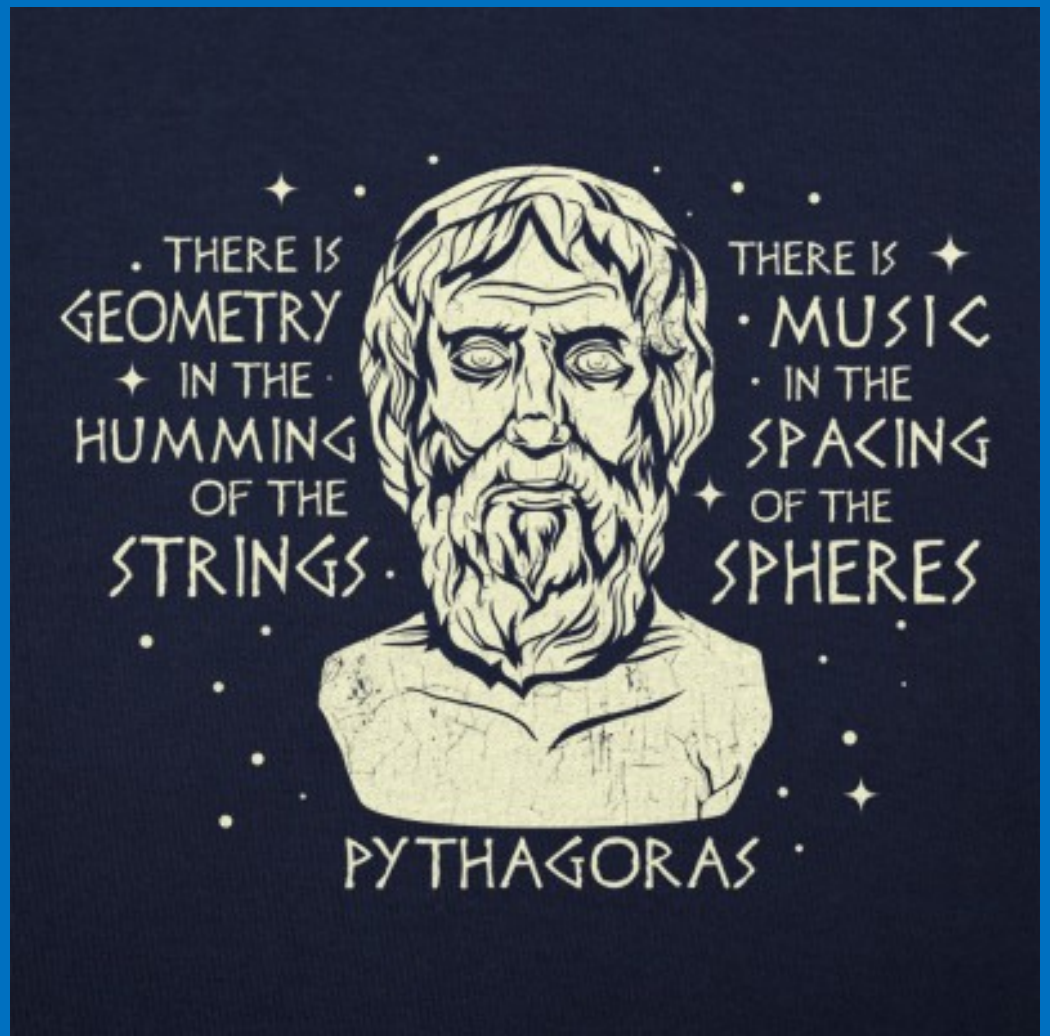
C-MAJOR CHORDS



CHORD NUMBERING SYSTEM

KEY	I	ii	iii	IV	V	vi	vii°	I
	1	2m	3m	4	5	6m	7°	1
C	C	Dm	Em	F	G	Am	B°	C
D	D	Em	F#m	G	A	Bm	C#°	D
E	E	F#m	G#m	A	B	C#m	D#°	E
F	F	Gm	Am	Bb	C	Dm	E°	F
G	G	Am	Bm	C	D	Em	F#°	G
A	A	Bm	C#m	D	E	F#m	G#°	A
B	B	C#m	D#m	E	F#	G#m	A#°	B

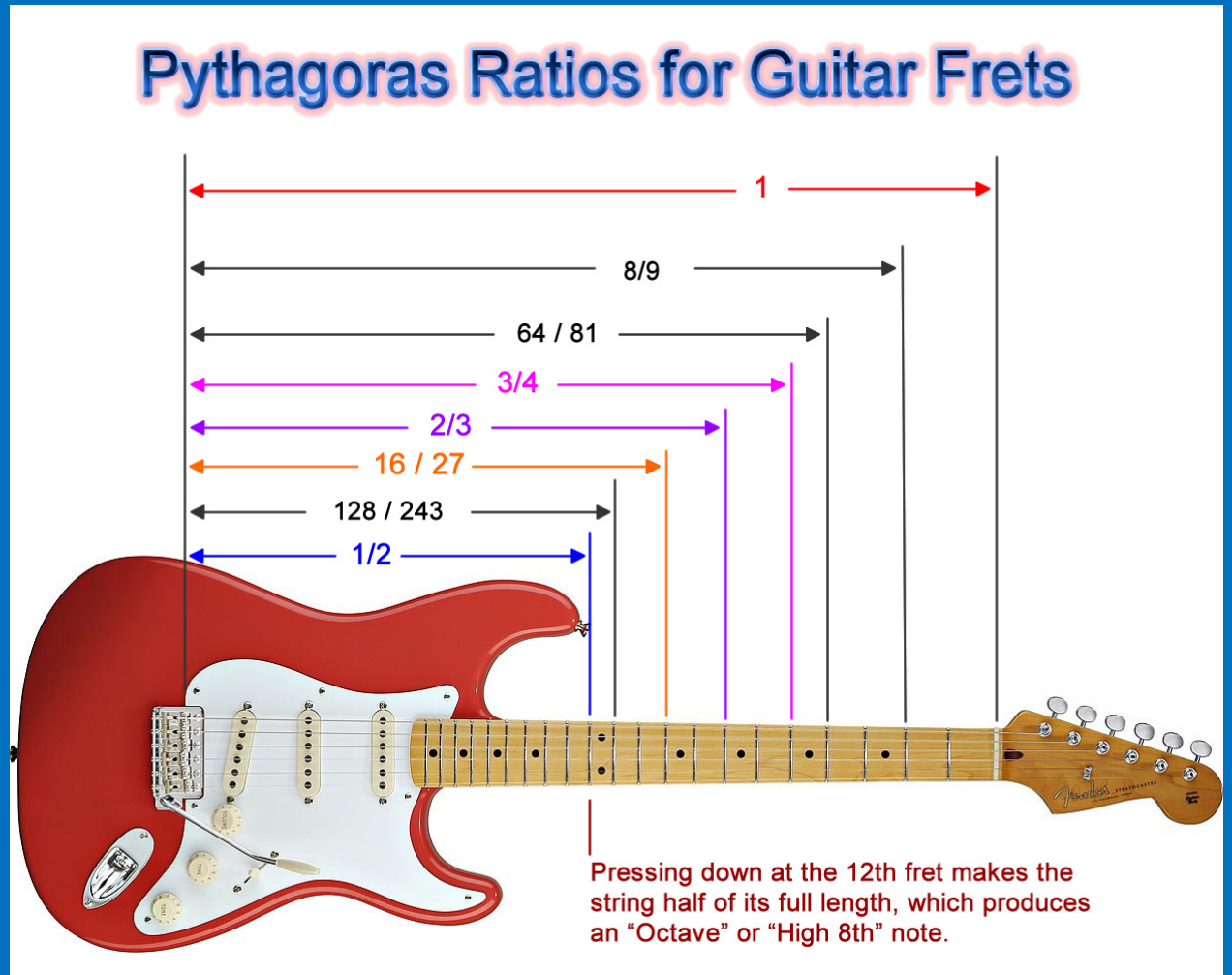
PYTHAGORAS AND MUSIC



PYTHAGOREAN INTERVALS

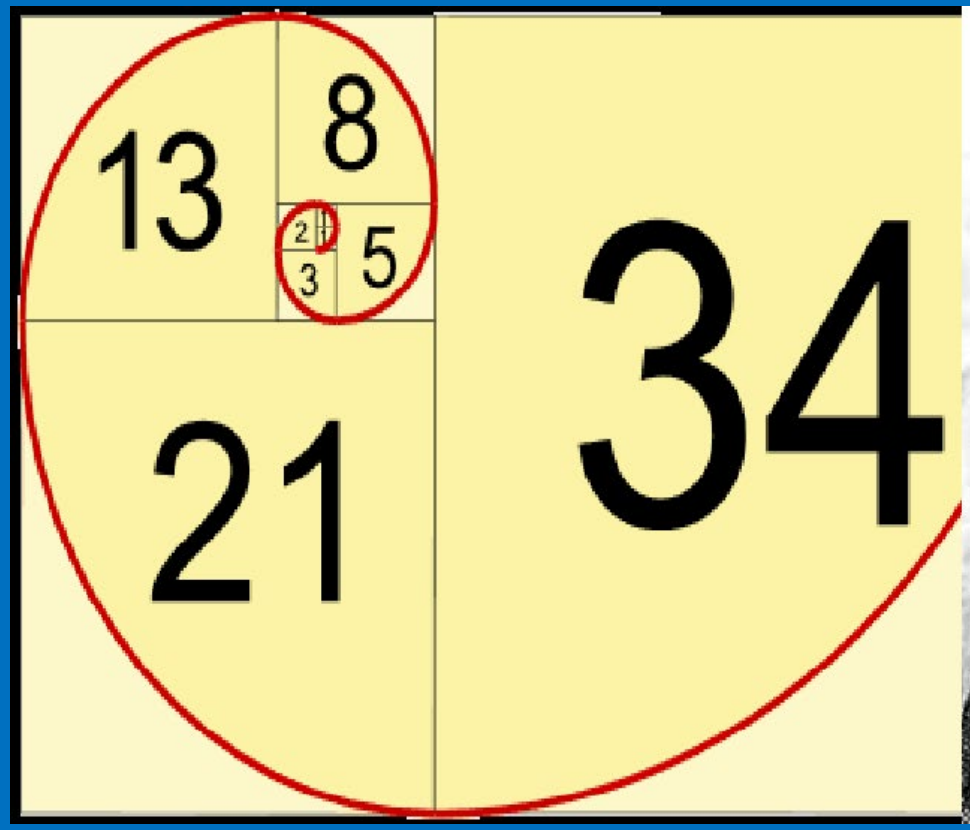
INTERVAL	PYTHAGOREAN RATIO
unison	1
minor second	256/243
major second	9/8
minor third	32/27
major third	81/64
perfect fourth	4/3
diminished fifth	1024/729
perfect fifth	3/2
minor sixth	128/81
major sixth	27/16
minor seventh	16/9
major seventh	243/128
octave	2/1

PYTHAGORAS AND THE GUITAR

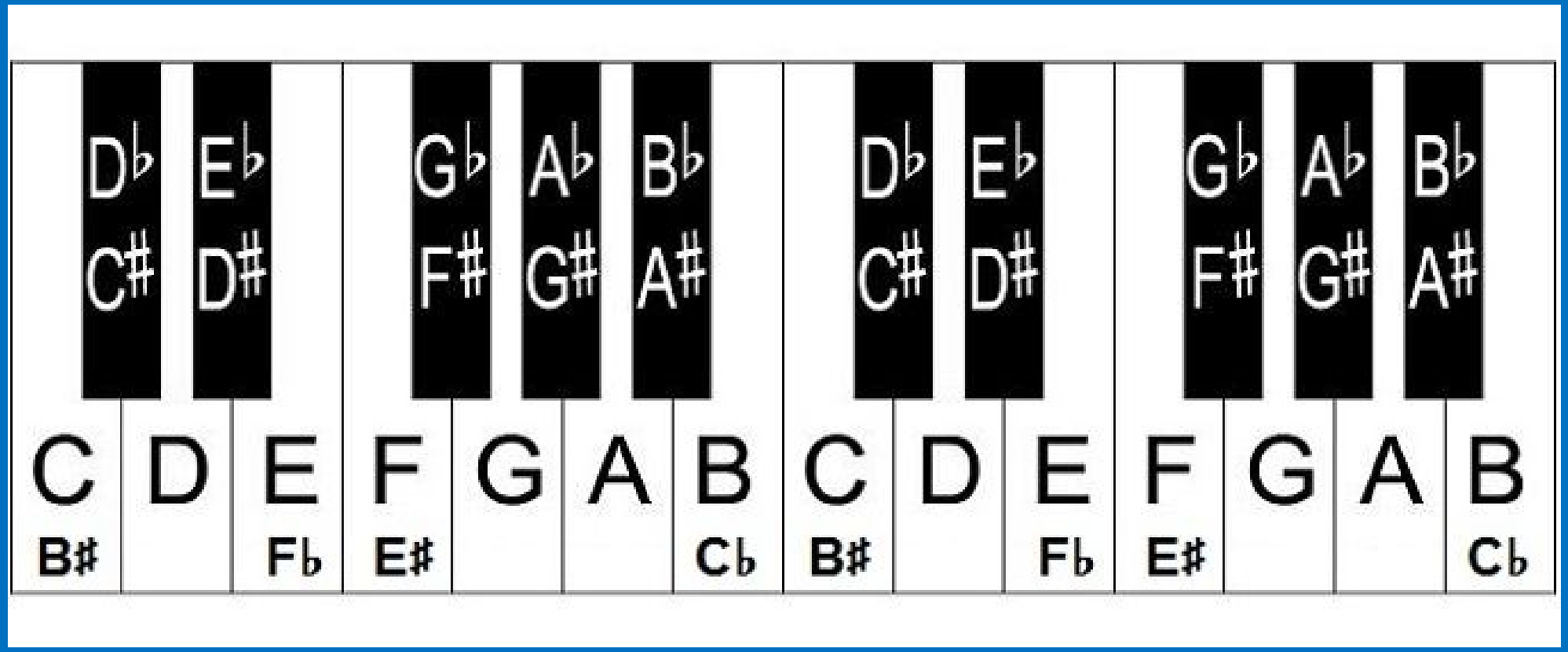


CELEBRATED BY "THE DONALD"

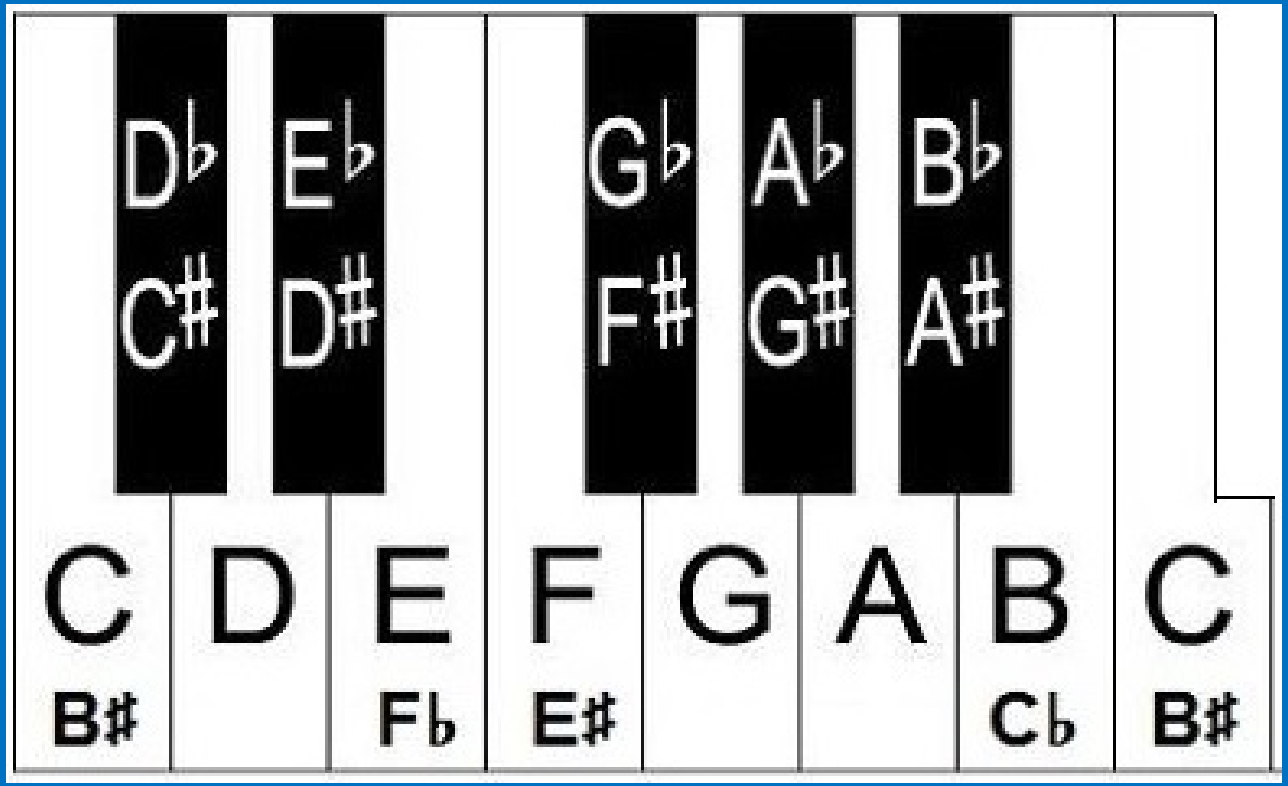
FIBONACCI ON KEYBOARDS



THE KEYBOARD



FIBONACCI "IN" THE KEYBOARD



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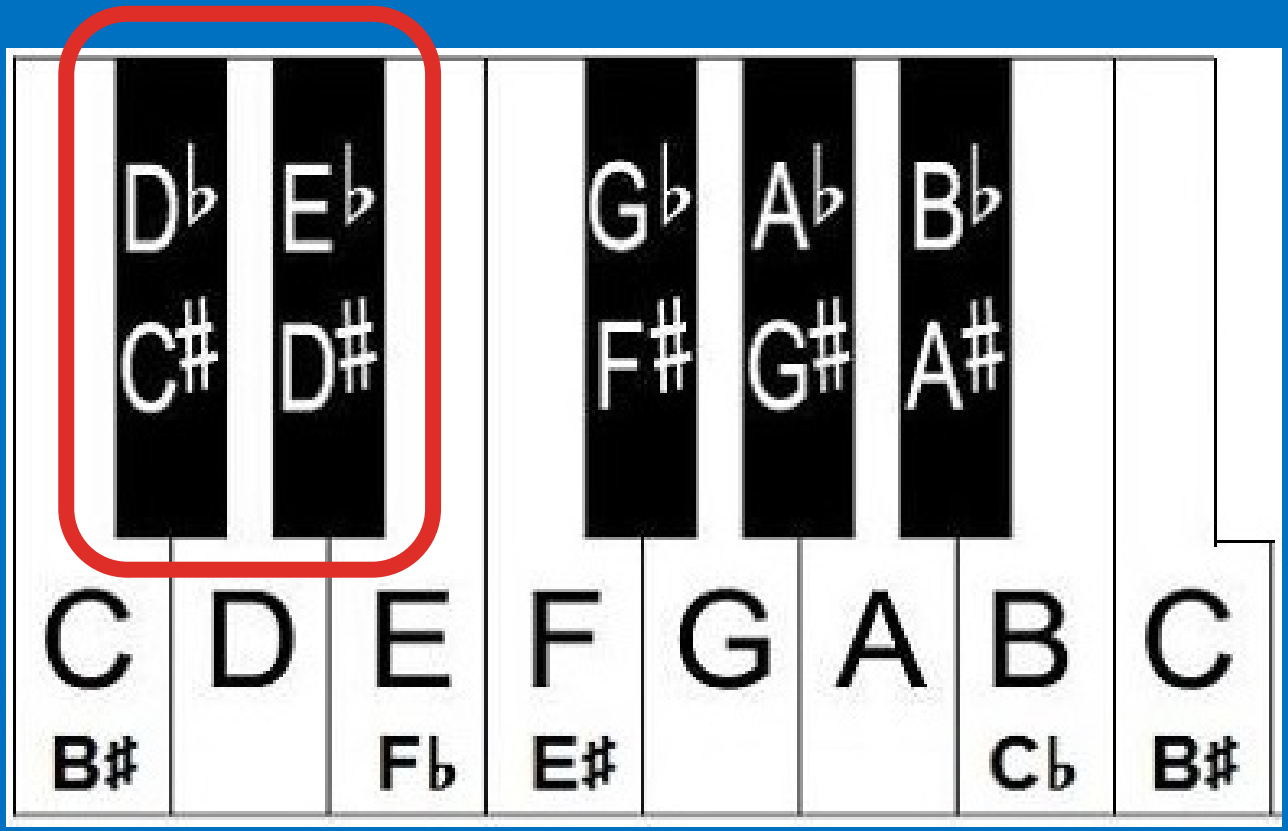

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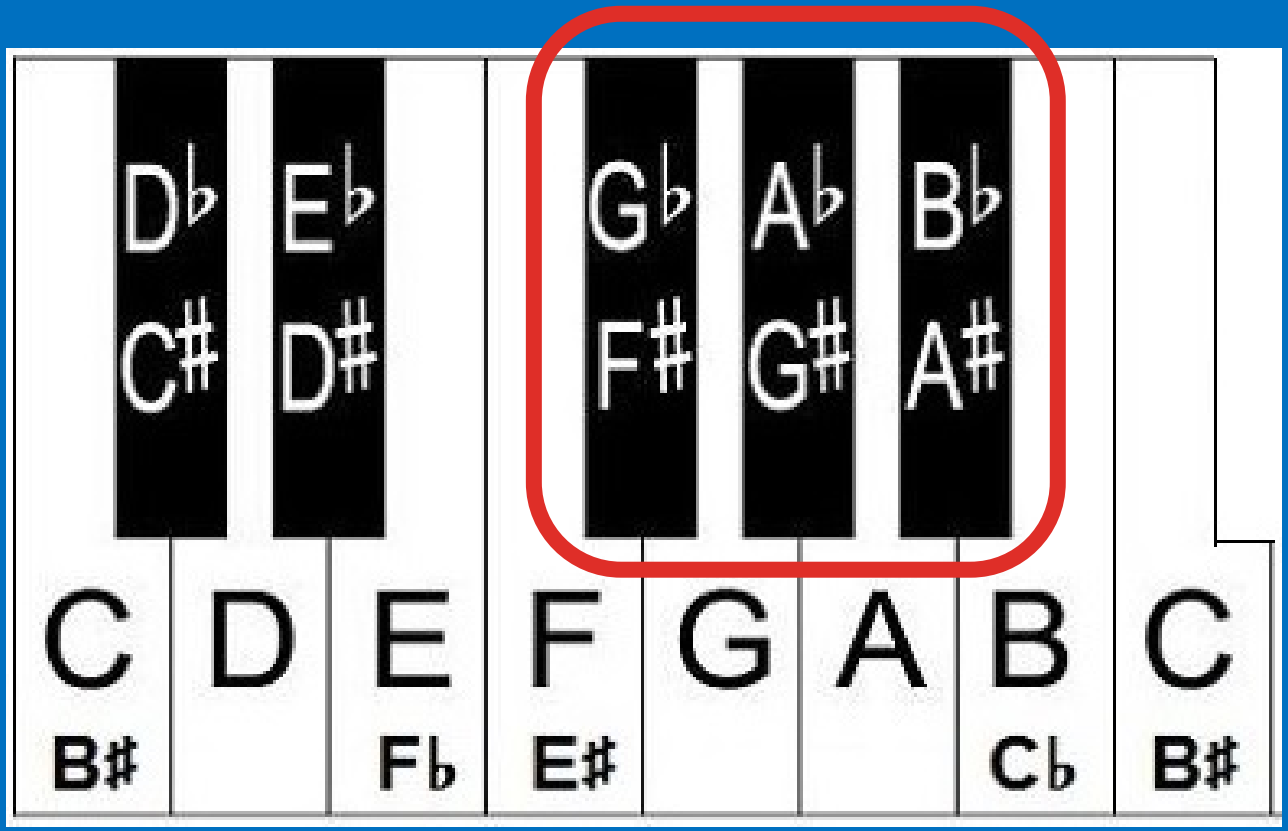

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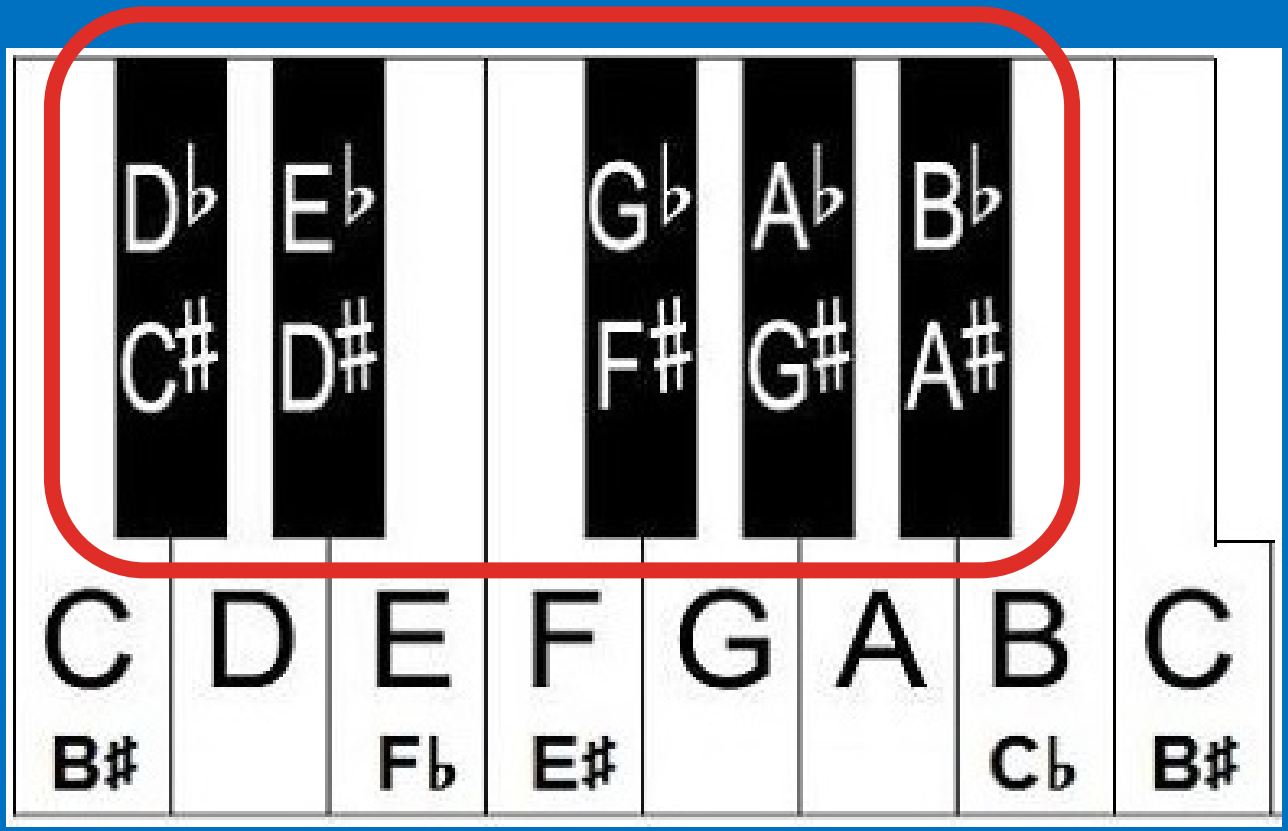

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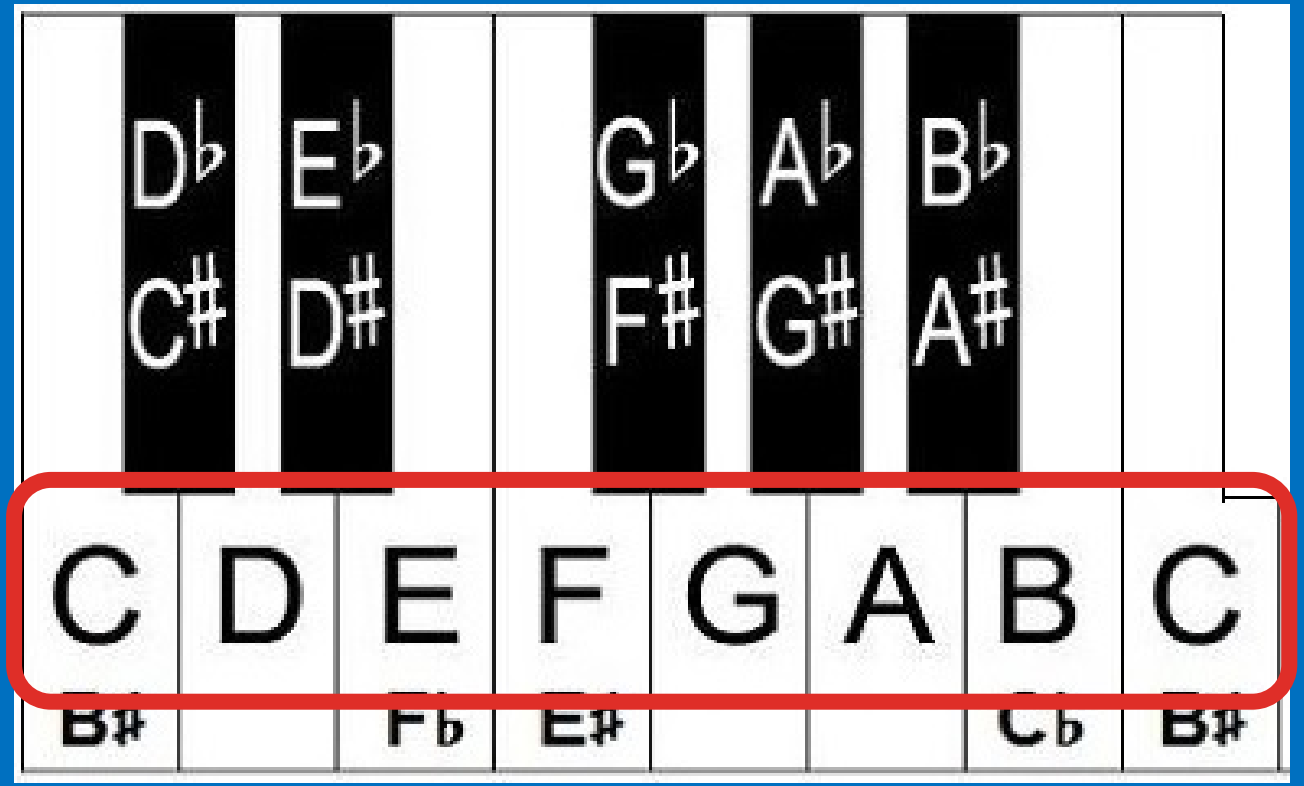

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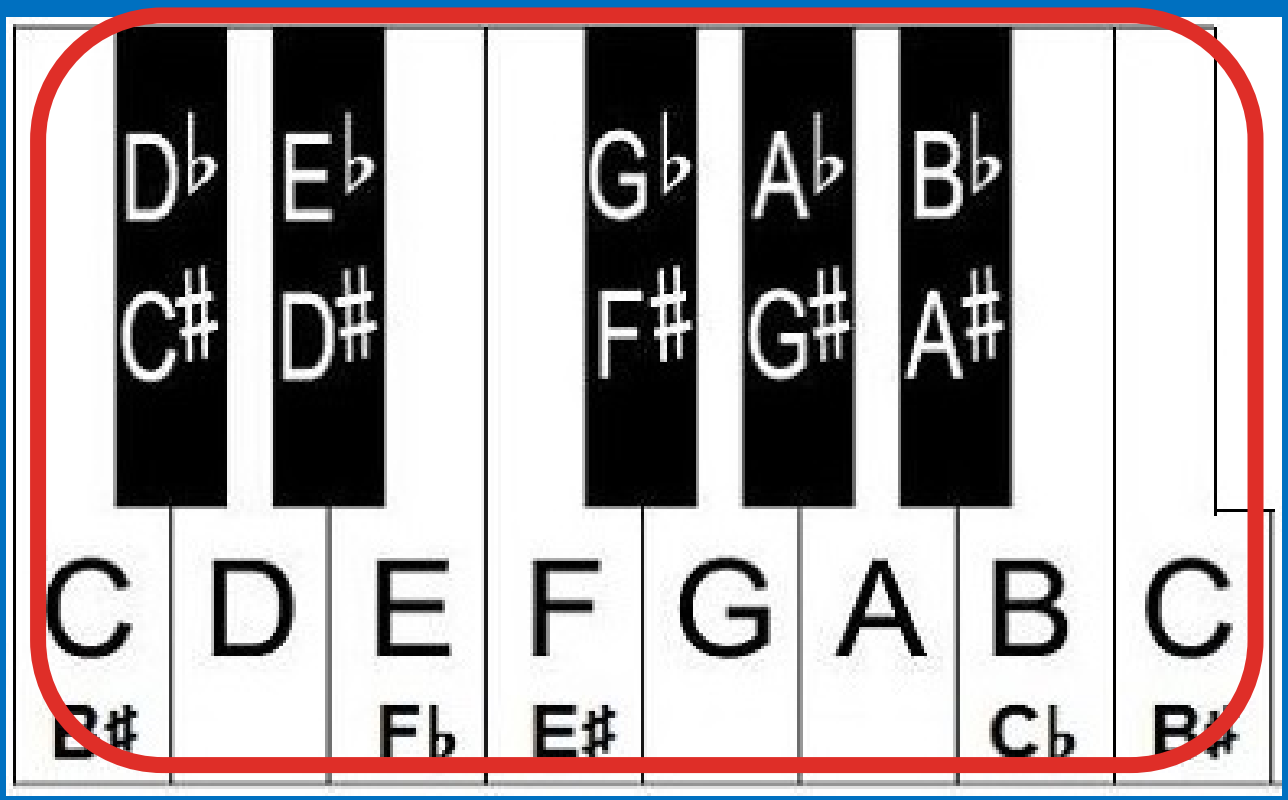

EUCLID ON DRUMS

EUCLID ON DRUMS

The *Euclidean* Algorithm Generates Traditional Musical Rhythms

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Abstract

The *Euclidean* algorithm (which comes down to us from Euclid's *Elements*) computes the greatest common divisor of two given integers. It is shown here that the structure of the Euclidean algorithm may be used to generate, very efficiently, a large family of rhythms used as timelines (*ostinatos*), in sub-Saharan African music in particular, and world music in general. These rhythms, here dubbed *Euclidean* rhythms, have the property that their onset patterns are distributed as evenly as possible. *Euclidean* rhythms also find application in nuclear physics accelerators and in computer science, and are closely related to several families of words and sequences of interest in the study of the combinatorics of words, such as Euclidean strings, to which the *Euclidean* rhythms are compared.

EUCLIDEAN (3,8) BEAT

- Spread 3 beats as evenly as possible over 8 spaces.
- Visually:

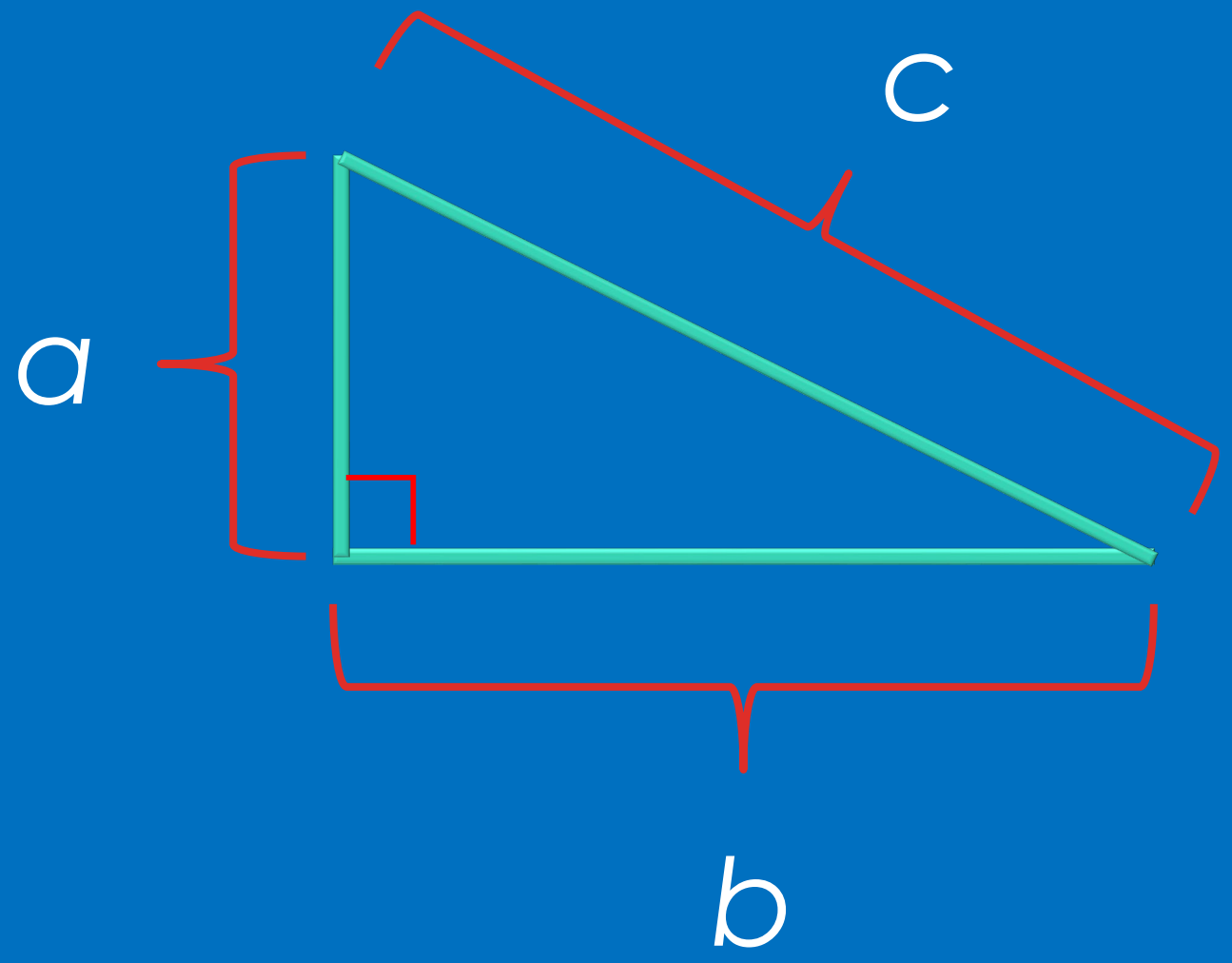
1		2	&	3		4	

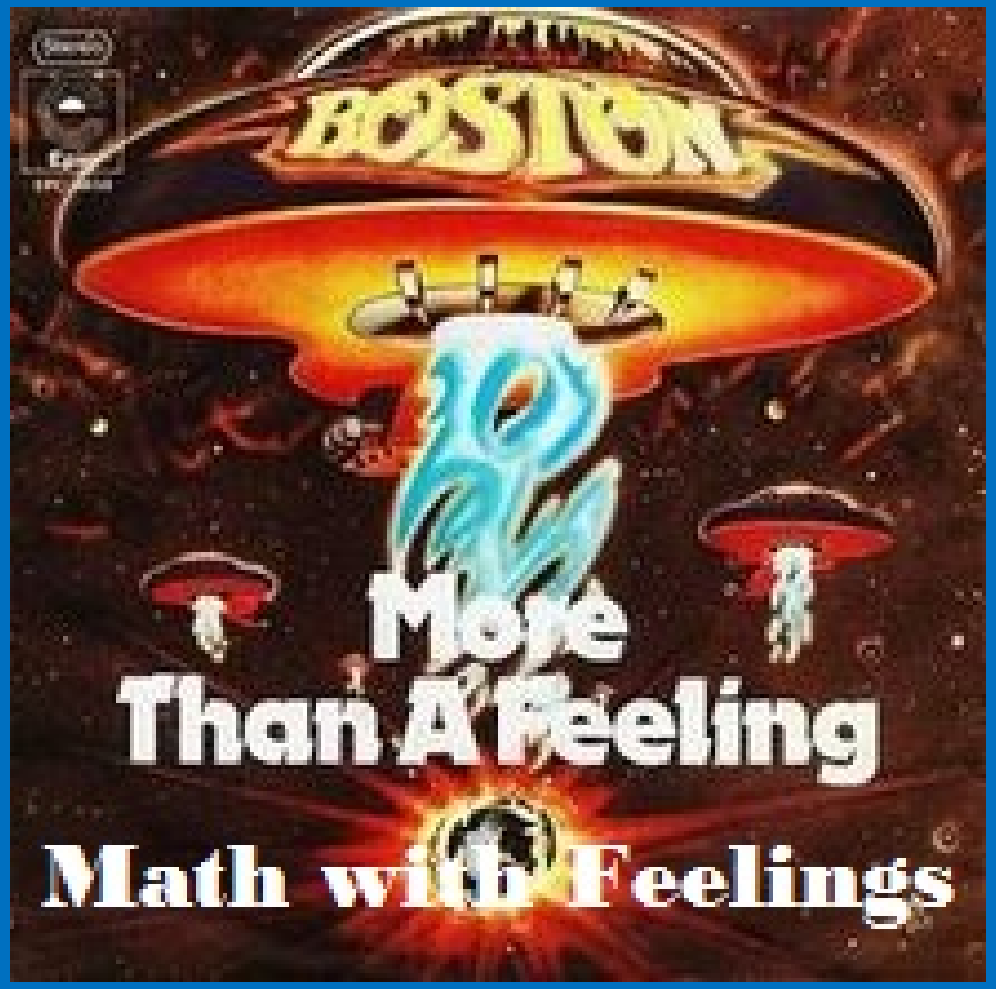
- Rhythmically: Dotted quarter, eighth-quarter tie, quarter
- *E*(3,8): Cuban Tresillo, also American Rockabilly, e.g., Elvis Presley's *Hound Dog*

YOUR TURN TO IMAGINE …

- Euclid on drums
- Fibonacci on keyboards
- Pythagoras on lead guitar
- Yours truly out front
- I call us the Euclidean Fibonacci Pythagorean Sword-Play Band, (or EFPSPB) and it goes something like this …

$$c^2 = a^2 + b^2$$





Math with Feelings

HOW DOES THIS RELATE TO CLASS?

Some things to do:

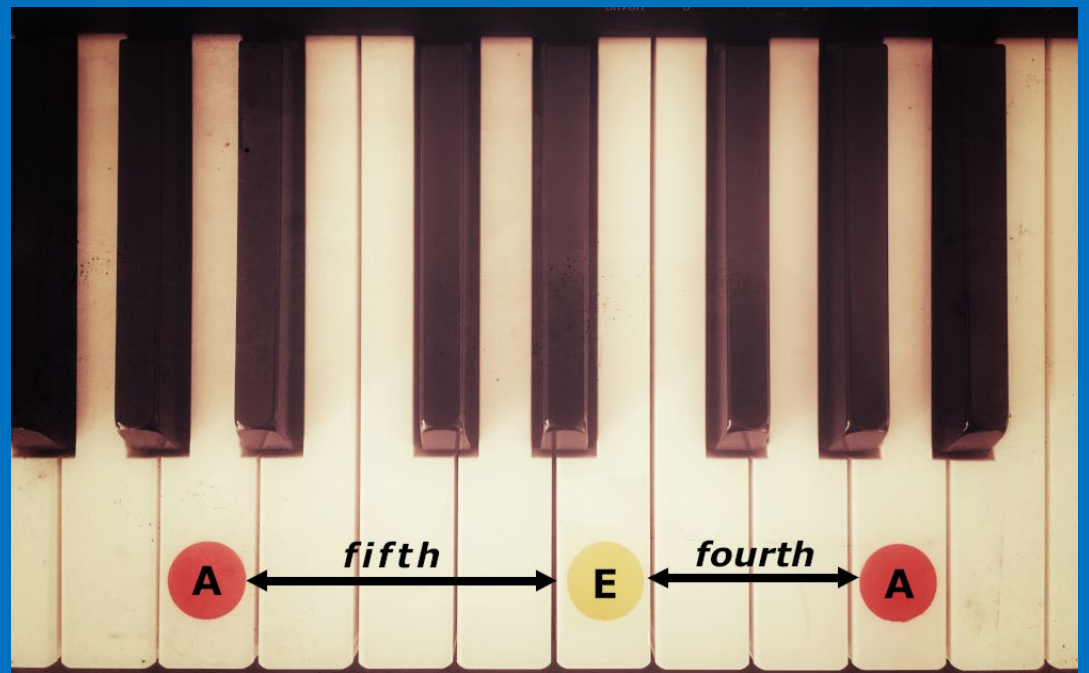
- Songwriting and Math Concepts
- Exploring Pythagorean Ratios
  - “Perfect” vs “Non-perfect” Intervals
  - Guitar Fret Measurement Investigation
- Fun With Fibonacci
  - Finding Fibonacci, The Golden Rectangle, and The Golden Spiral
  - The Calculator, Fibonacci, and the GR
  - For the Metalheads: Lateralus by Tool
- The Euclidean Algorithm
  - The Division Algorithm and .9 repeating …

SONGWRITING AND MATH CONCEPTS

- Example:
  - The Pythagorean Rap/Country/Rock/Disco/Whatever …
  - There are tons of easy-to-find background tracks (kids are much better at finding these than we are!)
- Assignment:
  - Let students choose a concept and write a song or rap to explain it
  - Humor and creativity are highly encouraged!
  - The fewer the guidelines the better, other than the obvious (no vulgarity, no insults, etc.)

EXPLORING PYTHAGOREAN RATIOS

- The "perfect" ratios:
  - Octave, 1:2, or 2:1 (or, any power of 2 or ½)
  - Fifth, 2:3, or 3:2 (whether going up or down)
  - Fourth, 3:4, or 4:3
  - String Length vs. Frequency Ratio
- "Perfect" in a lot of ways!
  - Up a perfect fourth from E is A
  - Down a perfect fifth from E is A
  - So, going up a fourth produces the same note as going down a fifth an octave apart (and vice versa)
  - But, … why is that?



UP A FOURTH AND DOWN A FIFTH

- On any instrument:

Start at E: approx. 659.26 Hz → Go up a perfect fourth interval, the frequency is 880 Hz → Get to a higher A note at frequency ratio of 4/3

Start at the same E: 659.26 Hz → Go down a perfect fifth interval, the frequency 440 Hz → Get to a lower A note at a frequency ratio of 2/3

Up an Octave: Frequency times 2

UP A FIFTH AND DOWN A FOURTH

- On the guitar A string, the E note is at the 7th fret:

Start at E: 7th fret approx. 17.019 inches → Go up a perfect fifth in string length: Open string is 25.5 inches → Get to a lower A at a string ratio of 3/2

Start at the same E: 7th fret approx. 17.019 inches → Go down a perfect fourth in string length: 12th fret approx. 12.75 inches → Get to a higher A at a string ratio of 3/4

Down an Octave: String Length times 2

NOT JUST FOR E AND A

- The Perfect 4th/5th frequency and string ratios also hold when moving up and down from E to B
- So up a Perfect Fourth and down a Perfect Fifth (and vice versa) produces the same tone, an octave apart

Note (Guitar A String)	Freq (Hz)	St Length (in)
B (Fret 2)	493.88	22.718
E (Fret 7)	659.26	17.019
B (Fret 14)	987.77	11.359

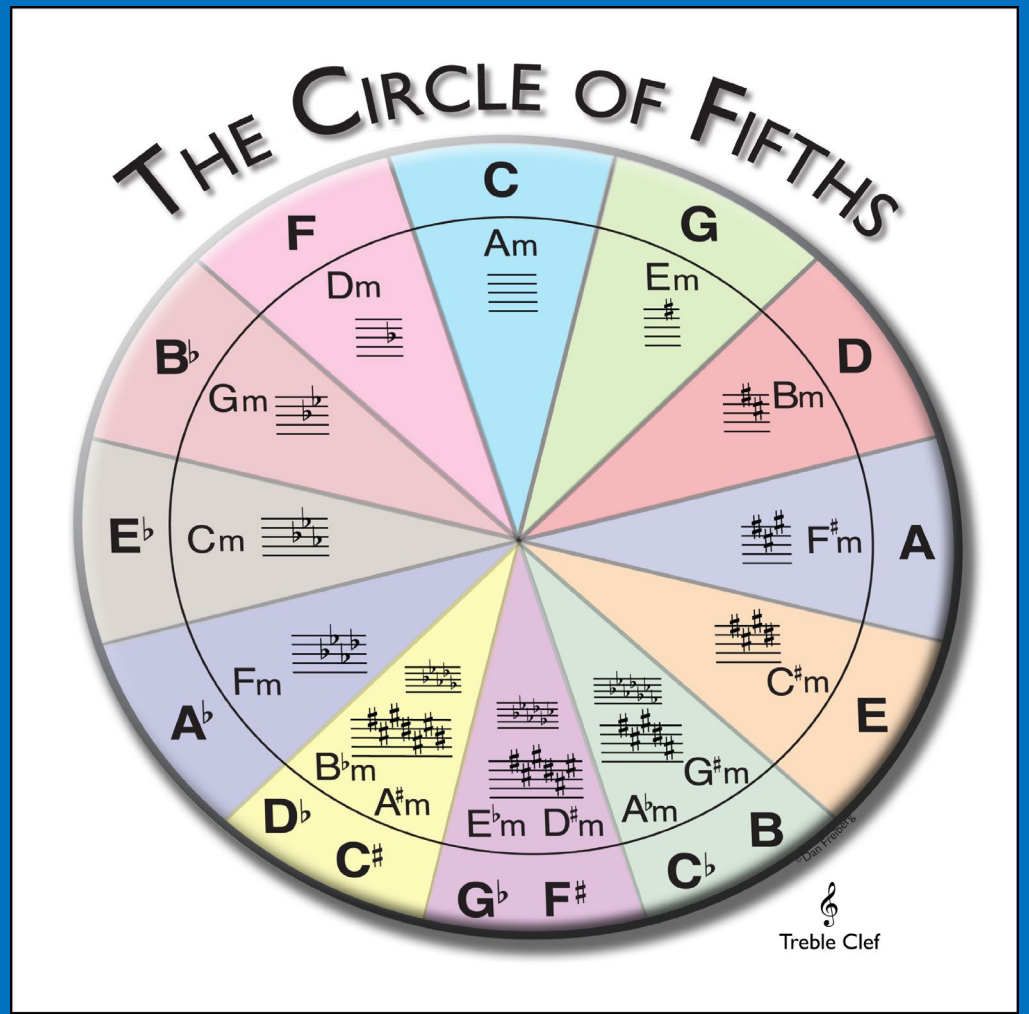
UP DOWN CHALLENGE

- Use Pythagorean Ratios to find another up/down interval pair.
  - One solution: Up a major third, and down a minor sixth
- Math: up is a fraction greater than 1, and down is a fraction less than 1, so find two fractions in which one is twice the reciprocal of the other
- Bonus: note the sum of the number half-steps contained in each pair is 12, which is the total number of half-steps in the chromatic scale

INTERVAL	PYTHAGOREAN RATIO
unison	1
minor second	256/243
major second	9/8
minor third	32/27
major third	81/64
perfect fourth	4/3
diminished fifth	1024/729
perfect fifth	3/2
minor sixth	128/81
major sixth	27/16
minor seventh	16/9
major seventh	243/128
octave	2/1

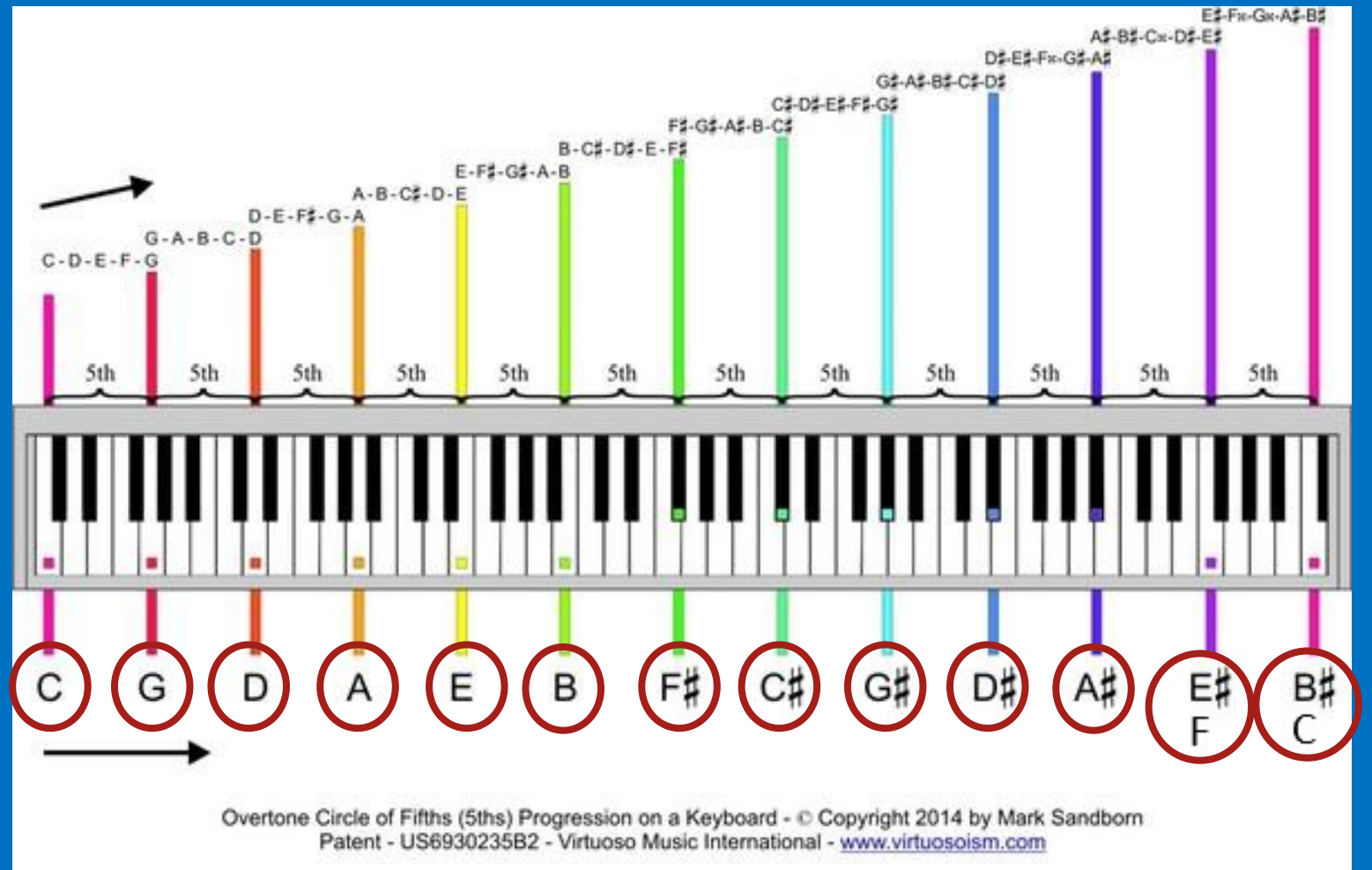
"PERFECT" MUSICAL CIRCLES

- The Circle of Fifths (Clockwise), or The Circle of Fourths (CCW):
- Infinite loops:
  - CW: up a 5th
  - CCW: up a 4th (or down a 5th)
  - Works for Major or minor keys
- Key Signatures:
  - CW: 1 sharp, 2 sharps, etc.
  - CCW: 1 flat, 2 flats, etc.
- For any key (M or m), the "five" chord is one "slice" CW, the "four" chord is one "slice" CCW.
- There is still much more … !



"PERFECT" MUSICAL CIRCLES

- The Circle of Fifths:

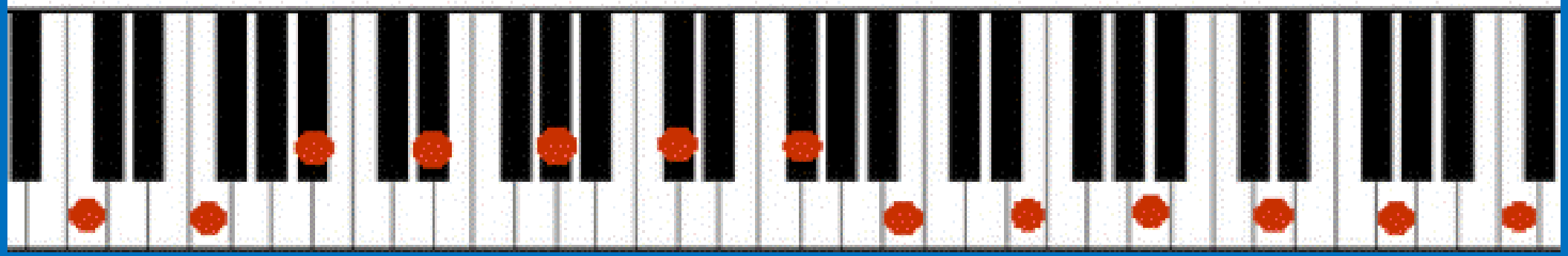


“PERFECT” MUSICAL CIRCLES

- The Circle of Fourths:

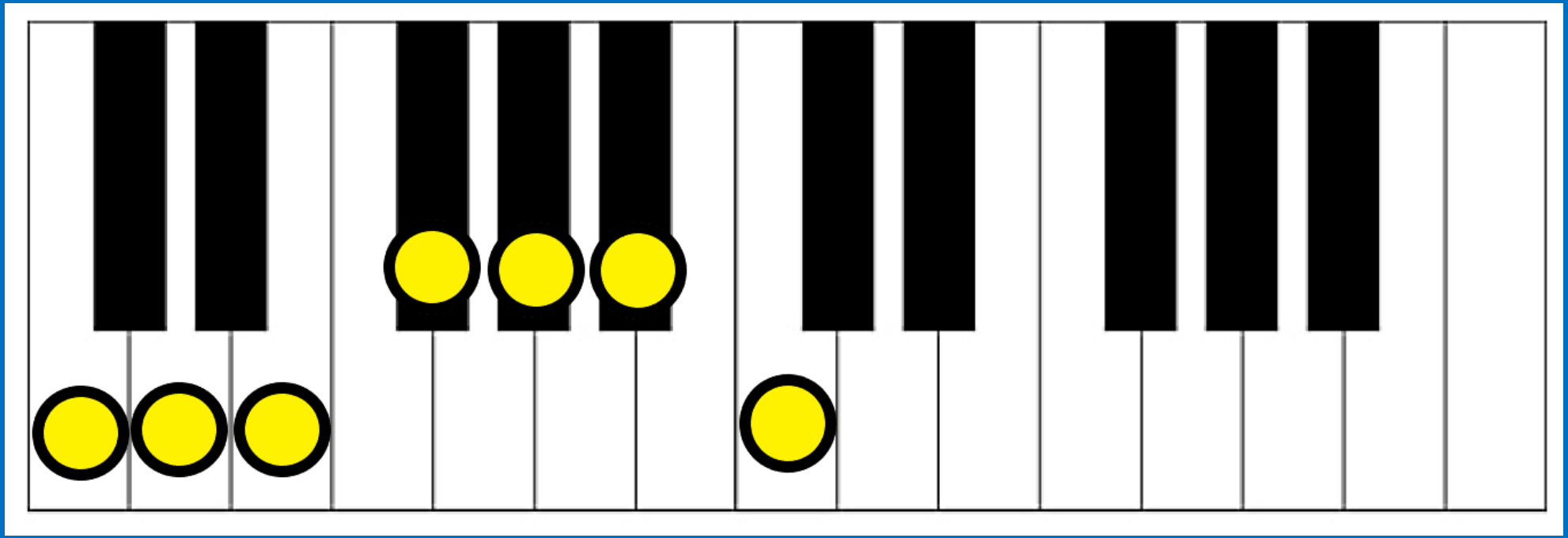
C – F – Bb – Eb – Ab – Db – Gb

= F# – B – E – A – D – G – C



“NON-PERFECT” MUSICAL CIRCLES

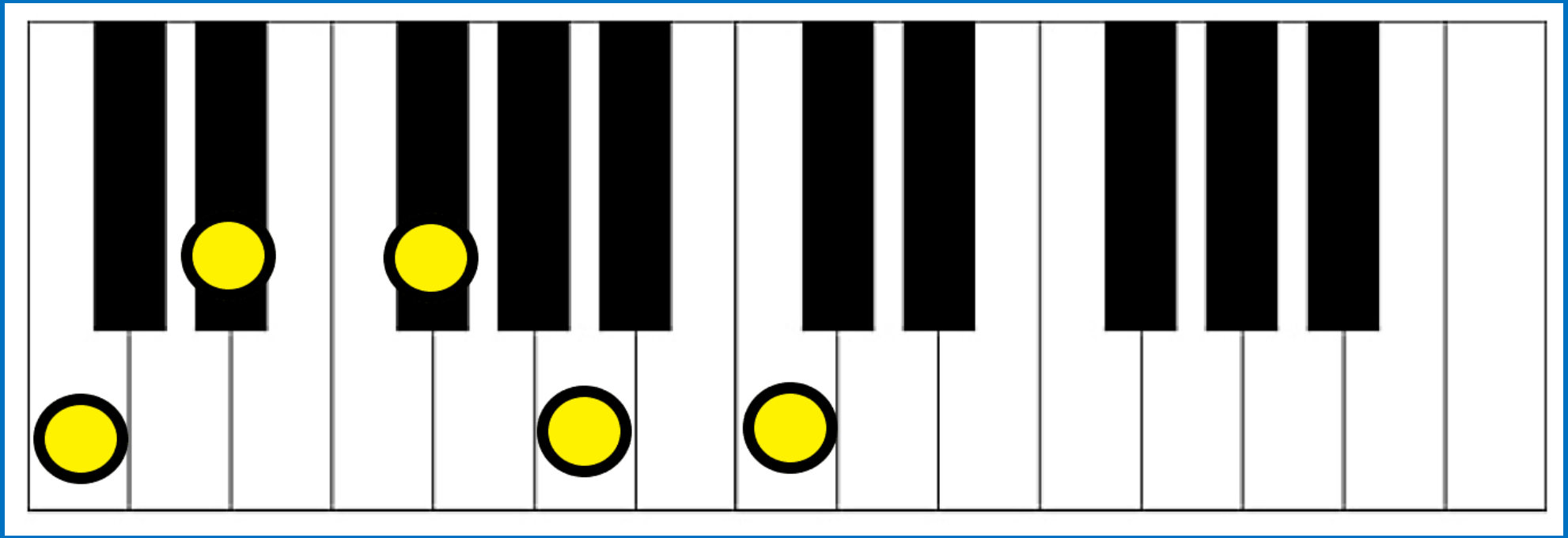
- The Circle of Major 2nds:
- (“Pretty pretty please, don’t you ever ever feel ..”)



C D E F#G#A# C

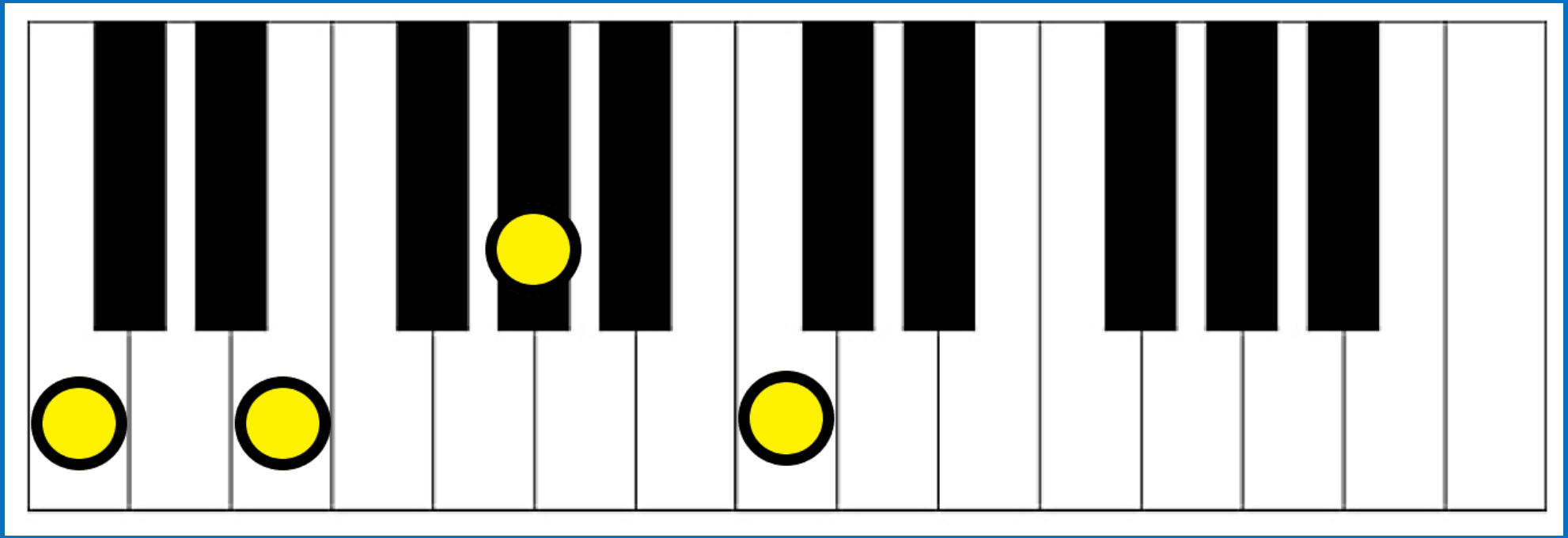
“NON-PERFECT” MUSICAL CIRCLES

- The Circle of Minor 3rds:



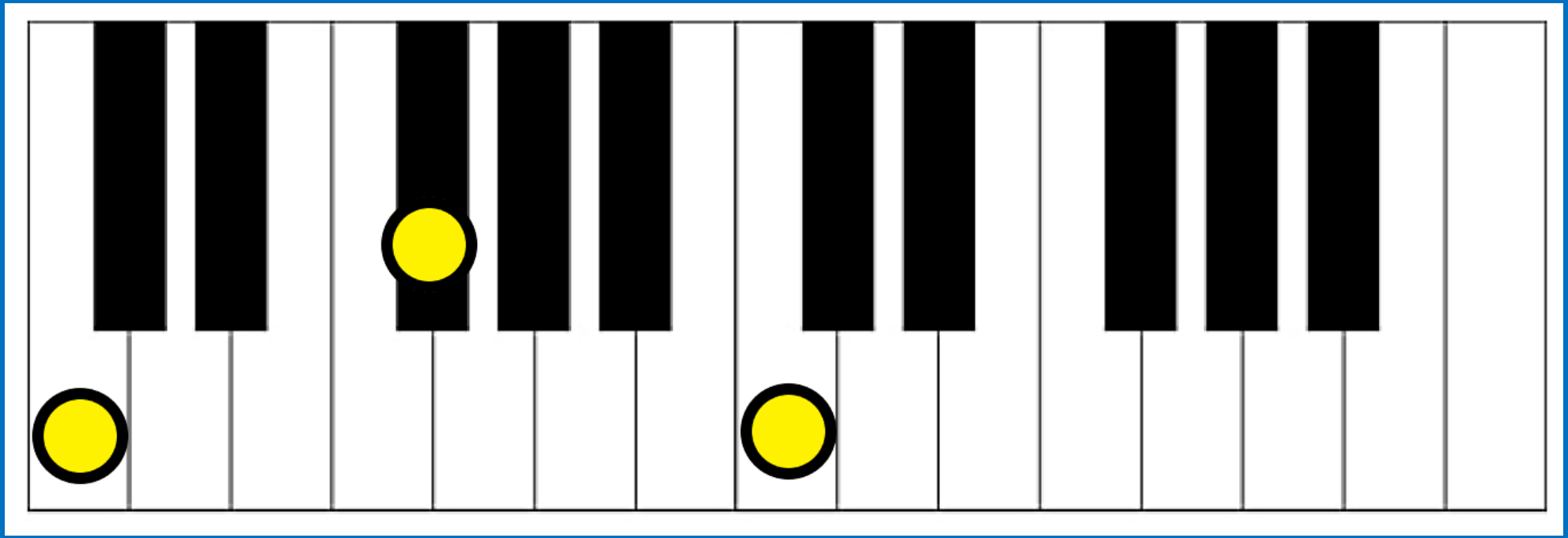
“NON-PERFECT” MUSICAL CIRCLES

- The Circle of Major 3rds:



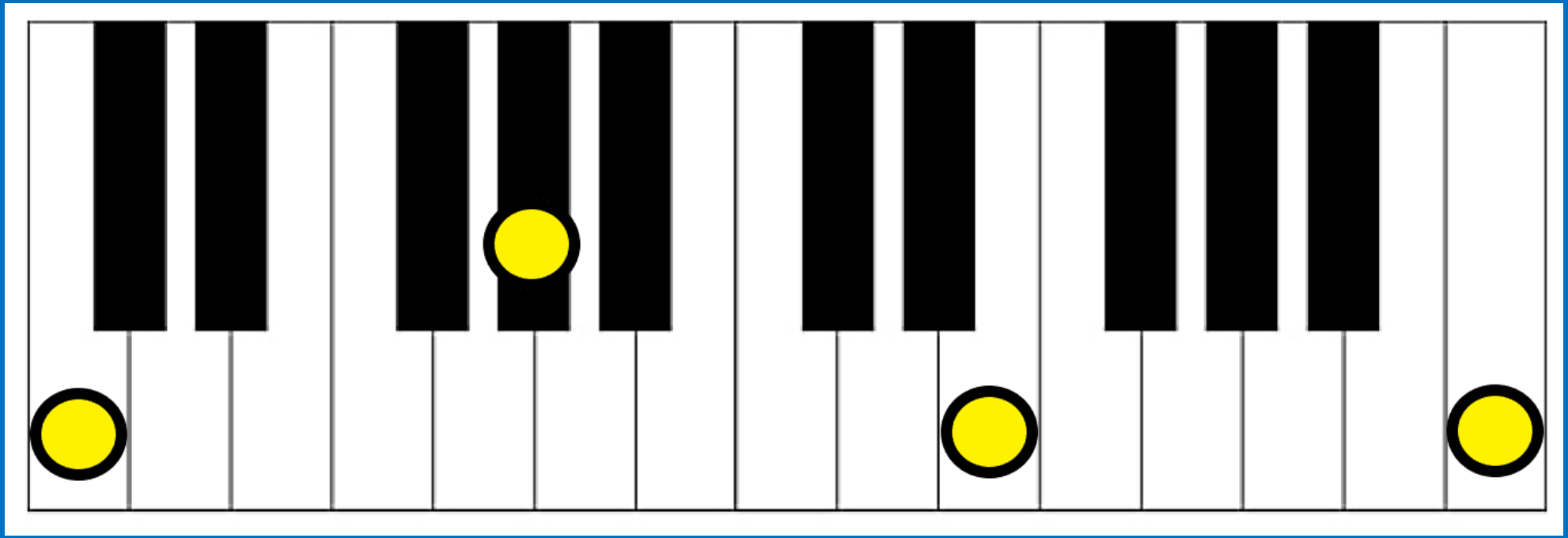
"NON-PERFECT" MUSICAL CIRCLES

- The Circle of Augmented 4th/Diminished 5ths:



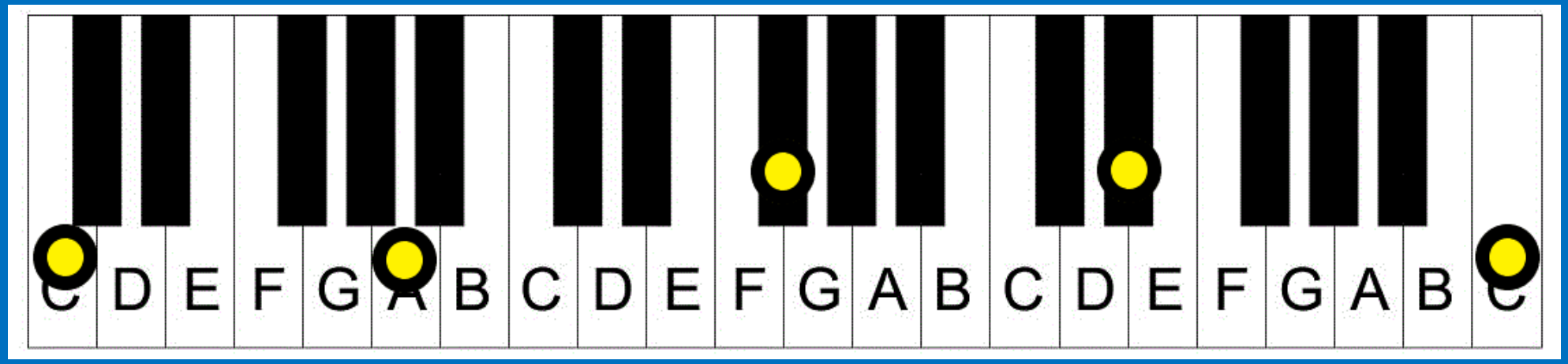
“NON-PERFECT” MUSICAL CIRCLES

- The Circle of Minor 6ths (2 Octaves):



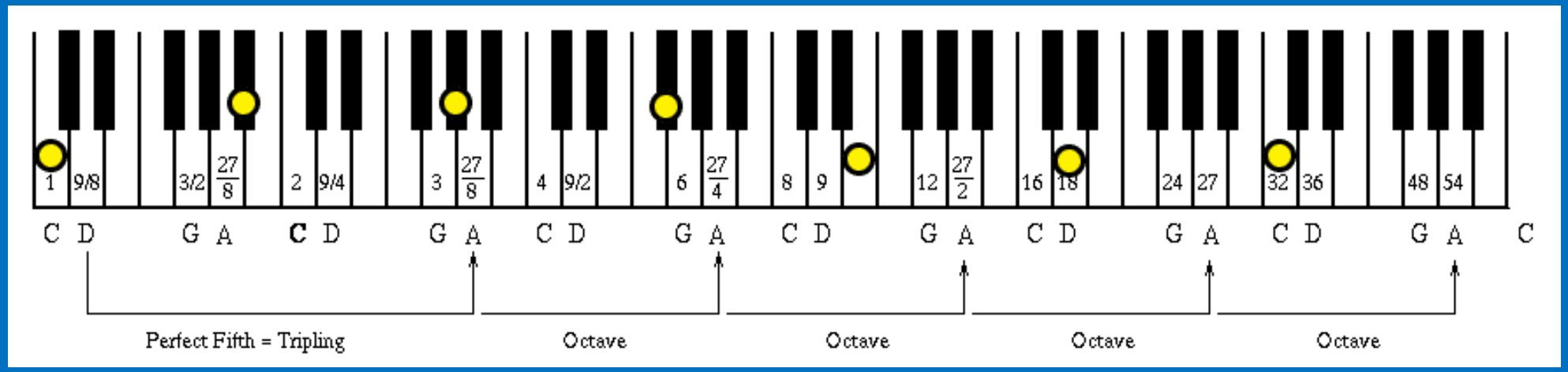
"NON-PERFECT" MUSICAL CIRCLES

- The Circle of Major 6ths (3 Octaves):



"NON-PERFECT" MUSICAL CIRCLES

- The Circle of Minor 7ths (5 Octaves):



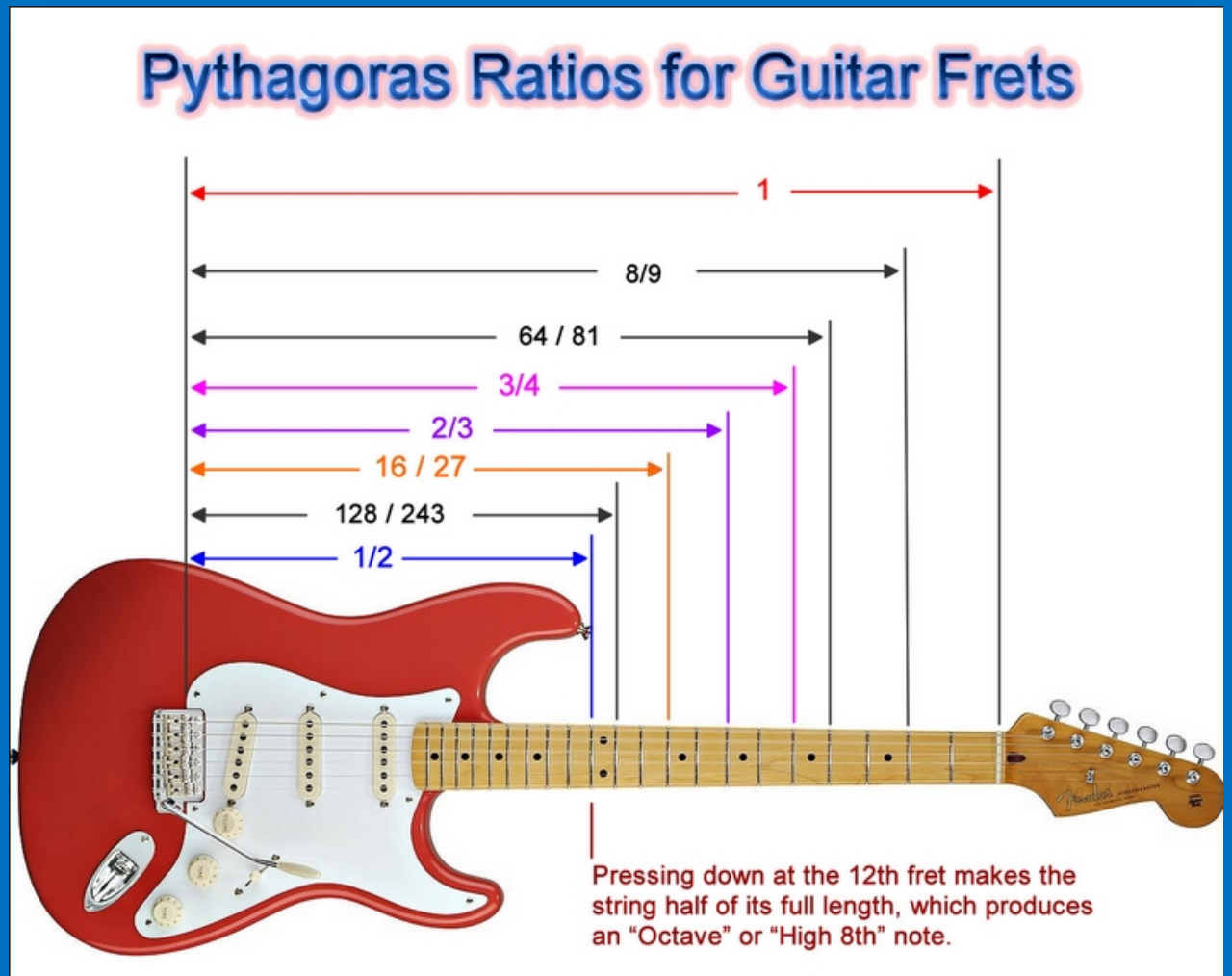
PERFECT VS NON-PERFECT

Perfect Circles	Num of half-tones per interval
Perfect 4th	5
Perfect 5th	7
Non-Perfect Circles	
Major 2nd	2
Minor 3rd	3
Major 3rd	4
Aug4/Dim5	6
Minor 6th	8
Major 6th	9
Minor 7th	10
Trivial Cases	
Minor 2nd	1
Major 7th	11

Relatively prime to 12

Not relatively prime to 12

GUITAR FRET MEASUREMENT INVESTIGATION



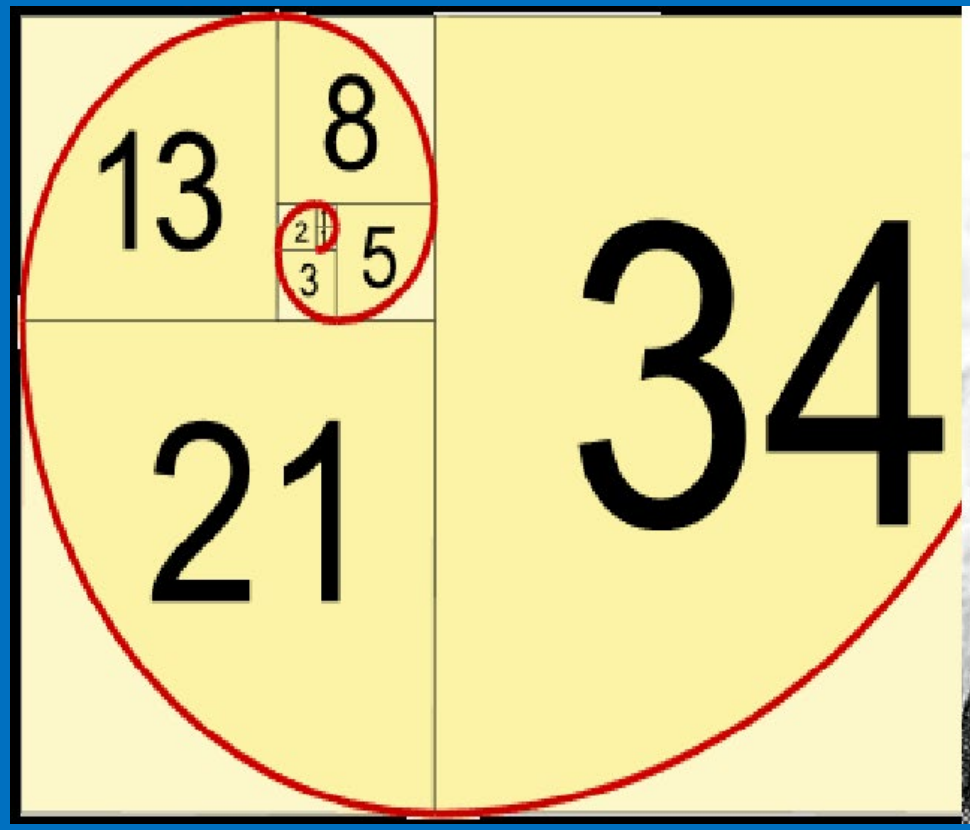
GUITAR FRET MEASUREMENT INVESTIGATION

GUITAR FRET MEASUREMENT INVESTIGATION

	A fret	B elecin	C eleccm	D bassin	E basscm
=	=seq(n,n,(				
1	0	25.4688	64.7	34.0625	86.5
2	1	24	61	32.125	81.65
3	2	22.6563	57.6	30.3438	77.05
4	3	21.375	54.35	28.625	72.75
5	4	20.1875	51.3	27.0313	68.7
6	5	19.0313	48.4	25.5	64.8
7	6	17.9688	45.7	24.0625	61.15
8	7	16.9375	43.1	22.7188	57.75
9	8	16	40.7	21.5625	54.5
10	9	15.0938	38.4	20.25	51.4
11	10	14.25	36.2	19.125	48.5
12	11	13.4375	34.2	18.0625	45.8
13	12	12.6875	32.25	17.0313	43.25
14	13	11.9688	30.4	16.0938	40.85
15	14	11.2813	28.7	15.1875	38.55
16	15	10.6563	27.1	14.3125	36.4
17	16	10.0625	25.6	13.5313	34.35

A1 =0

FIBONACCI ON KEYBOARDS



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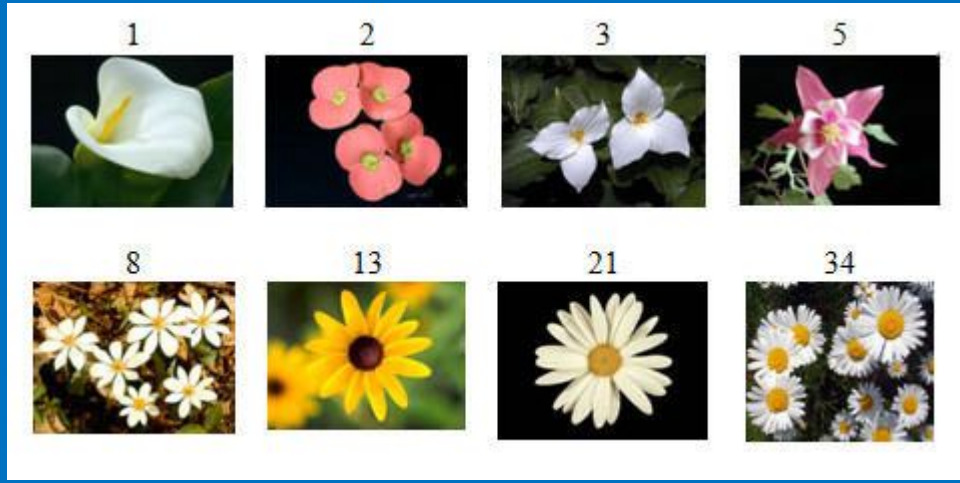

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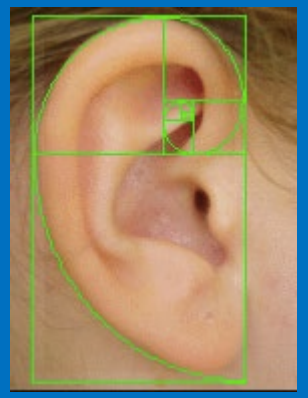
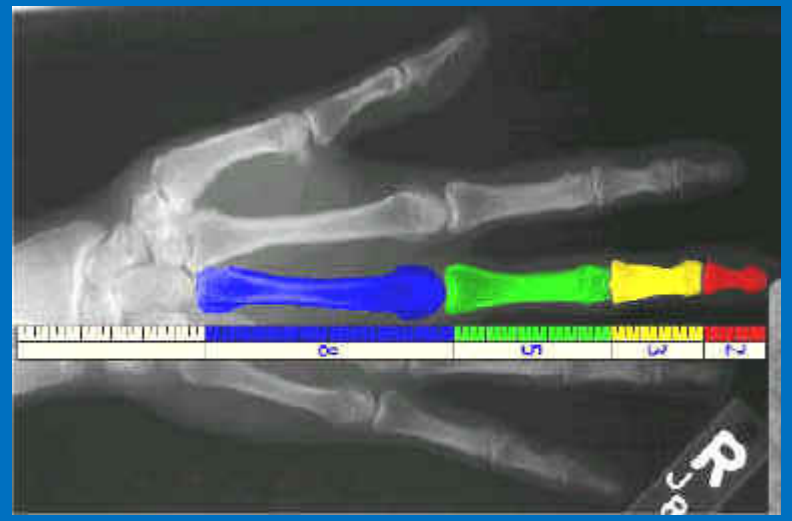
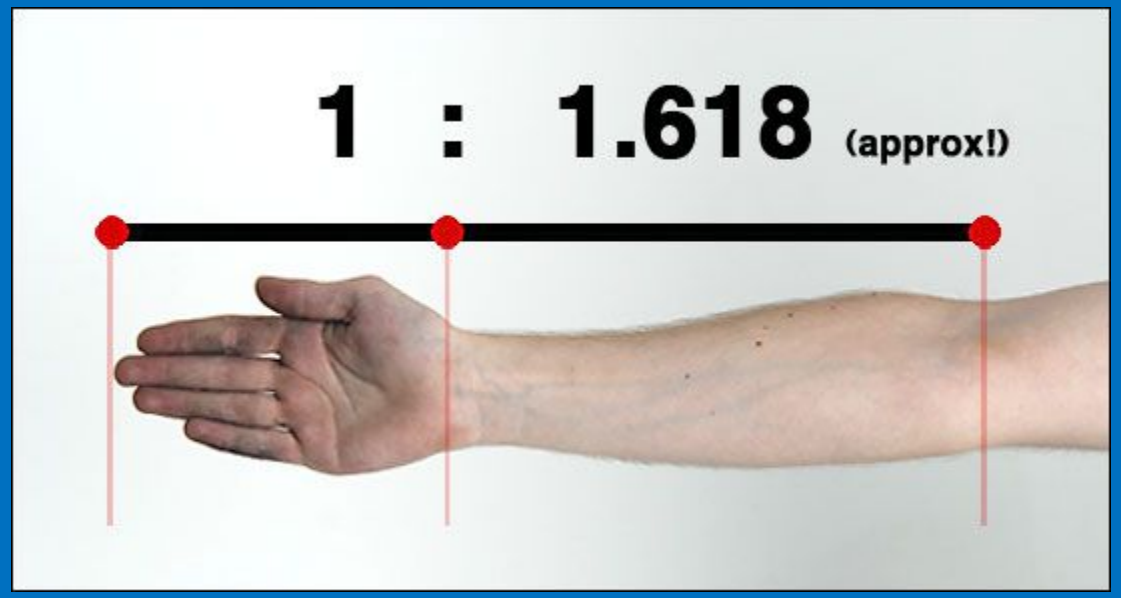
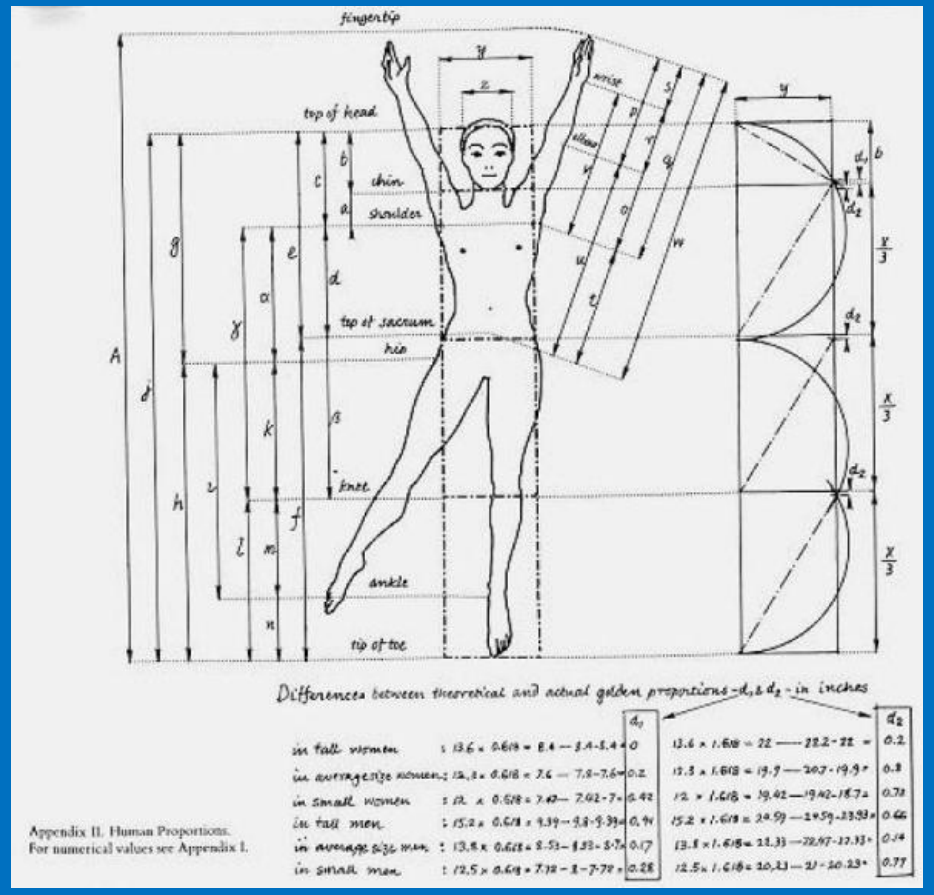
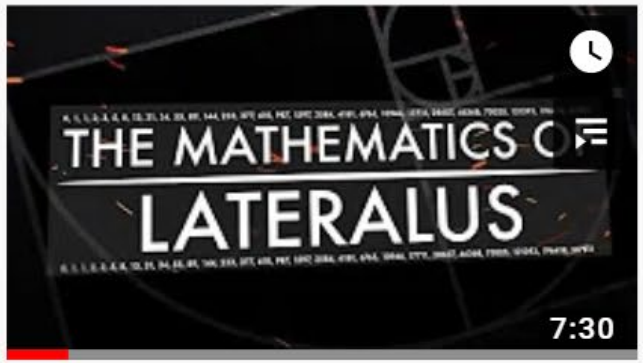








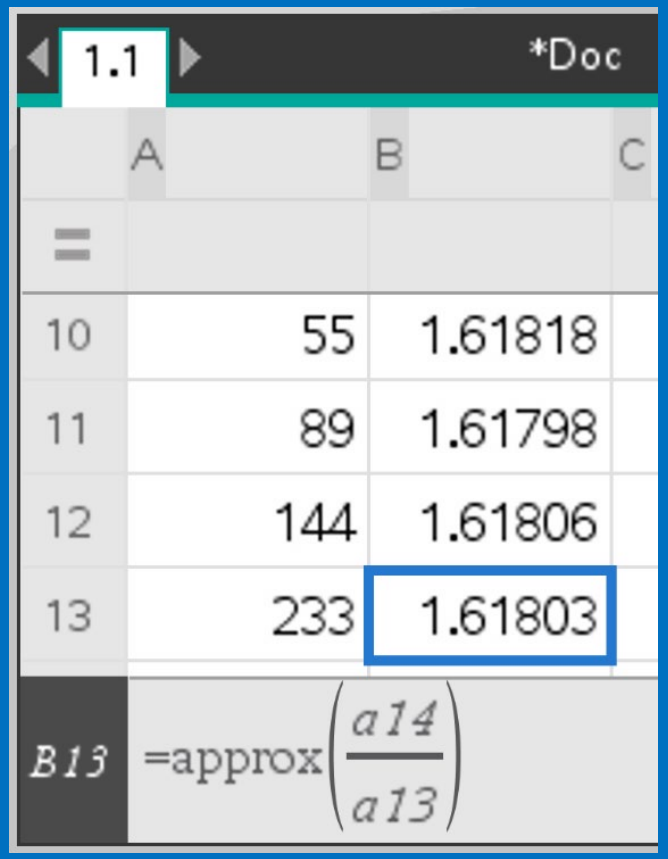
FUN WITH FIBONACCI



I call this a Haikunacci …

1: Black
1: Then
2: White are
3: All I see
5: In my infancy
8: Red and yellow then came to be
5: Reaching out to me
3: Let's me see

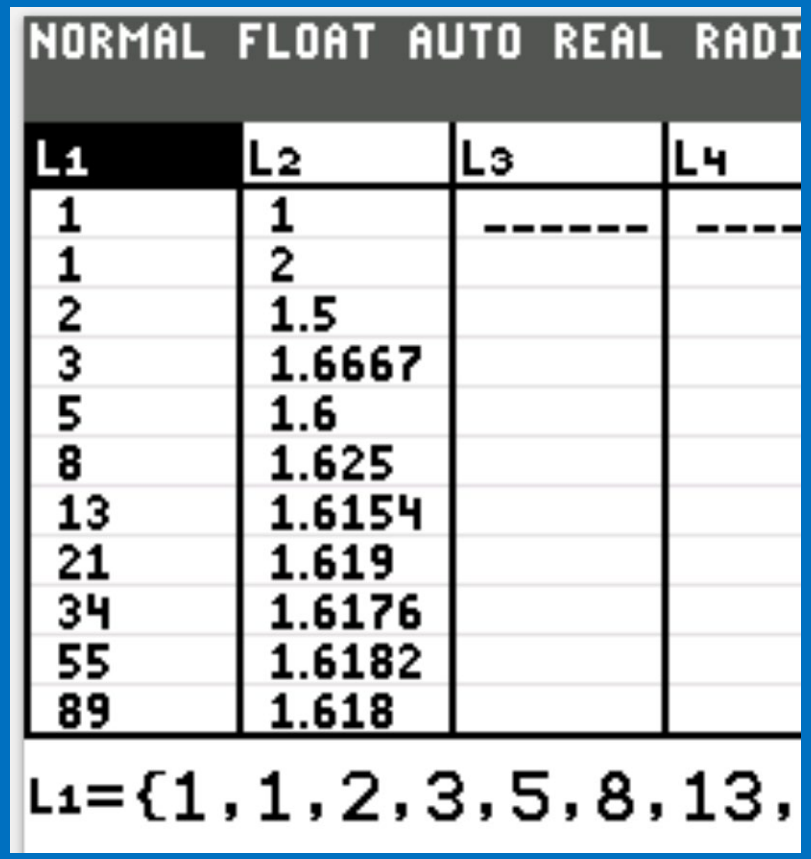
FIBONACCI ON THE CALCULATOR



1.1 *Doc

	A	B	C
=			
10	55	1.61818	
11	89	1.61798	
12	144	1.61806	
13	233	1.61803	

B13 =approx$\left(\frac{a14}{a13}\right)$



NORMAL FLOAT AUTO REAL RADI

L1	L2	L3	L4
1	1	------	----
1	2		
2	1.5		
3	1.6667		
5	1.6		
8	1.625		
13	1.6154		
21	1.619		
34	1.6176		
55	1.6182		
89	1.618		

L1={1,1,2,3,5,8,13,

EUCLID ON DRUMS

THE EUCLIDEAN ALGORITHM

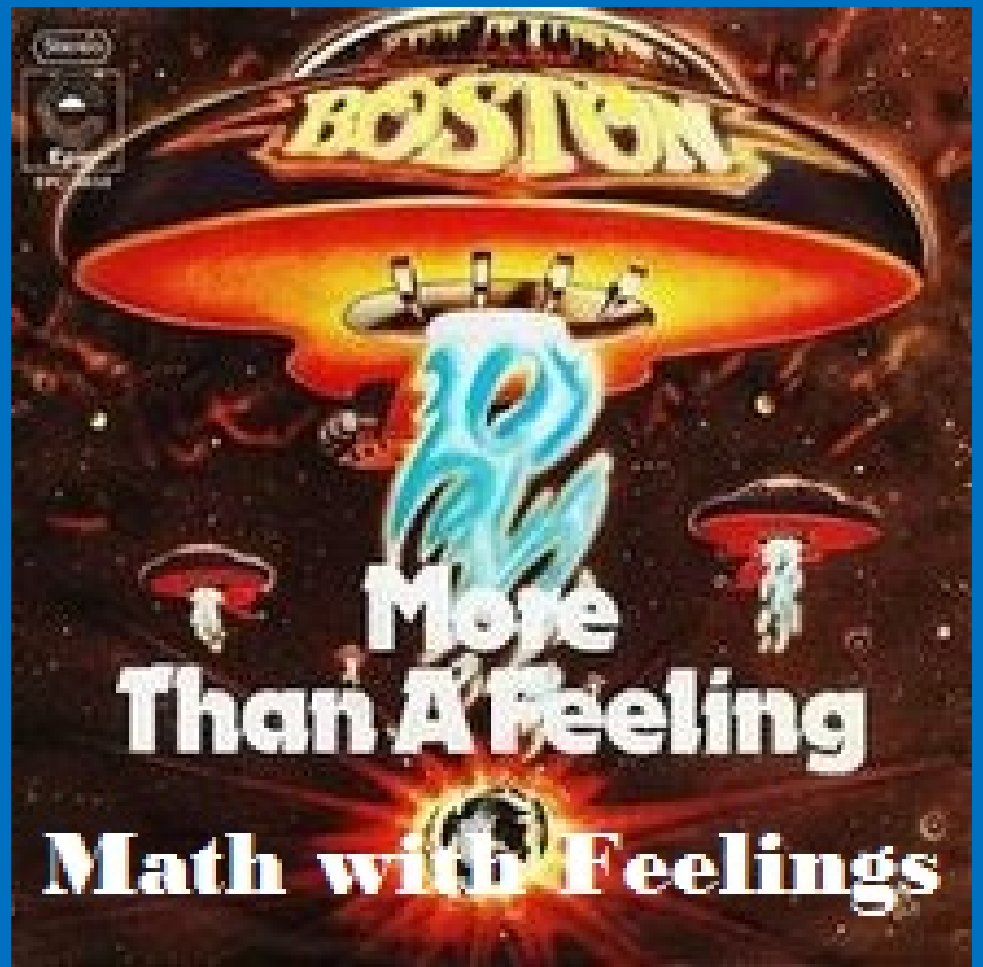
- The Euclidean Algorithm is based on the Division Algorithm: For any integers a and b, there exist integers q and r, such $a = bq + r$, $0 \leq r < b$.
- Note: this is an <u>existence</u> theorem, not a law or compulsory mandate.
  - Consider 4 and 11. The division algorithm tells me that there exist 2 and 3, such that $11 = 4*2 + 3$, where $0 \leq 3 < 4$.
  - But, $11 = 4*1 + 7$ (Note: this is a true statement. It just means I didn't use the division algorithm to make the statement.)
  - Using long division: 11 / 4 = 2R3, or 11/4 = 2¾
  - But 11/4 also equals 1R7, or 1 and 7/4
    - It's not wrong, but it feels very uncomfortable, because it is not what we usually write!
    - Why is it uncomfortable? Because we feel in our hearts like the division algorithm is a law. It is not a mandate – it is an existence theorem. Other things exist, too!

.9 REPEATING …

- The trichotomy property: for any real numbers a and b, exactly one of the following is true:
  - $a > b$
  - $a < b$
  - $a = b$
- So, what goes between .999… and 1? (>, <, =)
- Just like 11/4 can be represented as 1 and 7/4, 1/1 can be represented as 0.99999999…

SOME SUPPORTING LINKS

- NCTM Illuminations: Guitar Fret Activity
- Ted Talks
  - Roger Antonsen (9 minute mark): Pythagorean Ratios in Music, and 4/3 in Euclidean Rhythms
  - Authur Benjamin: The Magic of Fibonacci Numbers
- YouTube
  - Vi Hart .9 Repeating
  - Euclidean Rhythms Explained
  - How Tool uses Fibonacci in the song Lateralus
  - Fibonacci, Art, and Life

THANK YOU

- David Sword,
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