Discovering Pi

Summary: Students will estimate the measure of the radius, diameter, circumference and area of common circular objects. They will use these values to calculate ratios that demonstrate the relationships between these measures. They will relate their constant ratios to the value of pi.

Objective: Students will be able to make measurements on circular objects and calculate ratios to develop an understanding of the relationships among radius, diameter, area, and circumference. They will be able to generate an approximate value of pi.

Management: Students will work with a partner while making measurements. Each student will submit an individual lab report. Whole class discussions will be used for introductory and concluding activities.

Materials: Circular objects (cups, bottle caps, cylinders, rolls of tape, etc.), graph paper, rulers, scissors, non-stretchy string or tape measures, calculators, lab report sheets.

Activity:

Prerequisite skills:

Definitions of radius and diameter of a circle, measuring in centimeters, calculating ratios, graphing points and lines on a coordinate plane, estimating the line of best fit, calculating slope

Procedure:

The teacher introduces the activity by holding up an object and stating to the class that the activity will require students to make measurements of circular objects. The teacher solicits from the class the possible things that could be measured on the object. (Desired responses: diameter, radius, area, circumference or distance around.)

The teacher asks the class to brainstorm ways to measure these items with reasonable accuracy with the available materials. The teacher then demonstrates the desired methods after students complete their suggestions. Students should use Metric units (if metric graph paper available), otherwise use Inches (gives opportunity to review common fraction-decimal equivalents).

Circumference: Wrap a tape measure around the object, or wrap a length of string, then measure the string with a ruler.

Area: Trace the object on graph paper and estimate the number of square units used. Use this same tracing for diameter and radius below.

Diameter: Cut out the tracing and fold it exactly in half. Measure the length of the fold.

Radius: Fold the tracing again, so it is quarters. Measure the length of the double fold.

Pass out the lab sheets, graph paper, rulers, scissors, and tape measures or string. Have students pair off (either select their own partner or preassign partners). Students should select one circular object to measure first, then trade it for a different object. Note–some objects have two different sized circles for measurements (like a tapered plastic cup).

The teacher will circulate around the room as the students work. When the students have completed their lab sheets, the teacher solicits the attention of the class as a whole. The teacher asks students which ratios remained the same. (Desired responses: Diameter to radius, circumference to diameter, circumference to radius, area to radius^2, and area to diameter^2.) The teacher posts these results, with the actual ratios that students have calculated. (Desired responses will be approximately: D to R = 2, C to D = 3.14, C to R = 6.28, A to R^2 = 3.14, A to D^2 = .79).

Students then volunteer the equations they generated for each type of graph. The teacher displays the actual area and circumference formulas ($A = \pi r^2$ and $C = 2\pi r$) and discusses the relationships between the actual and the student formulas. The constant ratios that the students have discovered are approximations for the famous constant pi. The teacher should present other approximations (3.1415 and 22/7) and stress that pi is a non-terminating, non-repeating decimal.

Extension: (for advanced students) Make a graph of one of the ratios that did not remain constant, (A to R and A to D). Explain the shape of these graphs (parabolas).

Evaluation:
During activity:

The teacher will circulate around the classroom, checking students' work for understanding. Some common errors to look for are incorrect measurements with centimeters, calculating ratios in the reverse order, switching the x and y values on the graph, inconsistent scales on the axes, and incorrect calculations for the slope. The teacher can spot check some of the measurements to see if students are getting close to the desired values.

During discussion of conclusions:

The teacher can analyze the ratios and equations provided and clarify any student misunderstandings. There is likely to be a wide variation between the generated formulas and the desired values, which provides an opportunity to discuss measurement accuracy and inaccuracy.

Sample Data

Diameter	Radius	Circumference	Area
6	3	18.5	27.75
3.5	1.75	10.6	9
3.2	1.6	10.5	8
7.1	3.55	23.6	36.5
4.5	2.25	14.5	15
7	3.5	22	39
7.5	3.75	24	42
3.75	1.875	12	20
10.375	5.1875	32.8	79.5
6.875	3.4375	21.7	35

Resources: Metric Graph paper
https://mathbits.com/MathBits/StudentResources/GraphPaper/CentimeterFullPage.pdf