Using Photos, Videos and TI Technology to Connect Mathematics With the World Around Us

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e = 2.718...
email address
Ronnie

Takeaways:

Use photos and videos to introduce a topic or to provide students with an example of where math lives.

Develop math questions based on your photos.

Encourage your students to take their own photos and develop their own math questions related to them.

Takeaways:
Practice these habits of mind.

What do you notice?
What are you wondering about?
What questions do you have?

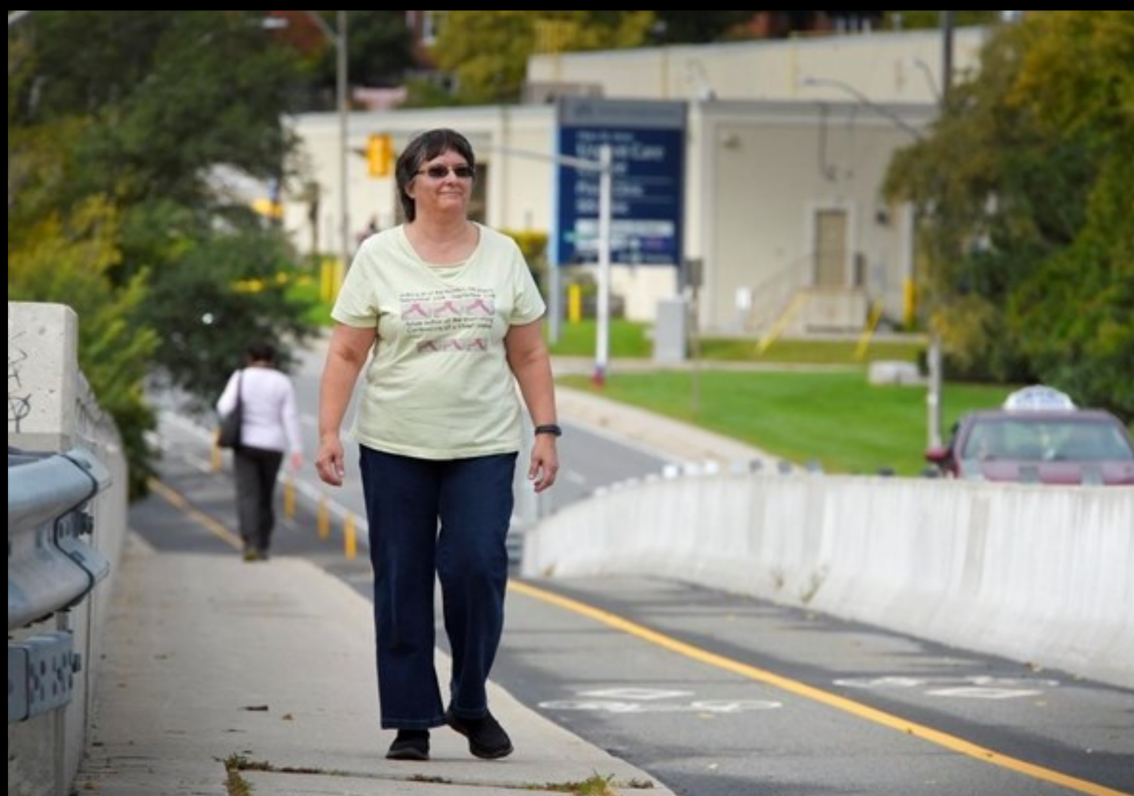
Takeaways:
A response to the question: "When am I ever going to use this?"

Part of the response to this question involves providing opportunities for students to learn where math lives and how it is used. This is not the entire response, but if we want to improve student achievement in mathematics, it seems to me that we are not going to get very far if students do not care about mathematics and if they feel it is a pointless exercise.

Everywhere I travel, I put on my mathematical pair of glasses and go for a walk. I enjoy coddiwompling – walking in a purposeful manner towards a vague destination.

Where the streets all have names — and Anita's walked every one
She walked down one street, then another, and in time the idea took shape, writes Jeff Mahoney
The Hamilton Spectator, October 29, 2018

https://www.thespec.com/opinion-story/8992706-where-the-streets-all-have-names-and-anita-s-walked-every-one/



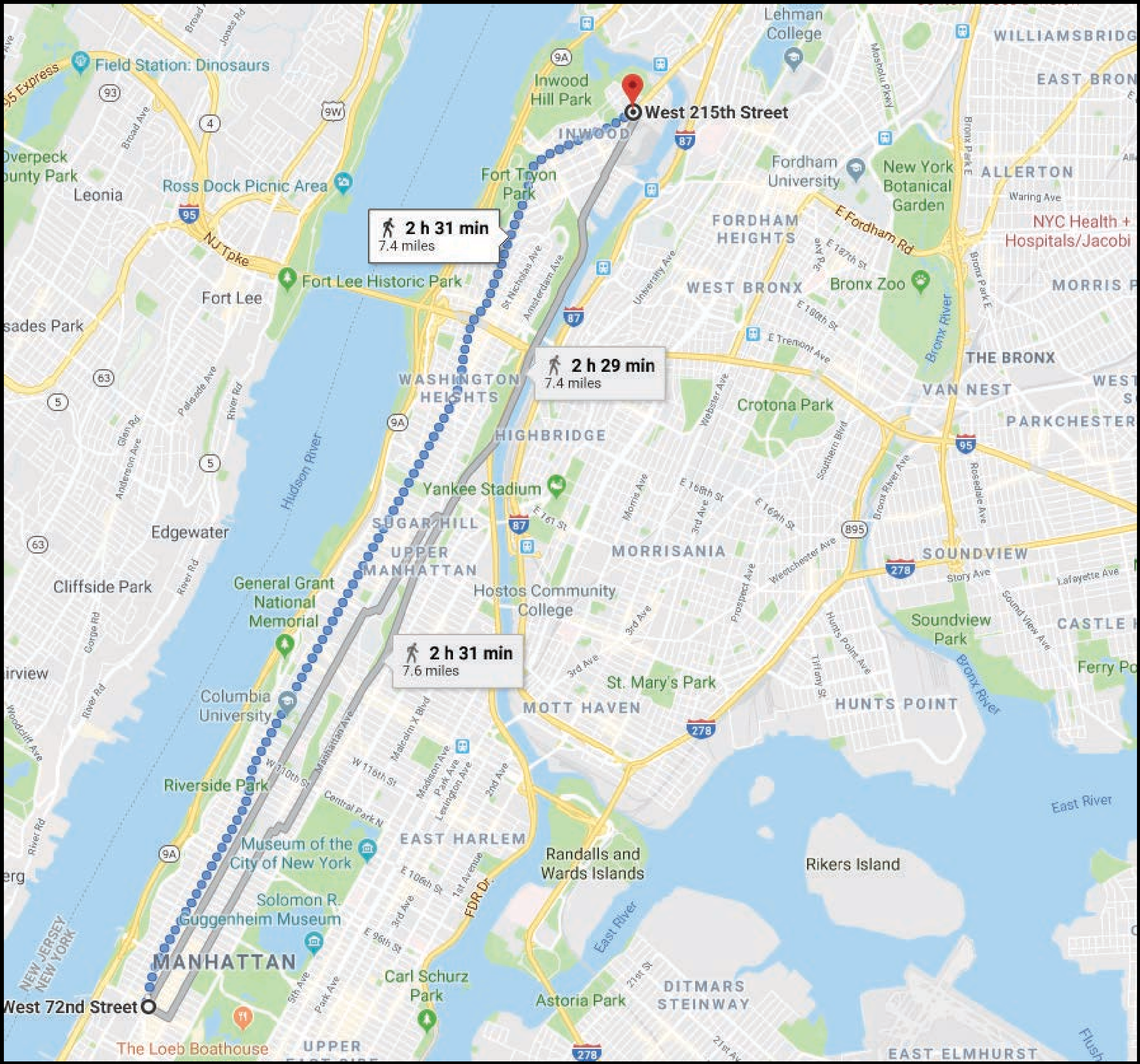
The World Before Your Feet

Documentary about Matt Green's mission to walk every block of all five of New York City's boroughs - for no real reason.

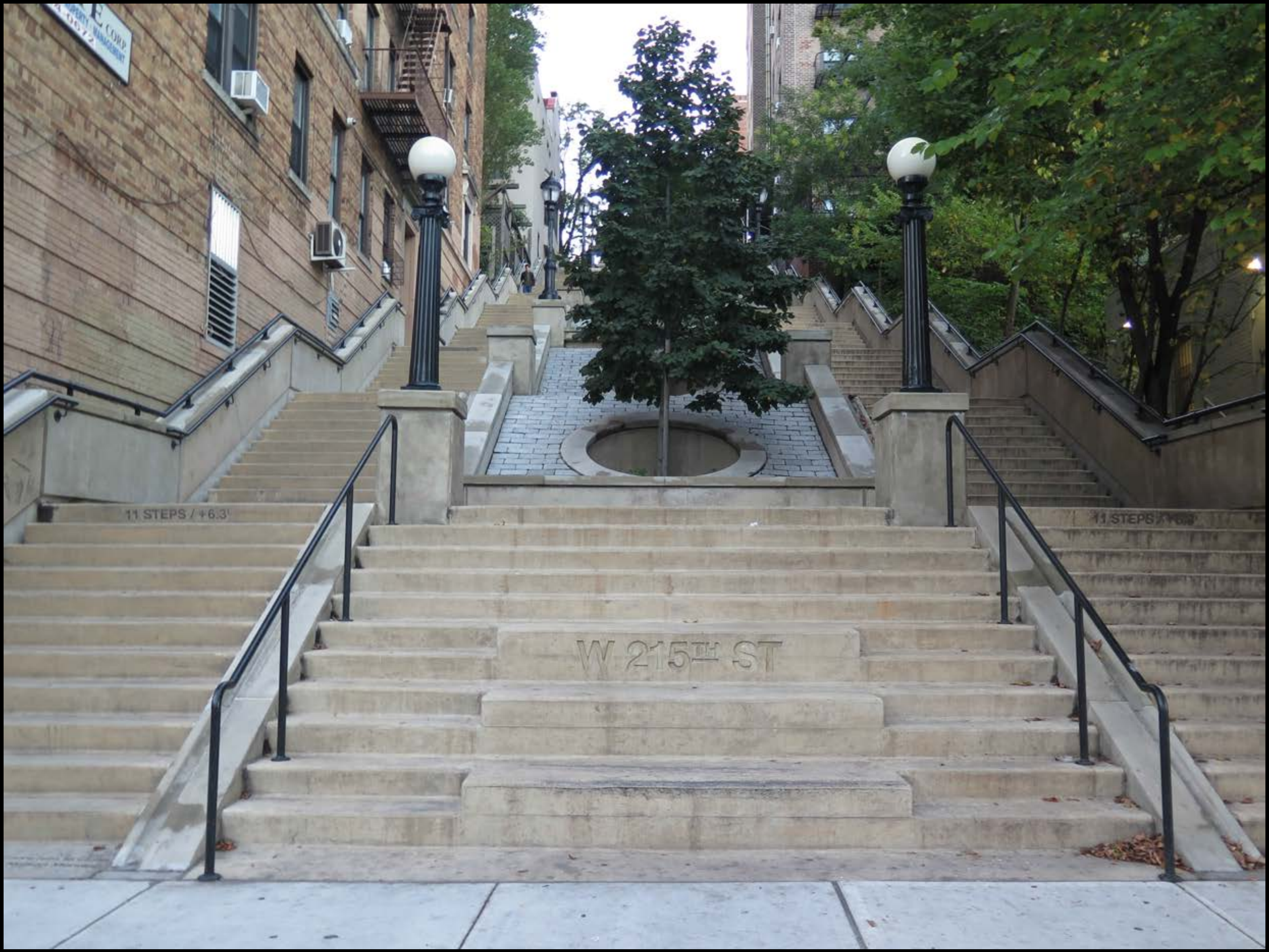
Walking is the best way to see a place, especially if you slow down your pace, stop, linger, be curious, and make it a habit of mind to ask these questions as you wander around.

What do you notice?
What are you wondering about?
What questions do you have?

One of my favourite walks is one I recently did along Broadway in Manhattan from West 72nd Street to West 215th Street.



The West 215th Step Street



Originally built in 1915, West 215th Step Street has since become one of the most iconic landmarks in the Inwood section of Manhattan.

The renovation of the 250-ft pedestrian walkway consisted of demolition and reconstruction of existing stairs, along with reconstruction of sidewalks and concrete benches.

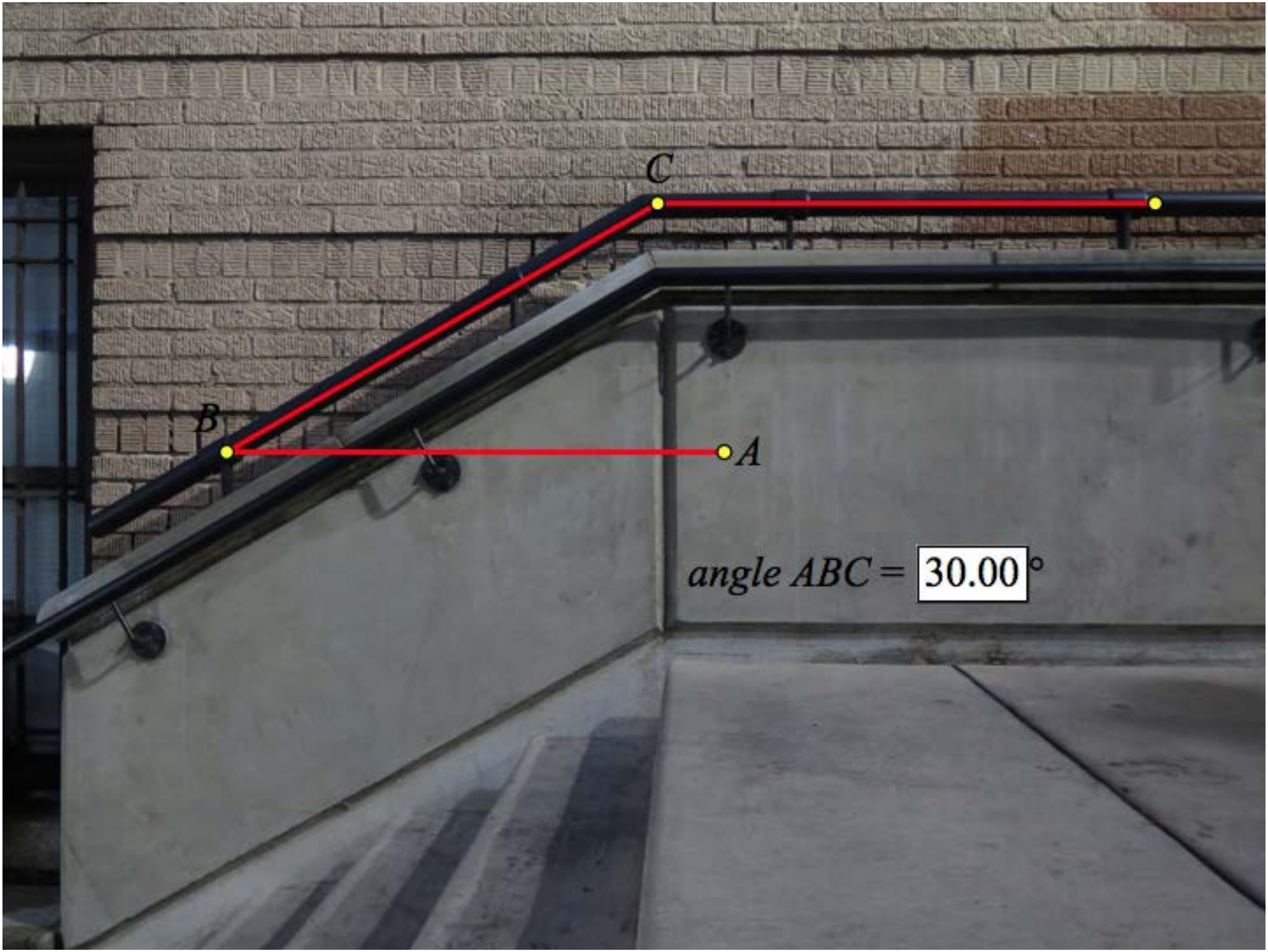
“The West 215th Step Street has connected Inwood residents to the subway and local businesses for a full century, and we are happy to continue that legacy with the reconstruction that was completed in February,” said Feniosky Peña-Mora, commissioner of the NYC Dept. of Design and Construction.

https://www.enr.com/articles/40408-landscapeurban-best-project-west-215th-step-street


11 STEPS / +6.3'
PARK TERRACE EAST
After climbing a great hill, one only finds that there are many more hills to climb.
— Nelson Mandela

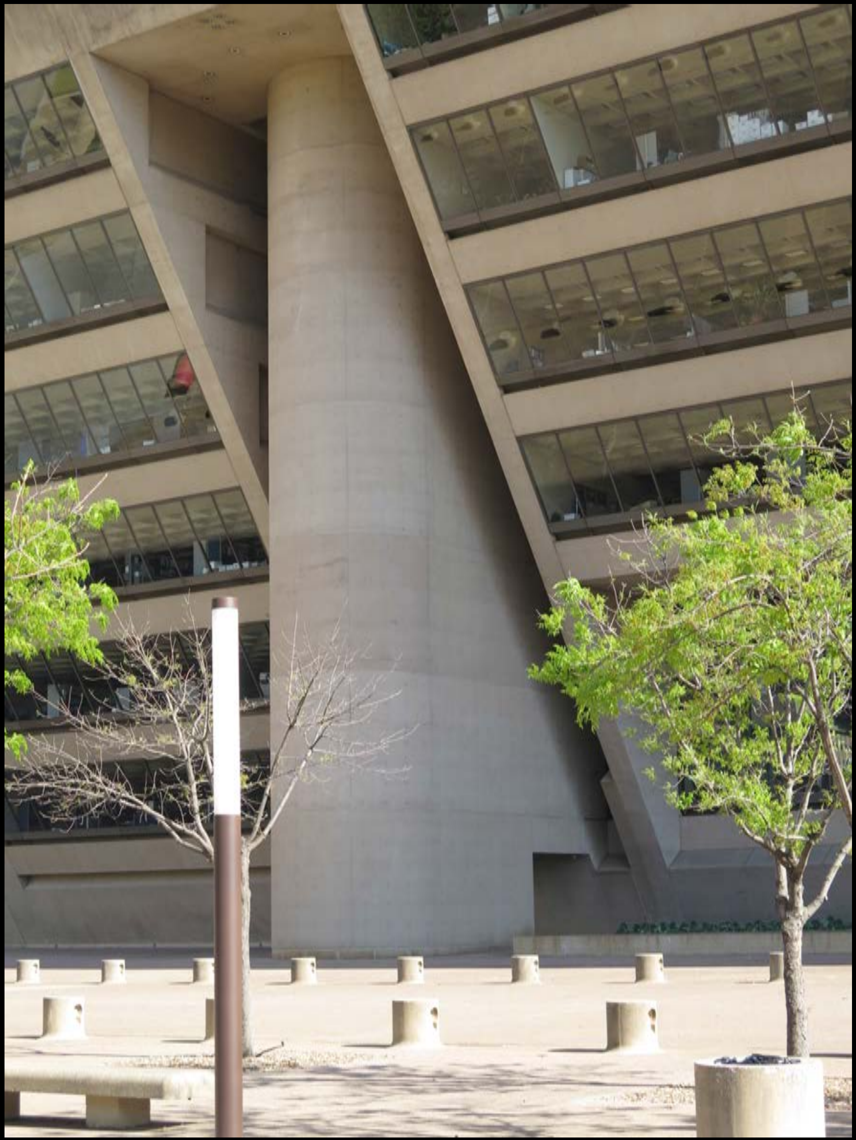

22 STEPS / +12.6'
33 STEPS / +18.9'
44 STEPS / +25.2'
55 STEPS / +31.5'
66 STEPS / +37.8'
77 STEPS / +44.1'
88 STEPS / +50.4'
99 STEPS / +56.7'
110 STEPS / +63.0'

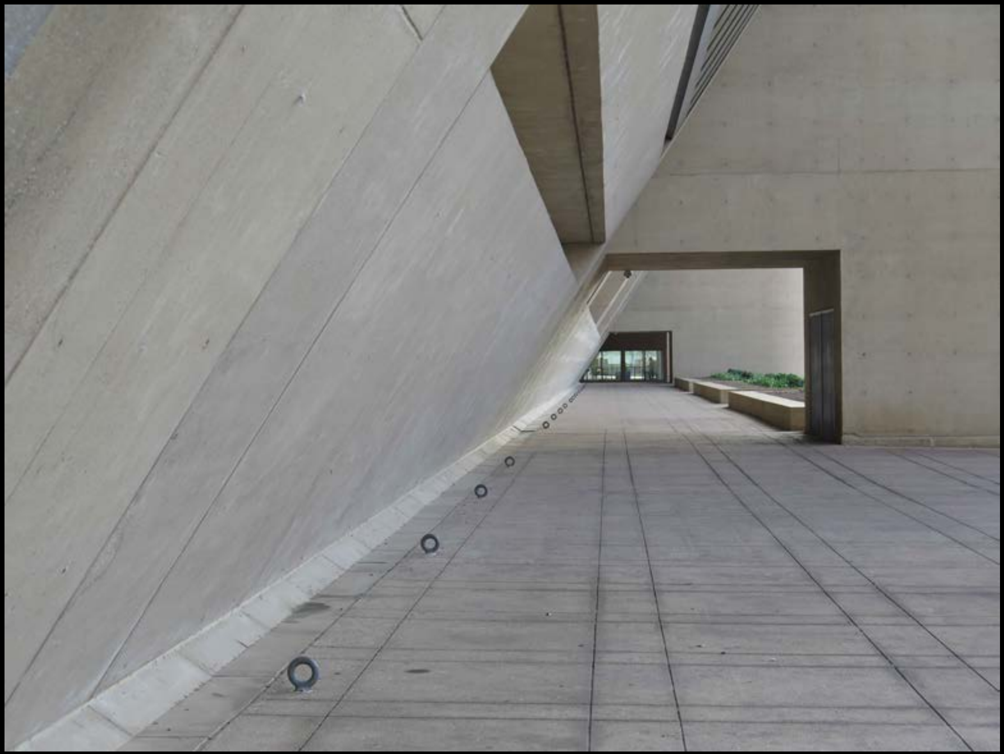

IVY Classic
16FEET


C
B
A
angle ABC = 30.00°

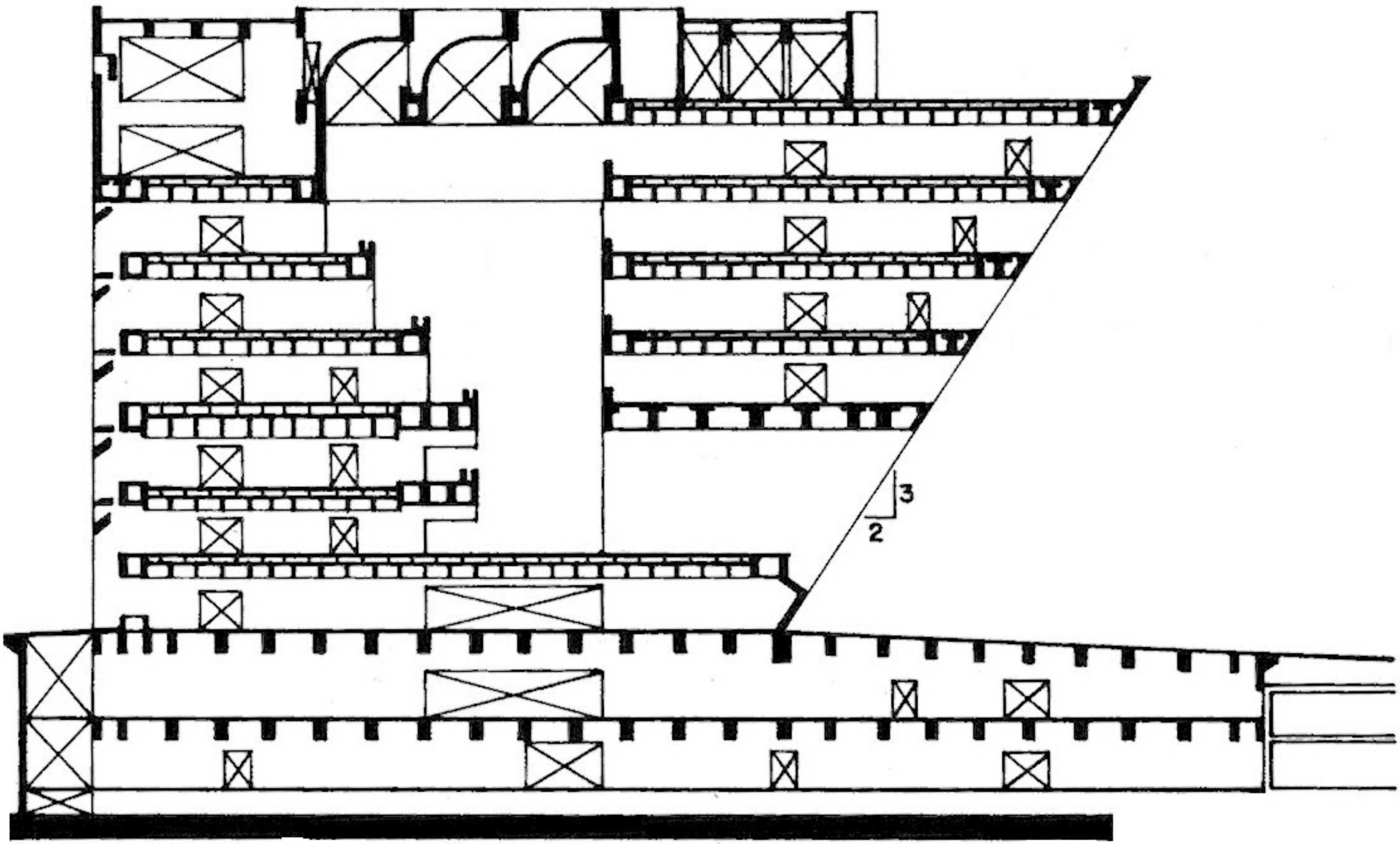
Dallas City Hall

Dallas City Hall



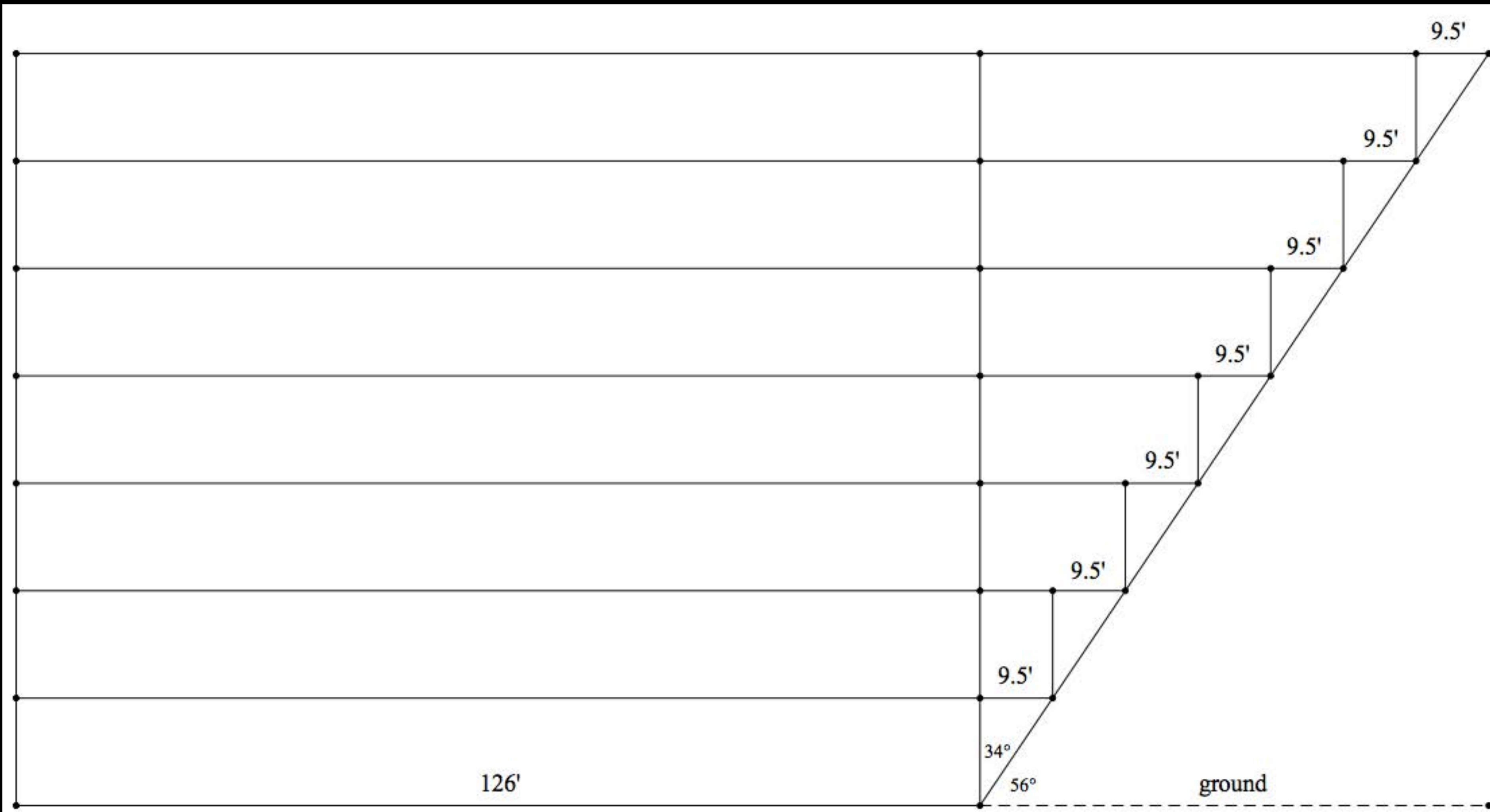


Dallas City Hall



Dallas City Hall is actually a skyscraper placed on its side, anchored at the southern end of the CBD. The most dynamic aspect of this 770,000 sq. ft., cast-in-place concrete structure is its trapezoidal shape, dramatically defined by the cantilevered front face of the building. This overscaled trapezoid, situated at the rear half of a rectangular, ten acre site stretches 560 ft. in length and rises to a height of 122 ft. from ground level. The sloping front of the building faces north to the vertical skyscrapers of the CBD. As the building's face angles 34 degrees out and away from perpendicular, it increases the depth of the roof area to 194 ft., versus the 126 ft. depth of the base area at ground level. This design increases the depth of each of the above grade floors by 9 1/2 ft. in arithmetic progression, enabling an agency and its support staff to be located on a common floor (or a part of one), which is in ratio to their collective size, and which accommodates their total space requirements. The design, thereby structurally imitates and facilitates the operations of the city's government, a lesson in form follows function instilled by Gropius. There are eight floors above grade, the first seven of which provide public areas and office space; the eighth houses the building's mechanical equipment.

Five Buildings in the Dallas Central Building District by I. M. Pei and Partner Henry N. Cobb: A Stamp on the City's Direction
Thesis presented to the Graduate Council of the North Texas State University in Partial Fulfillment of the Requirements For the Degree of Master of Arts by J. Barney Malesky, M. A., Denton, Texas, December, 1986
https://digital.library.unt.edu/ark:/67531/metadc500988/m2/1/high_res_d/1002775355-Malesky.pdf



9.5'
9.5'
9.5'
9.5'
9.5'
9.5'
9.5'
34°
56°
126'
ground

Let x represent the floor number. Let y represent the width of the xth floor. Graph y versus x.

floor number x	**width of the** x**th floor (ft)** y
1 (ground floor)	126
2	
3	
4	
5	
6	
7	
8	

Let x represent the floor number. Let y represent the width of the xth floor. Graph y versus x.

floor number x	**width of the xth floor (ft)** y
1 (ground floor)	126
2	$126 + 9.5$
3	$126 + 2 \times 9.5$
4	$126 + 3 \times 9.5$
5	$126 + 4 \times 9.5$
6	$126 + 5 \times 9.5$
7	$126 + 6 \times 9.5$
8	$126 + 7 \times 9.5$

Let x represent the floor number. Let y represent the width of the xth floor. Graph y versus x.

floor number x	**width of the xth floor (ft)** y
1 (ground floor)	126
2	135.5
3	145
4	154.5
5	164
6	173.5
7	183
8	192.5

Find a mathematical model for the width of the xth floor (y) as a function of the floor number (x).

Find a mathematical model for the width of the xth floor (y) as a function of the floor number (x).

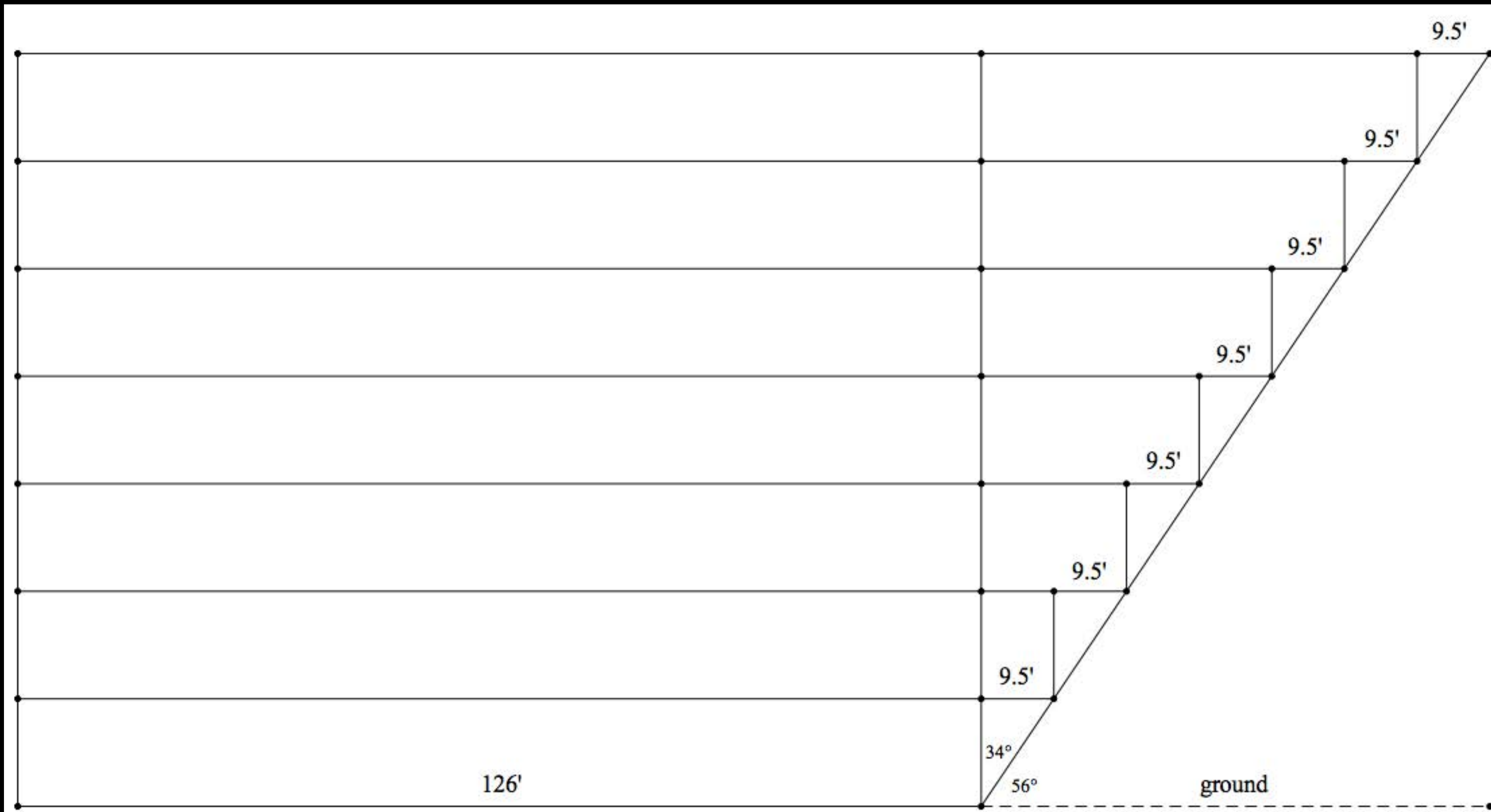
$$y = 126 + 9.5(x - 1)$$

Write the equation in standard form.
Why would we ask students to do this?

$$
\begin{aligned}
y &= 126 + 9.5(x - 1) \\
&= 116.5 + 9.5x
\end{aligned}
$$

How would the graph of y versus x and the mathematical model change if the angle between the face of the building and the perpendicular is

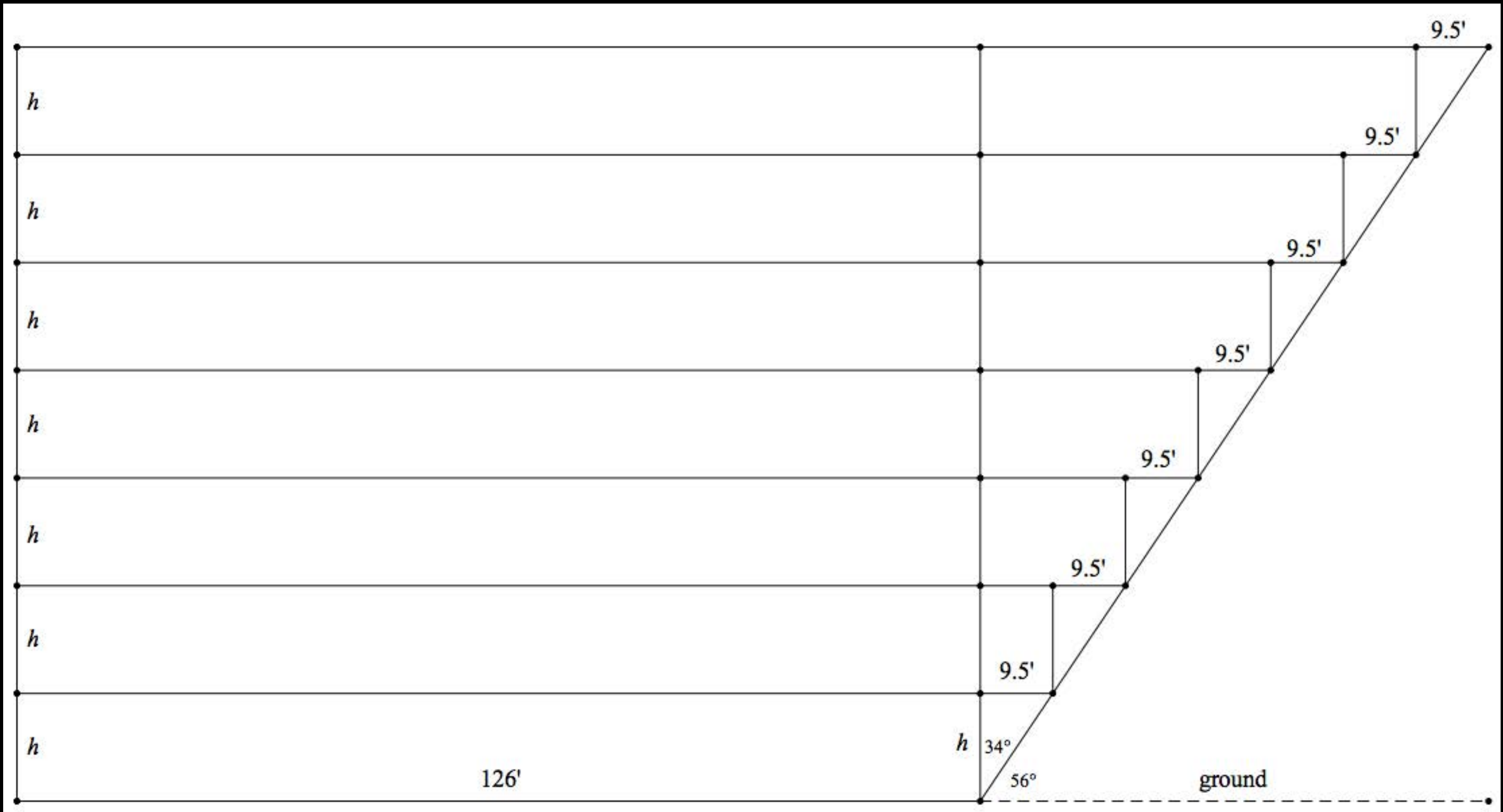
(a) 33^0 instead of 34° ?
(b) 35^0 instead of 34° ?

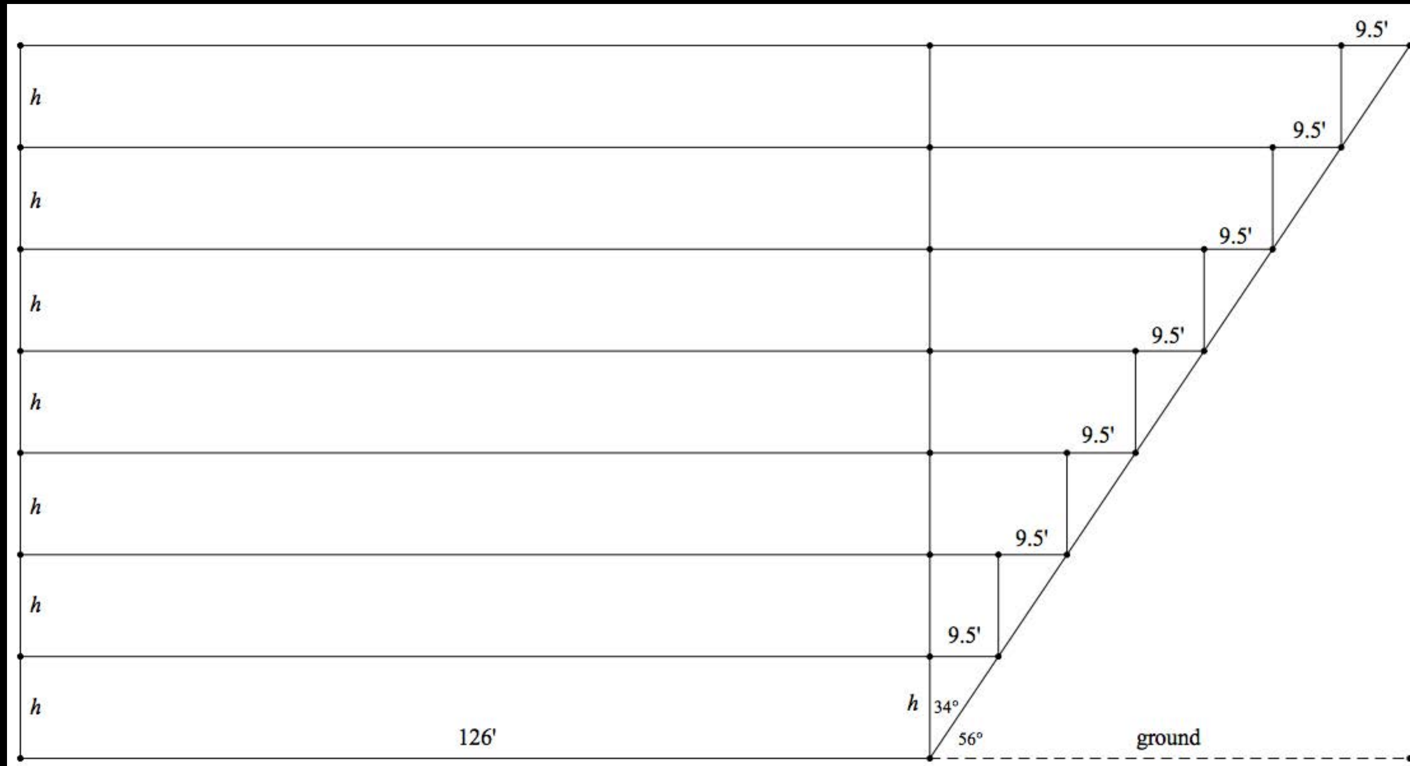


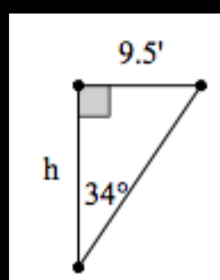
9.5'
9.5'
9.5'
9.5'
9.5'
9.5'
9.5'
34°
56°
126'
ground

Let θ be angle between the face of the building and the perpendicular. Let y be the width of the xth floor of the building. Find a mathematical model for y as a function of x and θ.

Assume that the width of the first floor and the height of each floor remains the same.


9.5'
9.5'
9.5'
9.5'
9.5'
9.5'
9.5'
h
h
h
h
h
h
h
h
h
34°
56°
126'
ground

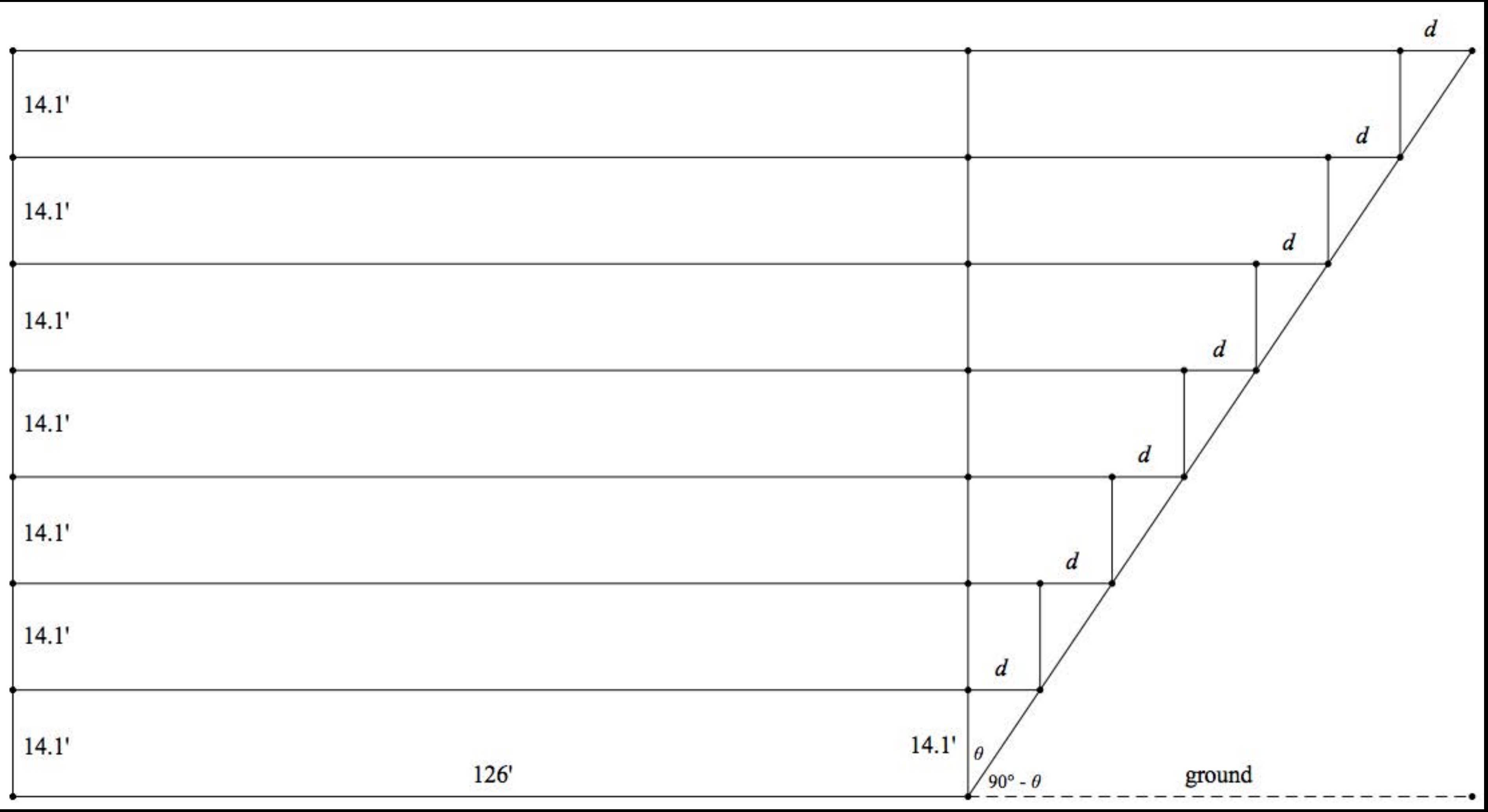


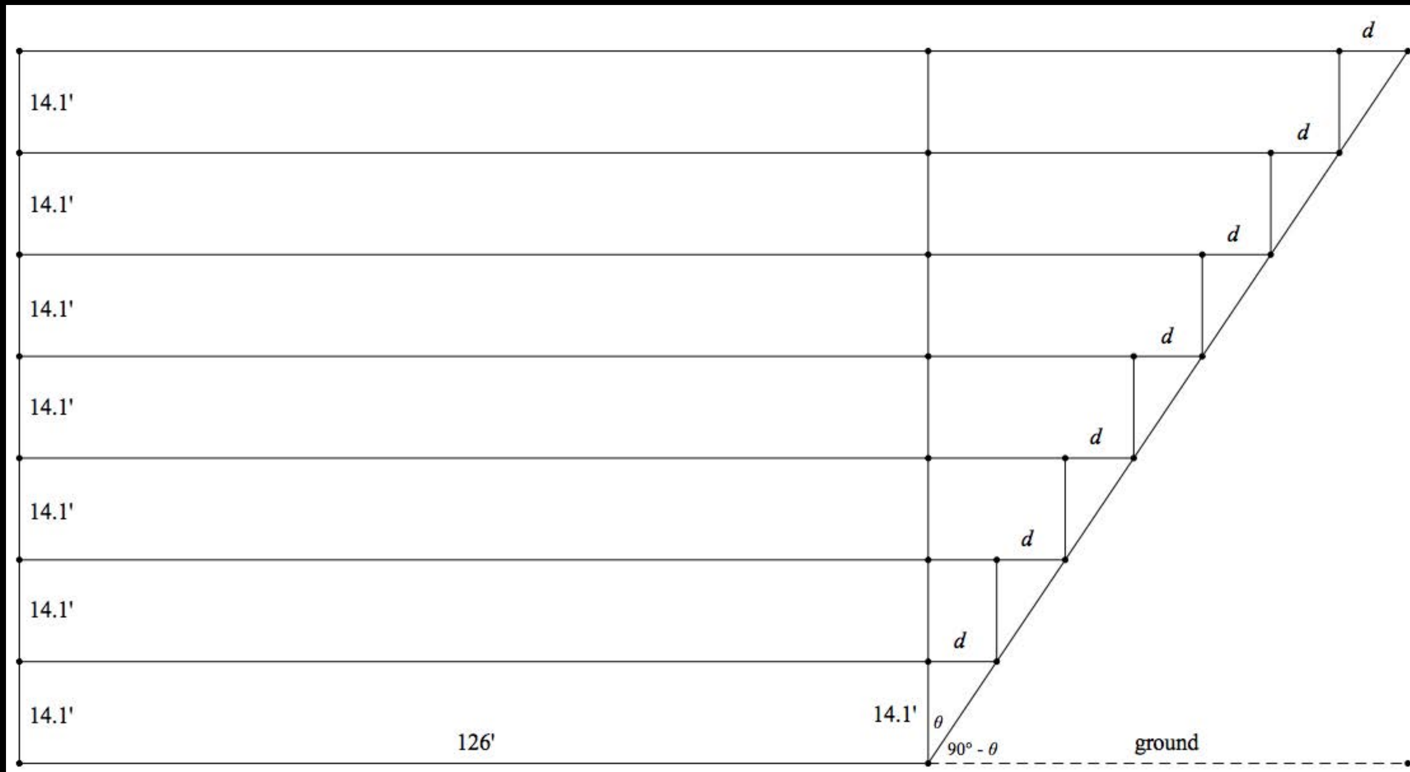


$$\frac{9.5}{h} = \tan\left(34^\circ\right)$$

$$h = \frac{9.5}{\tan\left(34^\circ\right)}$$

$$h \approx 14.1$$


d
14.1'
d
14.1'
d
14.1'
d
14.1'
d
14.1'
d
14.1'
d
14.1'
14.1'
θ
126'
90° - θ
ground



$$\frac{d}{14.1} = \tan\theta$$

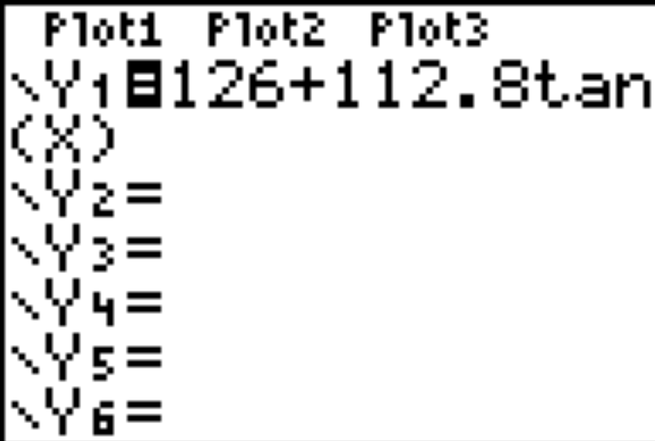
$$d = 14.1\tan\theta$$

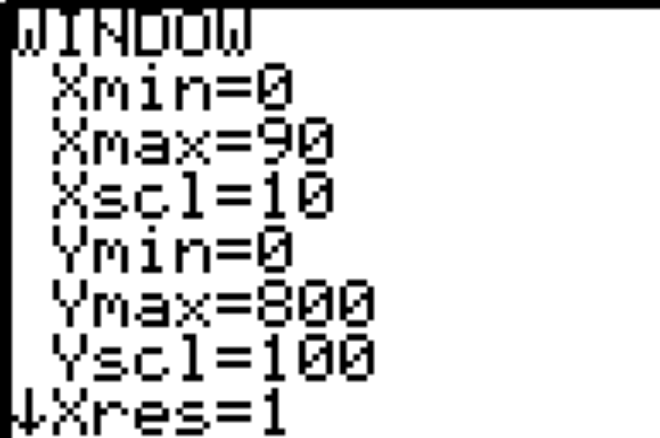
$$y = 126 + 14.1\tan\theta\left(x - 1\right)$$

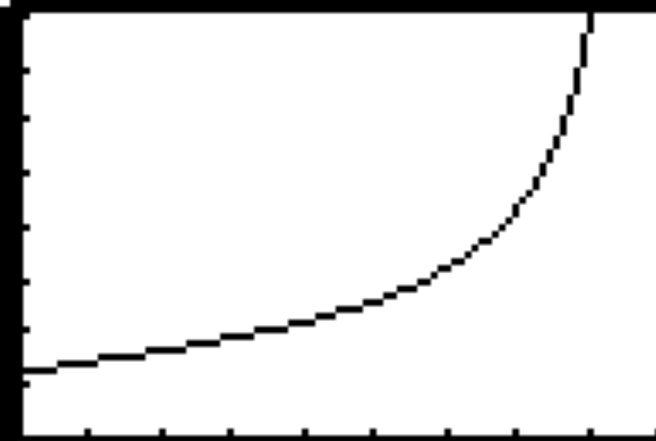
Graph the width of the 8th floor versus θ. Describe how the width changes as θ increases from 0° to 90°.

$$y = 126 + 14.1 \tan\theta (8 - 1)$$

$$= 126 + 98.7 \tan\theta$$



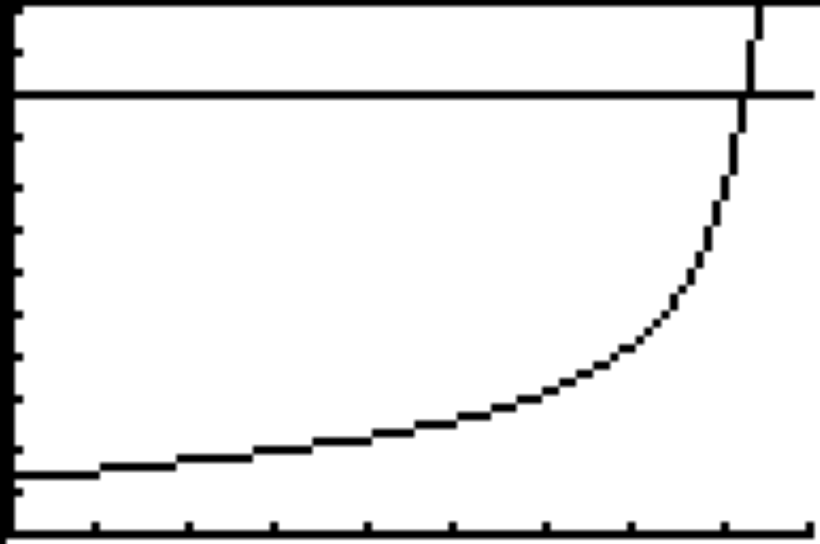


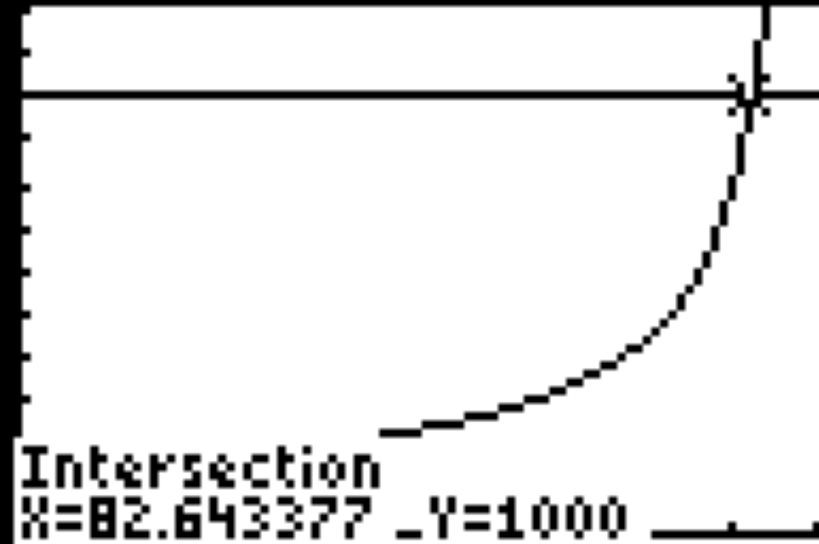


Graph of the the width of the 8th floor versus θ.

Is it possible to find a value of θ for which the width of the 8th floor is equal to 1000 feet (about 5 times the width of actual 8th floor)? Would it be possible to build this version of City Hall? Why or why not?

$$126 + 98.7 \tan\theta = 1000$$





EQUATION SOLVER
eqn:0=Y_1(X)-1000

Y_1(X)-1000=0
■ X=82.643377334…
bound={$^-$1E99,1…
■ left-rt=0

Let's Go To San Francisco
The Flower Pot Men

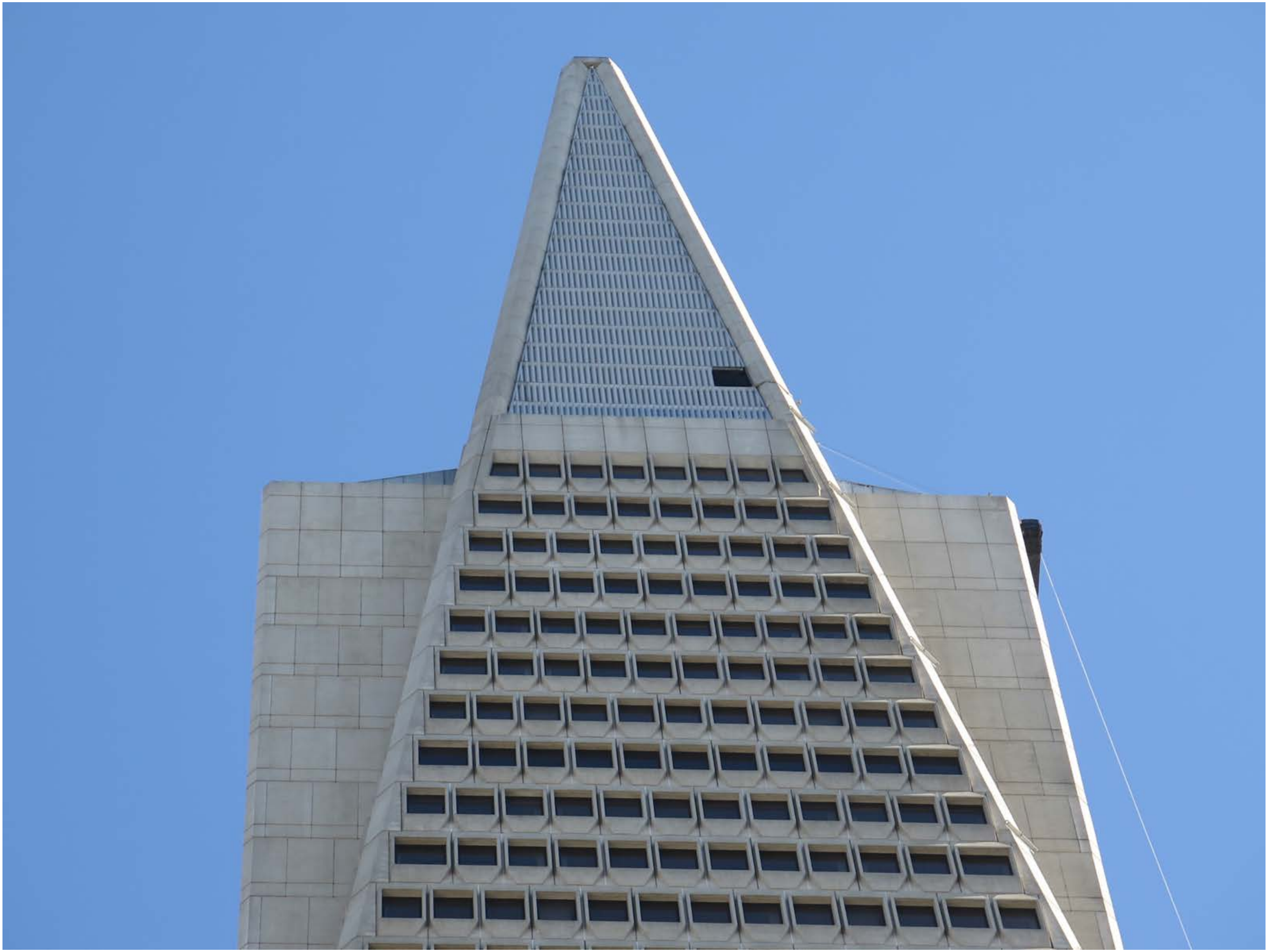
Let's Go To San Francisco
The Transamerica Building

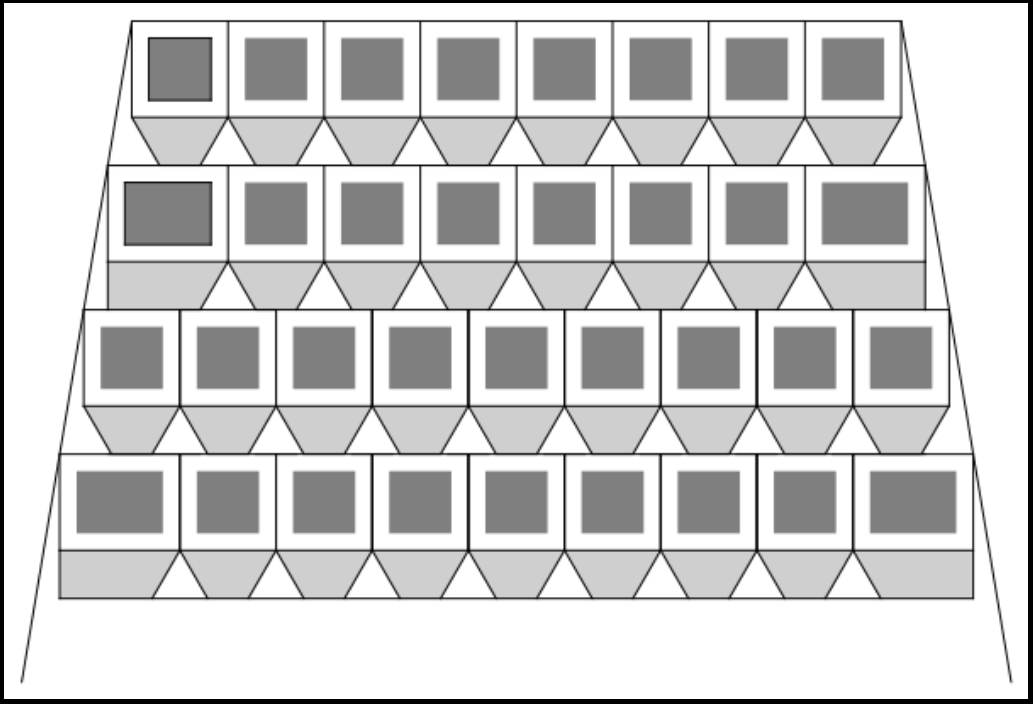
The Transamerica Pyramid building is San Francisco’s most distinctive and well-known building. The obelisk-shaped building was controversial when it opened in 1972, but today it is hard to imagine the skyline of San Francisco without it. The building was designed to minimize its impact on the neighborhood by allowing light and fresh air to circulate much as it would in an area surrounded by tall trees. In fact, this is what John R. Beckett, the president of the Transamerica Corporation, imagined when the building was in its planning stages in 1968. Unfortunately, there is currently no public access to the building. A public observation area on the 27th floor was closed as a result of the attacks of September 11, 2001. A conference room on the 48th floor has outstanding views of the city, but entry to this area is limited to tenants and their guests.

William Leonard Pereira was the architect of the Transamerica building. He designed hundreds of buildings including the Theme Building at LAX.

He also designed the Geisel Library at the University of California, San Diego.

Geisel Library is named in honor of Audrey and Theodor Seuss Geisel. Theodor is better known as children's author Dr. Seuss.





Skyscrapers in the shape of a rectangular prism generally have the same number of windows per floor. This is not the case with the Transamerica Pyramid; its windows form a pattern that unfolds from the top to the bottom. The number of windows per floor, starting on the 48th floor and going down, is partially shown in table 1.

Source:
The Mathematical Lens by Ron Lancaster and Brigitte Bentele
The Mathematics Teacher, November/December 2018
The National Council of Teachers of Mathematics (nctm.org)

floor number	number of windows
48	8
47	8
46	9
45	9
44	10
43	10
42	12
41	12

(a) If this pattern continues, how many windows are on the 6th floor? How many windows are on the nth floor, where n is an integer between 1 and 48?

(b) Calculate the total number of windows on the four sides of the building from the first floor to the 48th floor. List all the assumptions you make.

(c) According to the website given below and confirmed by numerous other sites, there are 3,678 windows in the Transamerica Pyramid. Is your answer to part (b) equal to 3,678? If not, provide reasons why your answer differs from what is widely reported. Transamerica Pyramid's architectural details can be found at http://www.pyramidcenter.com/tourism/pyramid-facts/.

floor number	number of windows
48	8
47	8
46	9
45	9
44	10
43	10
42	12
41	12

(a) If this pattern continues, how many windows are on the 6th floor? How many windows are on the nth floor, where n is an integer between 1 and 48?

Answers:
There are 29 windows on the 6th floor. One way of seeing this is to simply extend the pattern in table 1. Let w_n be the number of windows on the nth floor, where n is an integer. Then,

$$w_n = \begin{cases} -0.5n + 32 & n \text{ is even, } 2 \le n \le 48 \\ -0.5n + 31.5 & n \text{ is odd, } 1 \le n \le 47 \end{cases}$$

floor number	number of windows
48	8
47	8
46	9
45	9
44	10
43	10
42	12
41	12

(b) Calculate the total number of windows on the four sides of the building from the first floor to the 48th floor. List all the assumptions you make.

Answers:
The total number of windows on all four sides is 3,744 (936 on each side). This total can be obtained by simply adding the number of windows, possibly using a spreadsheet. Alternatively, the total can be found by using a summation formula. These calculations assume that the pattern for the number of windows continues all the way down to the first floor and that the designs are the same on all four sides.

$$
\begin{aligned}
&2(8+9+10+\ldots+30+31) \\
&=2\{(1+2+\ldots+31)-(1+2+\ldots+7)\} \\
&=2\left(\frac{31\times 32}{2}-\frac{7\times 8}{2}\right) \\
&=936
\end{aligned}
$$

(c) According to the website given below and confirmed by numerous other sites, there are 3,678 windows in the Transamerica Pyramid. Is your answer to part (b) equal to 3,678? If not, provide reasons why your answer differs from what is widely reported. Transamerica Pyramid's architectural details can be found at http://www.pyramidcenter.com/tourism/pyramid-facts/.

Answers:

The Pyramid lobby features the work of many artists in a rotating exhibit. The building has 48 floors, with the 5th floor being the largest, measuring 145 feet per side and containing 21,025 square feet of space. The smallest floor is the 48th, measuring only 45 feet per side and containing 2,025 square feet of floor space. The edifice has a total of 530,000 square feet of floor space.
Source: http://www.u-shistory.com/pages/h2585.html

(a) Find the length of the side of the *n*th floor, where *n* is an integer between 5 and 48.

(b) Find the area of the *n*th floor, where n is an integer between 5 and 48.

(c) Calculate the total area of all the floors from the 5th to the 48th. Should your answer be less than, equal to, or more than the total stated above? Why?

(a) Find the length of the side of the nth floor, where n is an integer between 5 and 48.

Answers:

Let s be the length of the side of the nth floor. Because we are dealing with a pyramid with a uniform slant, the equation for s can be determined by finding the equation of the line that contains the points $(5, 145)$ and $(48, 45)$.

$$\begin{aligned} s &= -\frac{100}{43}n + \frac{6735}{43} \\ &\approx -2.3n + 156.6 \end{aligned}$$

(b) Find the area of the nth floor, where n is an integer between 5 and 48.

$$a = \left(-\frac{100}{43} n + \frac{6735}{43} \right)^2$$

(c) Calculate the total area of all the floors from the 5th to the 48th. Should your answer be less than, equal to, or more than the total stated above? Why?

Answers:
The total area of all the floors from the 5th to the 48th is equal to 435,472 square feet. A spreadsheet can be used to calculate this total. This answer should be less than the total cited in (530,000 square feet) because that total includes the bottom 4 floors. The total area can also be calculated using summation formulas, as shown below.

$$\sum_5^{48}\left(-\frac{100}{43}n+\frac{6735}{43}\right)^2$$

$$=\sum_5^{48}\left(\frac{10000}{1849}n^2-\frac{1,347,000}{1849}n+\frac{45,360,225}{1849}\right)$$

$$=\frac{1}{1,849}\left(\sum_5^{48}10,000n^2-\sum_5^{48}1,347,000n+\sum_5^{48}45,360,225\right)$$

$$=\frac{1}{1,849}\left(379,940,000-1,570,602,000+1,995,849,900\right)$$

$$\simeq 435,472$$

Triangular numbers in central Seoul
$1 + 2 + 3 + 4 + 5 + 6 = 21$

Triangular numbers in central Seoul

1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, ...

Triangular numbers

1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, ...

Formula for t_n, the nth triangular number

$$t_n = \frac{n(n+1)}{2}$$

Triangular numbers

1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, ...

Prove that the sum of any two consecutive triangular numbers is always a perfect square.

Prove that the sum of any two consecutive triangular numbers is always a perfect square.

$$\frac{n(n+1)}{2} + \frac{(n+1)(n+2)}{2}$$

$$= \frac{(n+1)(n+n+2)}{2}$$

$$= \frac{(n+1)(2n+2)}{2}$$

$$= (n+1)(n+1)$$

$$= (n+1)^2$$

Dragon Beard candy in central Seoul

Powers of 2

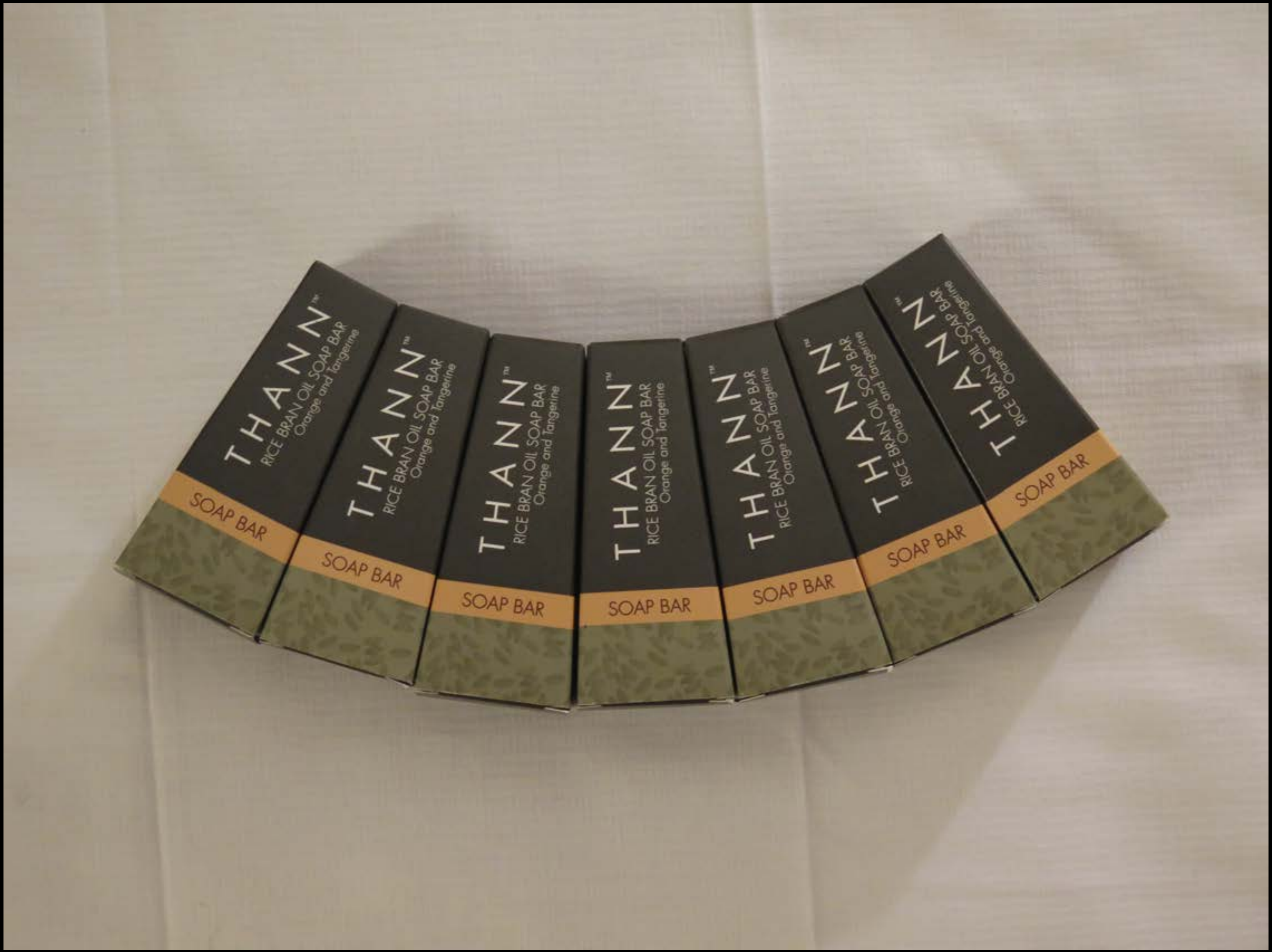
1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, 4096, 8192, 16384, ...

When will there be 1,000,000 strands?

During the 2019 T^3 International Conference I stayed at the Baltimore Marriott Waterfront Hotel. Look what I found in the bathroom...

Thann soap in a pyramidal frustum

Is it possible to add boxes to the seven shown below to form an *n*-gon that closely approximates a circle? If so, how many boxes are needed?

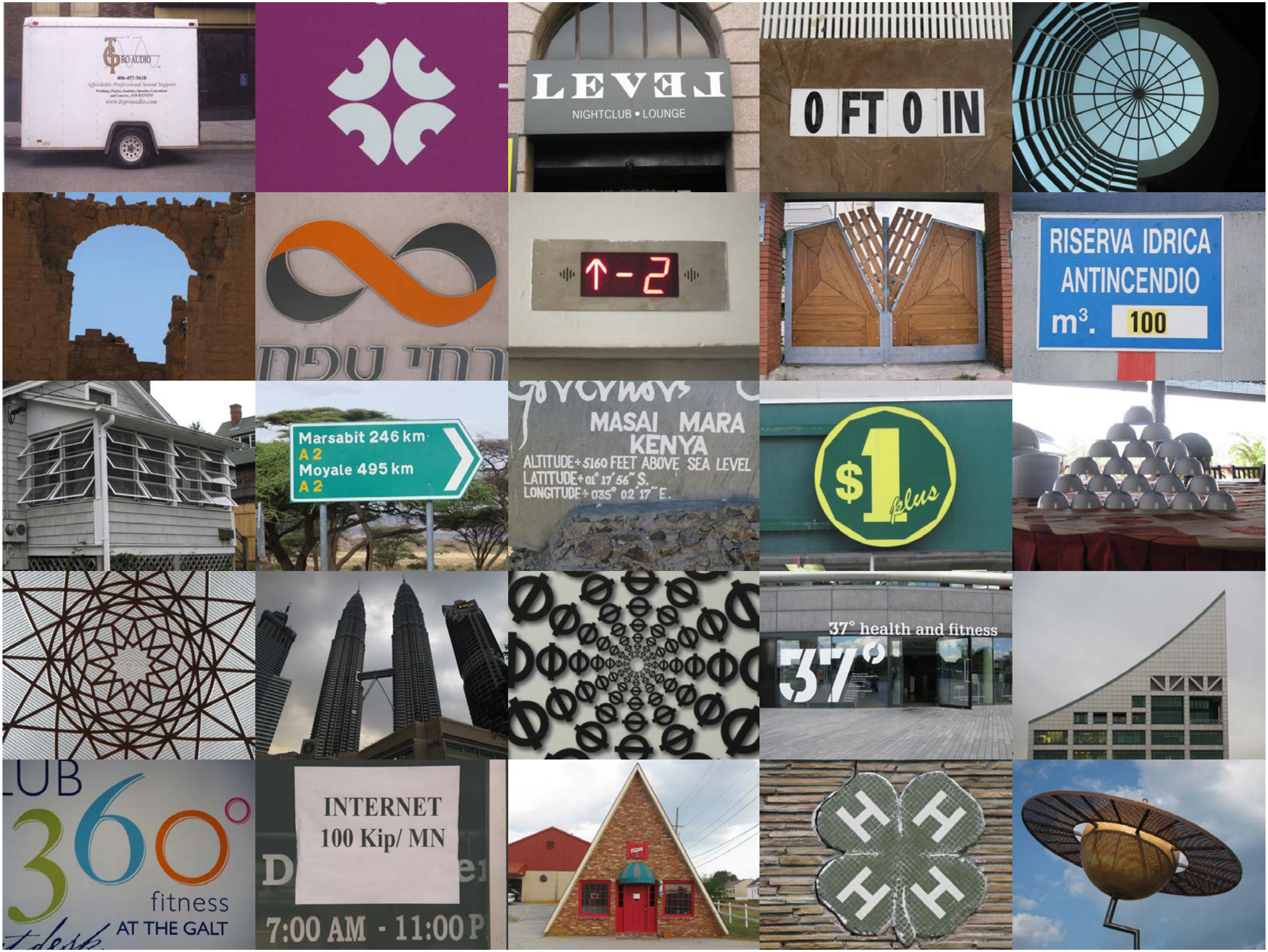



THANN™
RICE BRAN OIL SOAP BAR
Orange and Tangerine
SOAP BAR

Where does math live?

Where does math live?

EVERYWHERE


PRO AUDIO
LEVEL
NIGHTCLUB • LOUNGE
0 FT 0 IN
↑-2
RISERVA IDRICA
ANTINCENDIO
m3. 100
Marsabit 246 km
A 2
Moyale 495 km
A 2
MASAI MARA
KENYA
ALTITUDE÷5160 FEET ABOVE SEA LEVEL
LATITUDE÷01° 17' 56" S.
LONGITUDE÷035° 02' 17" E.
$1 plus
37° health and fitness
37°
UB
360°
fitness
AT THE GALT
INTERNET
100 Kip/ MN
7:00 AM - 11:00 P

If you would like to take a course with me or attend a talk, I will be presenting at the following conferences. It would be nice to meet you.

Mathematical Sciences Institute
June 10 – 12, 2019
Metairie Park Country Day School, New Orleans, Louisiana
https://www.mpcds.com/Math-Science-Institute-2019

Anja S. Greer Conference on Secondary School Mathematics, Science and Technology
June 23 - June 28, 2019
Phillips Exeter Academy, Exeter, New Hampshire
http://exeter.edu/programs-educators/summer-conference-general-information/anja-s-greer-conference-mathematics-and

Teaching Contemporary Mathematics Conference
January 24 – 24, 2020
North Carolina School of Science and Mathematics, Durham, North Carolina
https://www.ncssm.edu/tcmconference

Ron Lancaster
ron2718@nas.net

If you would like to take a course with me or attend a talk, I will be presenting at the following conferences. It would be nice to meet you.

Wisconsin Mathematics Council Annual Conference
May 1 – 3, 2019, Green Lake Conference Center, Green Lake, Wisconsin
http://www.wismath.org/2019-Annual-Conference

Ontario Association for Mathematics Education Annual Conference
May 16 - 18, 2019, University of Ottawa, Ottawa, Ontario
https://oame2019.ca/

MOVES Conference at the National Museum of Mathematics
August 4 – 6, 2019, New York, New York
https://momath.org/moves-conference/

USACAS
June 15 – 16, 2019, Highland Park High School, Highland Park, Illinois
https://www.meecas.org/usacas-2019

NCTM Regional Conference and Exhibition
September 25 – 27, 2019, Boston, Massachusetts
https://www.nctm.org/boston2019/

Weekend Workshops for International School Teachers in Shanghai (October 2019) and Osaka (November 2019)

Thanks a million for taking this webinar.