

# IMPLEMENT TASKS THAT PROMOTE REASONING AND PROBLEM SOLVING

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<http://bit.ly/T3-ImplementTasks>

# LEARNING INTENTIONS

- I can work within NCTM's Eight Mathematical Teaching Practices for strengthening the teaching and learning of mathematics.
- I can implement tasks that promote reasoning and problem solving.
- I can exercise mathematical flexibility to show what I know in more than one way.

# NORMS: HOPES & DREAM OF HOW WE WILL BE

**SAFE  
SPACE**

BE BRAVE VULNERABLE



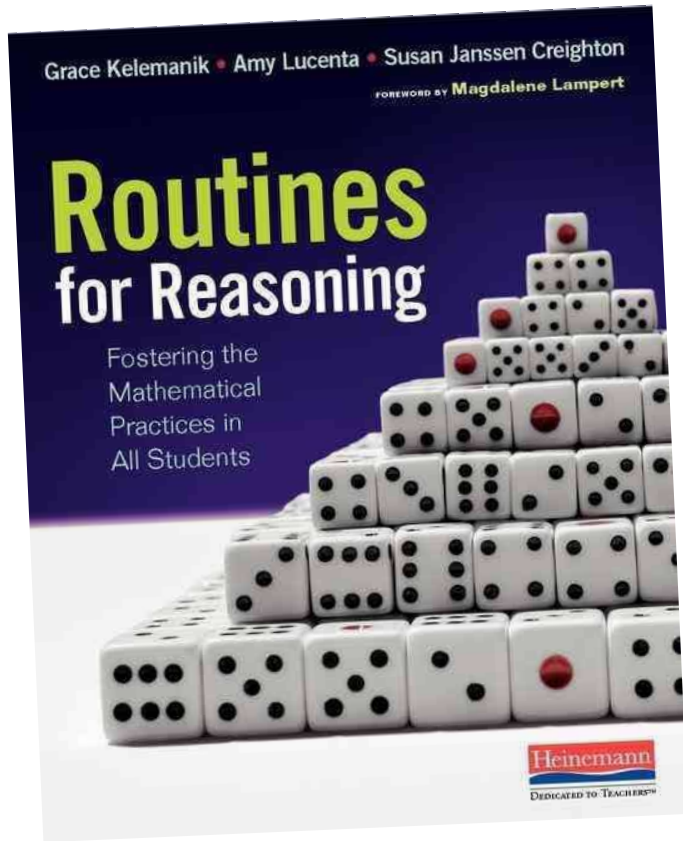
CONSIDERATE KIND

WELCOME  
\*KNOW\*  
\*DON'T KNOW\*

**CELEBRATE  
OPPORTUNITIES  
TO LEARN**



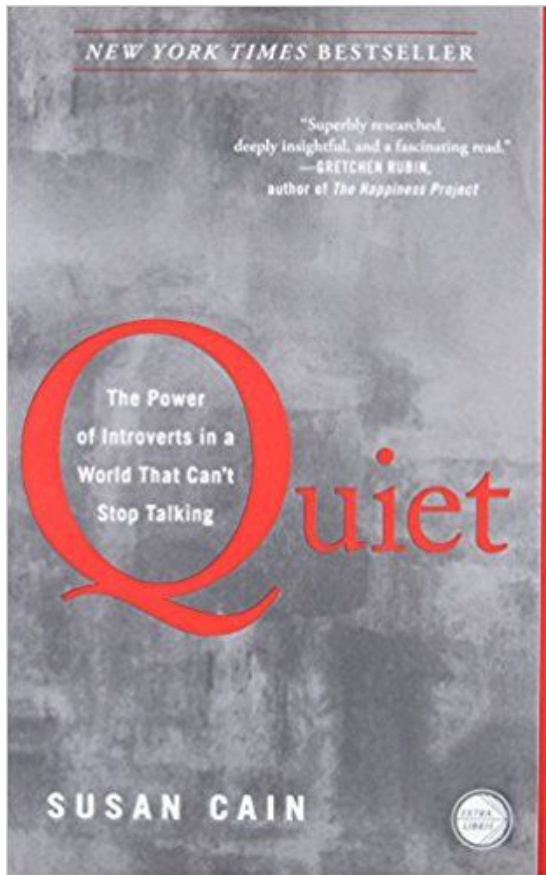
~~SHIP IT EARLY~~  
**LOW FIDELITY  
PROTOTYPE**



All students need time to think before responding to a teacher's question, talking with a partner, or following a classmate's strategy presented to the whole group.

Grace Kelemanik, Amy Lucenta, Susan Janssen

Kelemanik, Grace, Amy Lucenta, Susan Janssen. Creighton, and Magdalene Lampert. *Routines for Reasoning: Fostering the Mathematical Practices in All Students*. Portsmouth, NH: Heinemann, 2016. Print.



**Add a reflective pause because our extroverts need to learn to listen. Our introverts need to learn to talk.**

1. Establish mathematics goals to focus learning.
2. **Implement tasks that promote reasoning and problem solving.**
3. Use and connect mathematical representations.
4. Facilitate meaningful mathematical discourse.
5. Pose purposeful questions.
6. Build procedural fluency from conceptual understanding.
7. Support productive struggle in learning math.
8. Elicit and use evidence of student thinking.

# NCTM: PRINCIPLES TO ACTIONS

## **Implement Tasks that Promote Reasoning and Problem Solving.**

Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.

*Principles to Actions: Ensuring Mathematical Success for All*

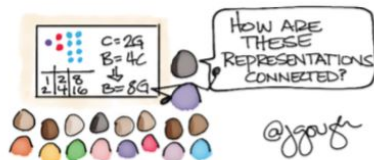
# Implement tasks that promote reasoning and problem solving

What are <i>teachers</i> doing?	What are <i>students</i> doing?
<p>Motivating students' learning of mathematics through opportunities for exploring and solving problems that build on and extend their current mathematical understanding.</p> <p>Selecting tasks that provide multiple entry points through the use of varied tools and representations.</p> <p>Posing tasks on a regular basis that require a high level of cognitive demand.</p> <p>Supporting students in exploring tasks without taking over student thinking.</p> <p>Encouraging students to use varied approaches and strategies to make sense of and solve tasks.</p>	<p>Persevering in exploring and reasoning through tasks.</p> <p>Taking responsibility for making sense of tasks by drawing on and making connections with their prior understanding and ideas.</p> <p>Using tools and representations as needed to support their thinking and problem solving.</p> <p>Accepting and expecting that their classmates will use a variety of solution approaches and that they will discuss and justify their strategies to one another.</p>

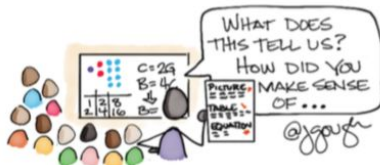


I can implement tasks that promote reasoning and problem solving.

#NCTMP2A - 2



**Level 4:**  
I can elicit and use evidence of student thinking to continually adjust instruction.



**Level 3:**  
I can implement tasks that promote reasoning and problem solving.



**Level 2:**  
I can empower learners to make sense of tasks and persevere in solving them.



**Level 1:**  
I can select low-floor high-ceiling tasks that motivate students to reach for their next independent level.



# PHASES OF THE MOON



# PHASES OF THE MOON



***FULL MOON***

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Fraction of the Moon Illuminated, 2018  
at Midnight  
Universal Time

Astron. Applications Dept.  
U. S. Naval Observatory  
Washington, DC 20392-5420

Day	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.
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03	0.99	0.92	0.99	0.93	0.91	0.83	0.81	0.68	0.51	0.43	0.25	0.19
04	0.95	0.85	0.95	0.87	0.85	0.76	0.73	0.58	0.40	0.32	0.16	0.12
05	0.88	0.76	0.89	0.80	0.78	0.67	0.63	0.48	0.29	0.22	0.09	0.06
06	0.80	0.67	0.82	0.71	0.69	0.58	0.53	0.37	0.19	0.13	0.04	0.02
07	0.70	0.57	0.73	0.62	0.60	0.48	0.43	0.26	0.11	0.06	0.01	0.00
08	0.60	0.47	0.64	0.53	0.51	0.38	0.33	0.17	0.04	0.02	0.00	0.01
09	0.49	0.37	0.55	0.44	0.41	0.28	0.23	0.09	0.01	0.00	0.02	0.03
10	0.39	0.28	0.45	0.34	0.32	0.19	0.14	0.03	0.00	0.01	0.06	0.07
11	0.30	0.20	0.36	0.26	0.23	0.11	0.07	0.00	0.02	0.04	0.11	0.12
12	0.22	0.13	0.27	0.17	0.15	0.05	0.02	0.01	0.07	0.10	0.18	0.19
13	0.14	0.08	0.19	0.11	0.08	0.01	0.00	0.04	0.14	0.17	0.26	0.27
14	0.08	0.03	0.12	0.05	0.03	0.00	0.01	0.10	0.22	0.25	0.35	0.36
15	0.04	0.01	0.07	0.02	0.01	0.02	0.05	0.17	0.31	0.34	0.44	0.45
16	0.01	0.00	0.03	0.00	0.01	0.07	0.12	0.27	0.41	0.43	0.54	0.55
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18	0.01	0.05	0.00	0.05	0.09	0.24	0.31	0.47	0.60	0.62	0.72	0.74
19	0.03	0.10	0.03	0.11	0.17	0.34	0.41	0.57	0.69	0.71	0.81	0.83
20	0.08	0.17	0.07	0.19	0.26	0.45	0.52	0.66	0.77	0.79	0.88	0.90
21	0.14	0.25	0.13	0.29	0.37	0.56	0.62	0.75	0.85	0.86	0.94	0.96
22	0.21	0.35	0.22	0.40	0.48	0.66	0.72	0.83	0.91	0.92	0.98	0.99
23	0.30	0.46	0.32	0.51	0.59	0.76	0.80	0.89	0.96	0.97	1.00	1.00
24	0.40	0.58	0.43	0.62	0.70	0.84	0.87	0.94	0.99	0.99	0.99	0.98
25	0.51	0.69	0.54	0.73	0.79	0.90	0.93	0.98	1.00	1.00	0.96	0.93
26	0.62	0.79	0.65	0.82	0.87	0.95	0.97	1.00	0.99	0.98	0.90	0.85
27	0.73	0.88	0.76	0.90	0.93	0.99	0.99	1.00	0.96	0.94	0.82	0.76
28	0.82	0.94	0.85	0.95	0.97	1.00	1.00	0.98	0.91	0.87	0.72	0.66
29	0.90		0.92	0.99	0.99	0.99	0.99	0.94	0.84	0.79	0.62	0.55
30	0.96		0.97	1.00	1.00	0.97	0.96	0.88	0.76	0.69	0.50	0.44
31	1.00		1.00		0.98		0.91	0.81		0.58		0.33



# PHASES OF THE MOON



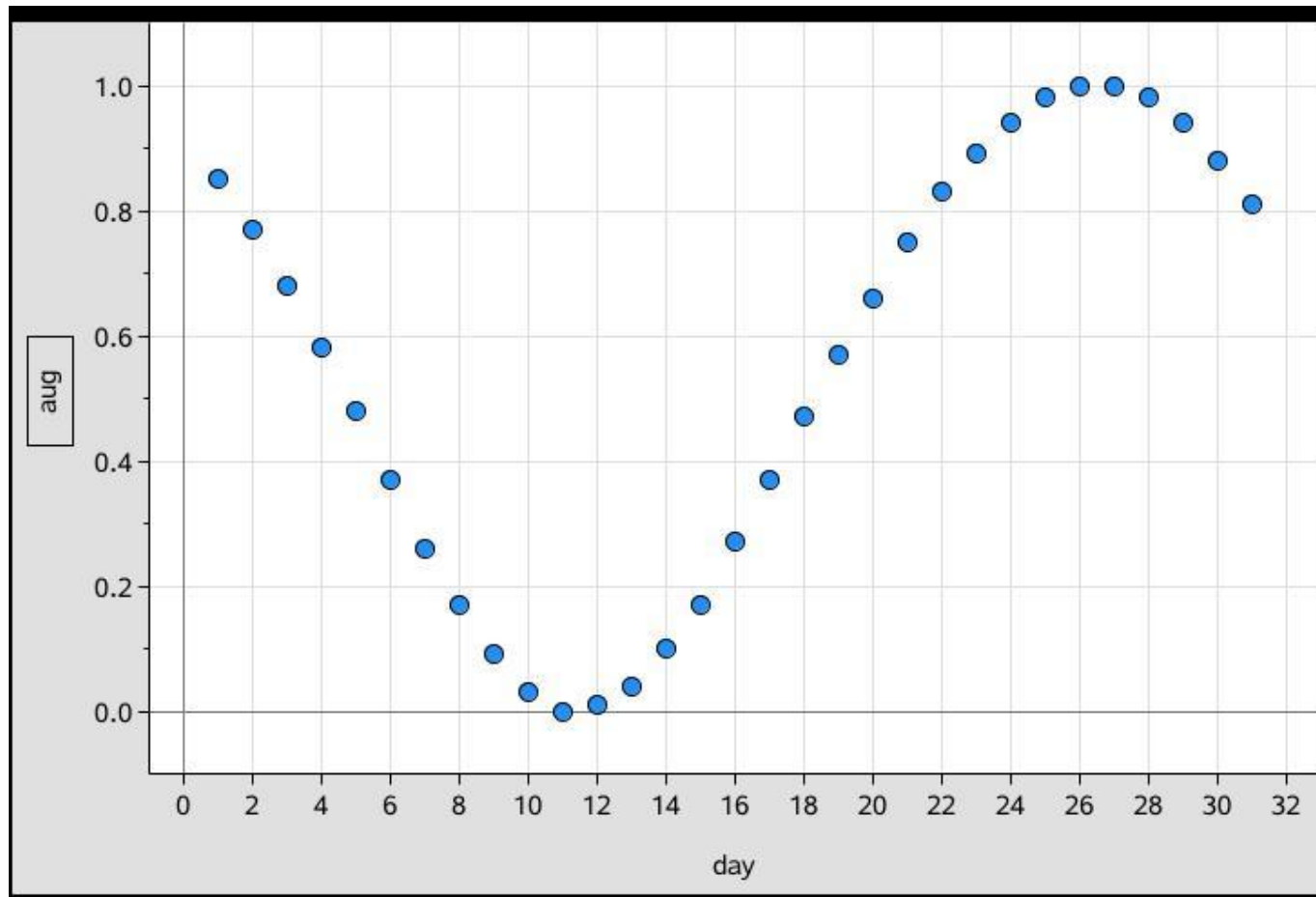
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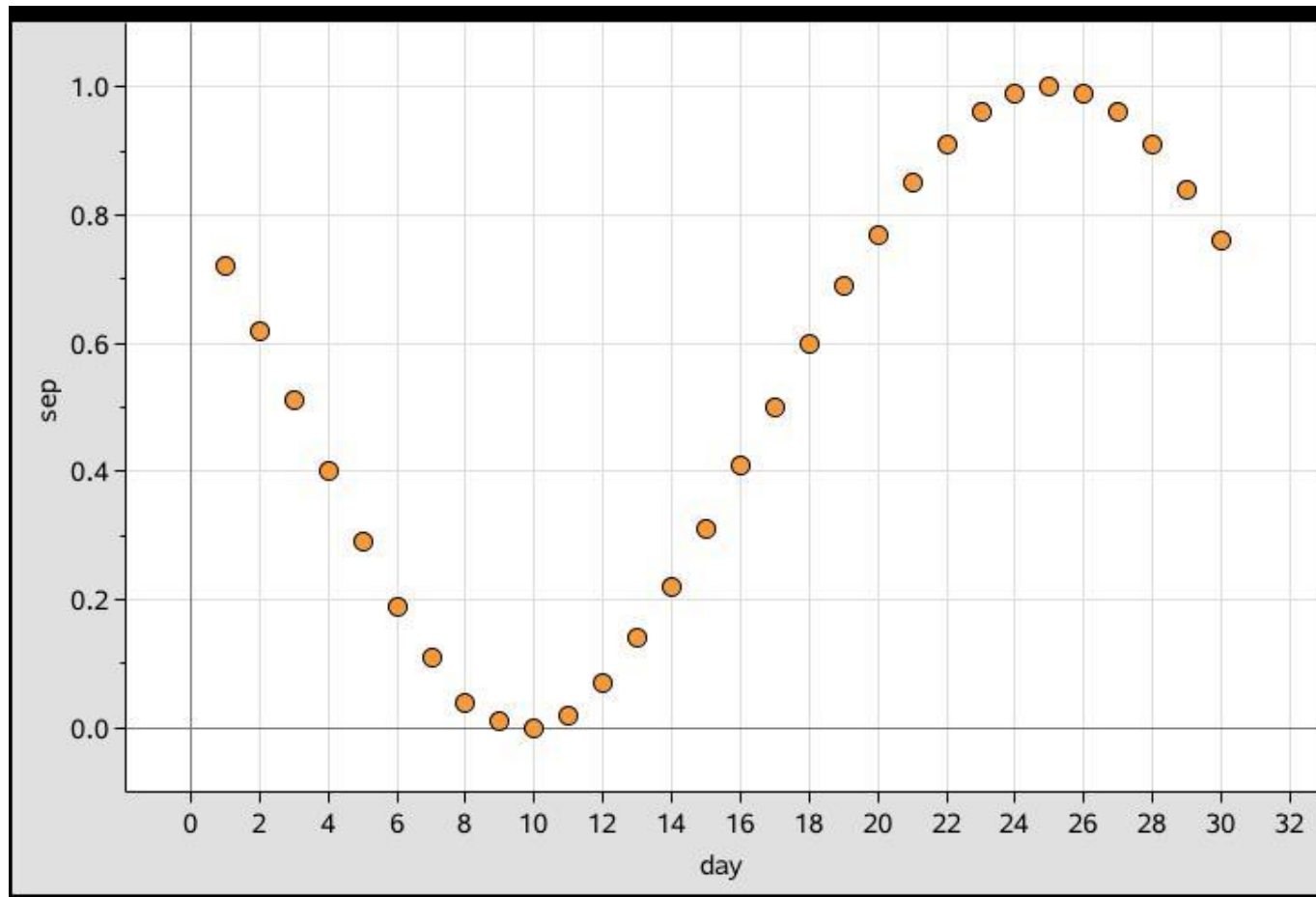
# PHASES OF THE MOON



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<http://aa.usno.navy.mil/data/docs/MoonFraction.php>

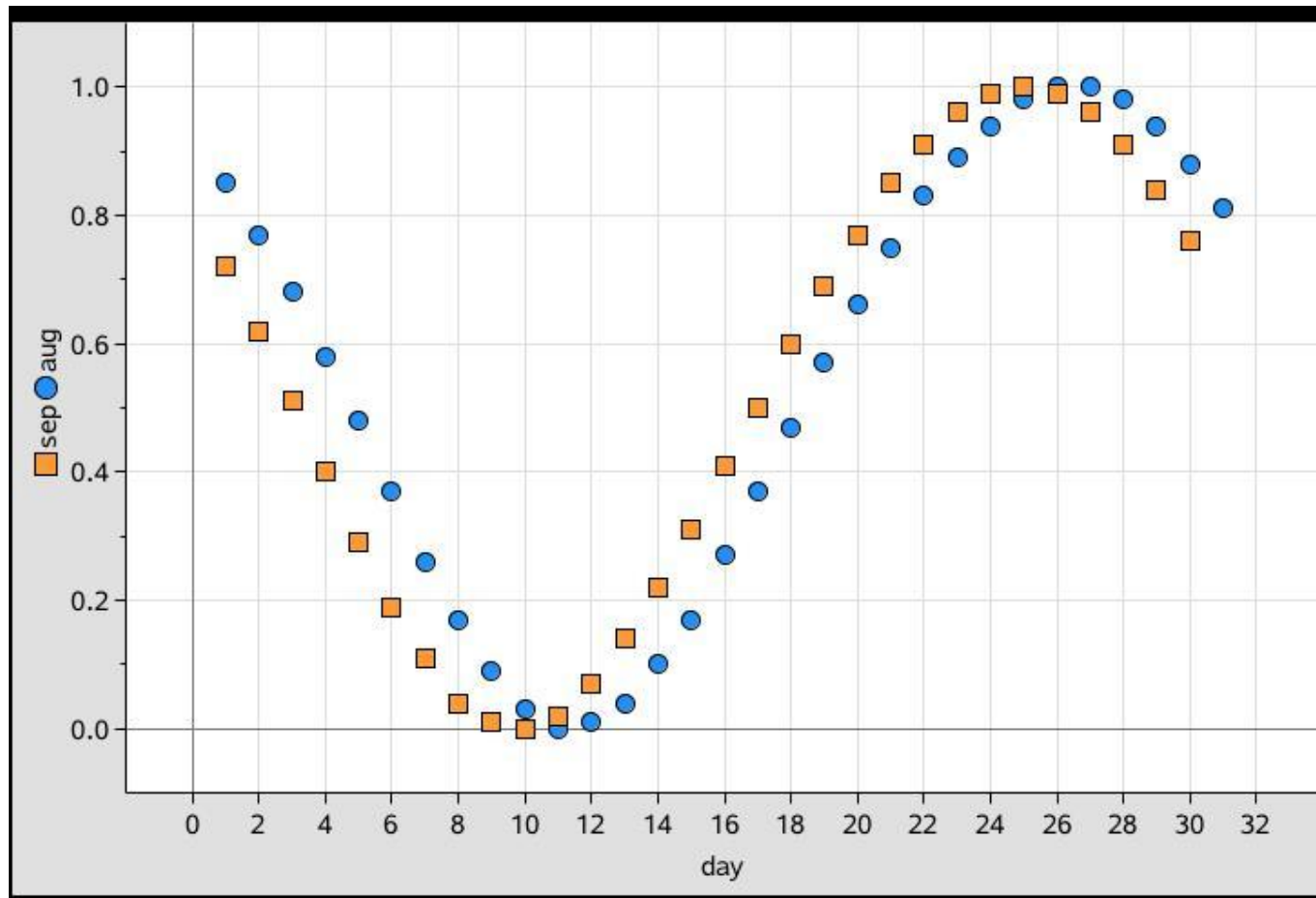
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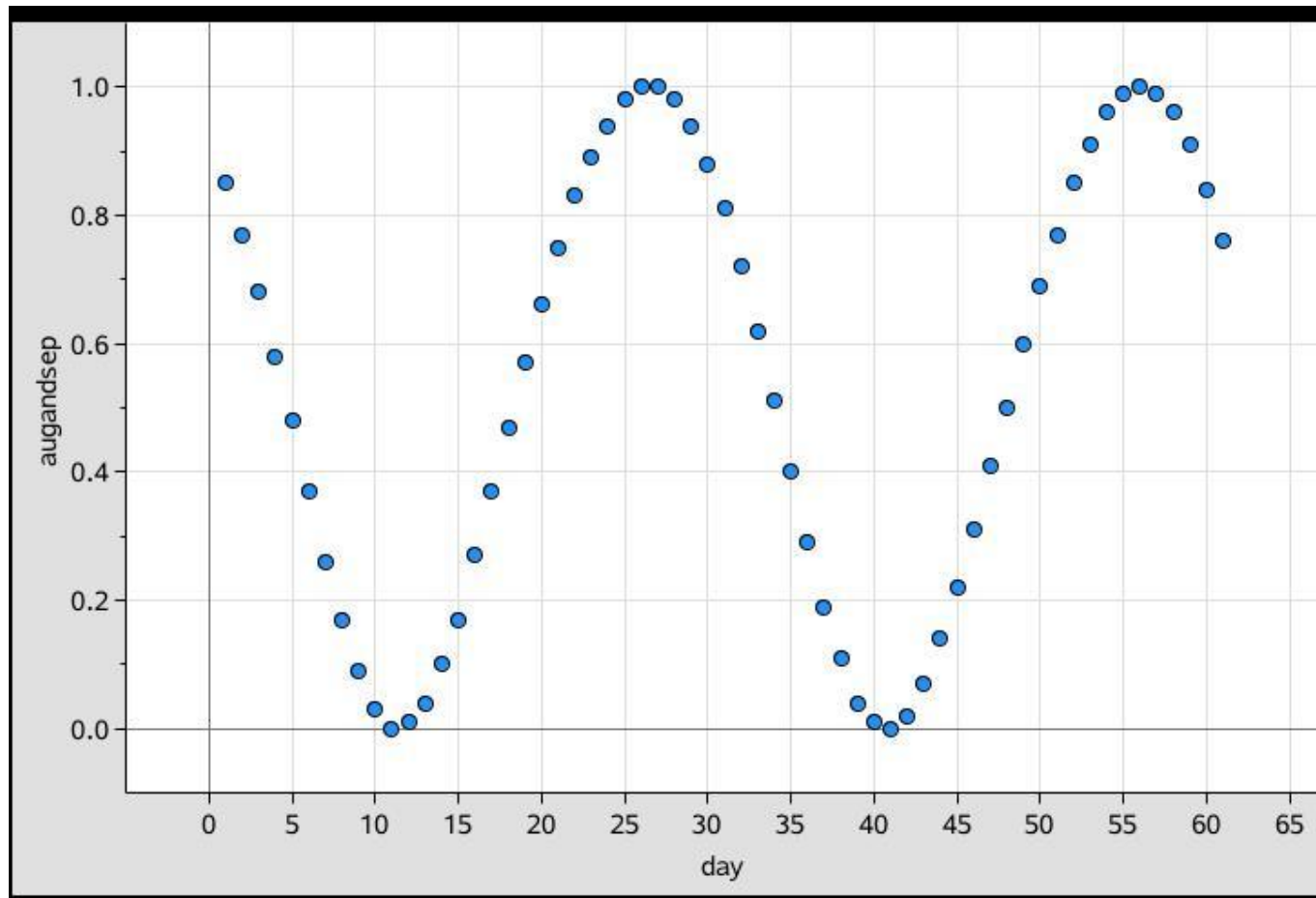
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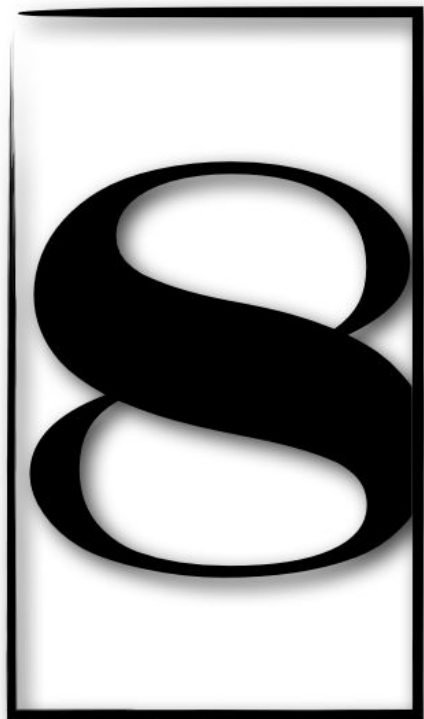
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**Look for and express regularity in repeated reasoning.**

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal.

By paying attention to the calculation of slope as they repeatedly check whether points are on the line through  $(1, 2)$  with slope 3, middle school students might abstract the equation  $(y - 2)/(x - 1) = 3$ . Noticing the regularity in the way terms cancel when expanding  $(x - 1)(x + 1)$ ,  $(x - 1)(x^2 + x + 1)$ , and  $(x - 1)(x^3 + x^2 + x + 1)$  might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

[bit.ly/ccssmp](http://bit.ly/ccssmp)



# SMP 8: I can look for and express regularity in repeated reasoning.

I can look for and express regularity in repeated reasoning.

SMP - 8



**Level 4:**  
I can attend to precision as I construct a viable argument to express regularity in repeated reasoning.



**Level 3:**  
I can look for and express regularity in repeated reasoning.



**Level 2:**  
I can identify and describe patterns and regularities, and I can begin to develop generalizations.



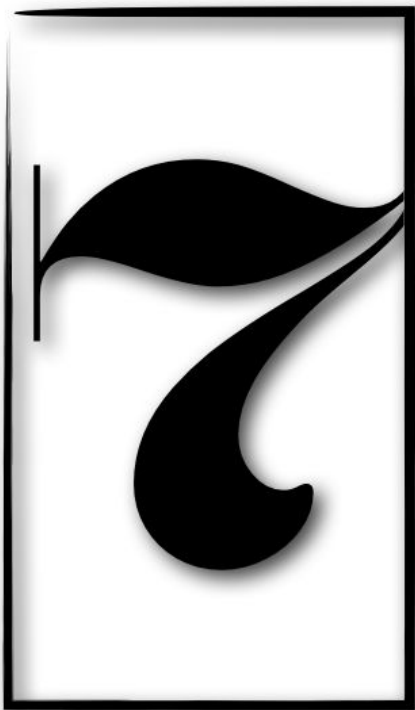
**Level 1:**  
I can notice and note what changes and what stays the same when performing calculations or interacting with geometric figures.





# Implement tasks that promote reasoning and problem solving

What are <i>teachers</i> doing?	What are <i>students</i> doing?
<p>Motivating students' learning of mathematics through opportunities for exploring and solving problems that build on and extend their current mathematical understanding.</p> <p>Selecting tasks that provide multiple entry points through the use of varied tools and representations.</p> <p>Posing tasks on a regular basis that require a high level of cognitive demand.</p> <p>Supporting students in exploring tasks without taking over student thinking.</p> <p>Encouraging students to use varied approaches and strategies to make sense of and solve tasks.</p>	<p>Persevering in exploring and reasoning through tasks.</p> <p>Taking responsibility for making sense of tasks by drawing on and making connections with their prior understanding and ideas.</p> <p>Using tools and representations as needed to support their thinking and problem solving.</p> <p>Accepting and expecting that their classmates will use a variety of solution approaches and that they will discuss and justify their strategies to one another.</p>

**Look for and make use of structure.**

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have.

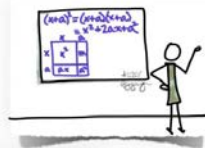
Later, students will see  $7 \times 8$  equals the well remembered  $7 \times 5 + 7 \times 3$ , in preparation for learning about the distributive property. In the expression  $x^2 + 9x + 14$ , older students can see the 14 as  $2 \times 7$  and the 9 as  $2 + 7$ . They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see  $5 - 3(x - y)^2$  as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers  $x$  and  $y$ .

[bit.ly/ccssmp](http://bit.ly/ccssmp)

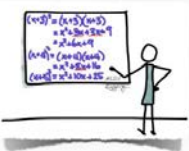
# SMP 7: I can look for and make use of structure.

I can look for and make use of structure.

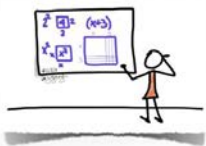
SMP - 7



**Level 4:**  
I can integrate geometric and algebraic representations to confirm structure and patterning.



**Level 3:**  
I can look for and make use of structure.



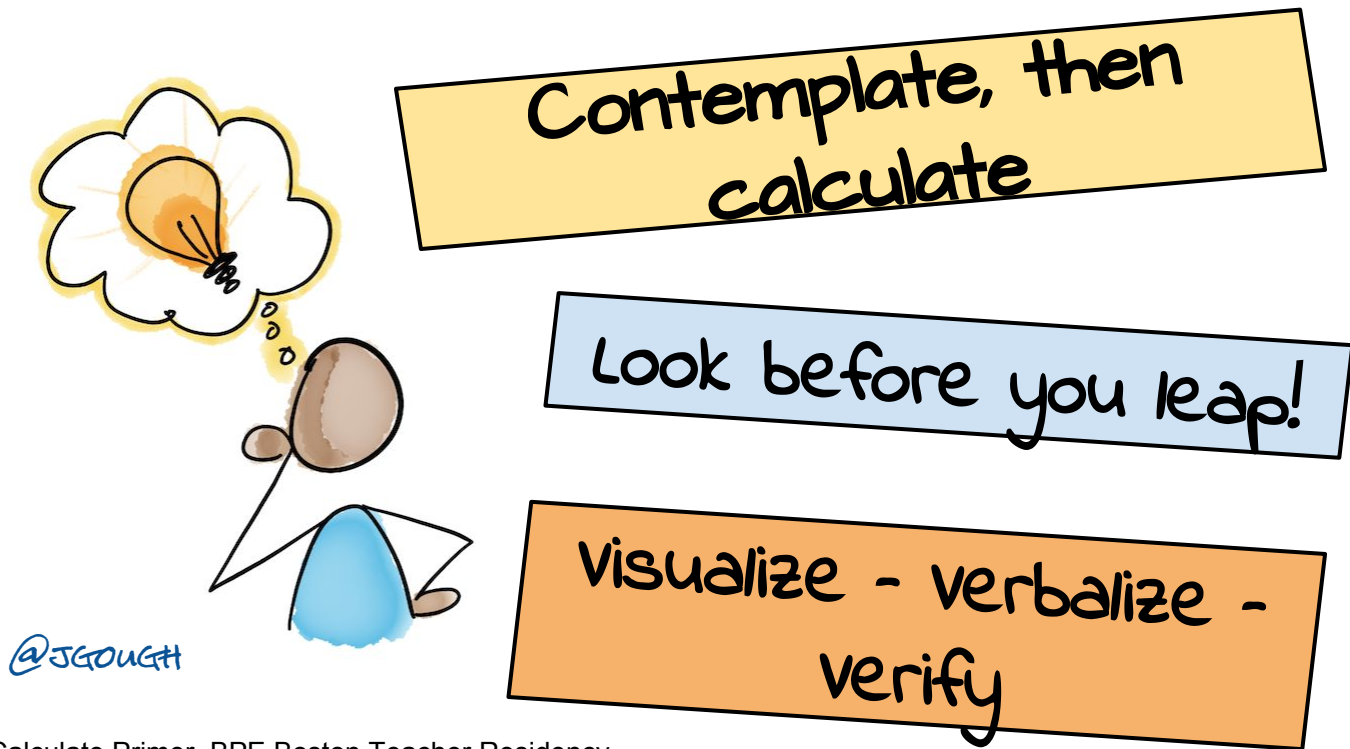
**Level 2:**  
I can rewrite an expression into an equivalent form, draw an auxiliary line, or identify a pattern to make what isn't pictured visible.



**Level 1:**  
I can compose and decompose numbers, expressions, and figures to make sense of the parts and of the whole.



# SMP 7: I can look for and make use of structure.



Contemplate then Calculate Primer, BPE Boston Teacher Residency.

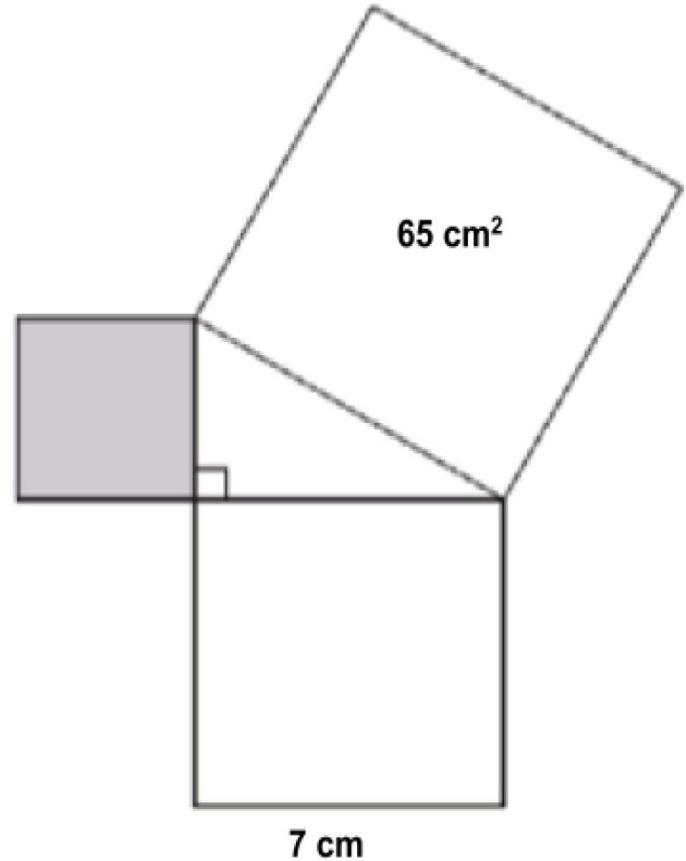
Look before you leap, Gail Burrill

Visualize, Verbalize, Verify, H/T Jules Bonine-Ducharme

# TASK

The three squares shown are joined at their vertices to form a right triangle.

What is the area of the shaded square?



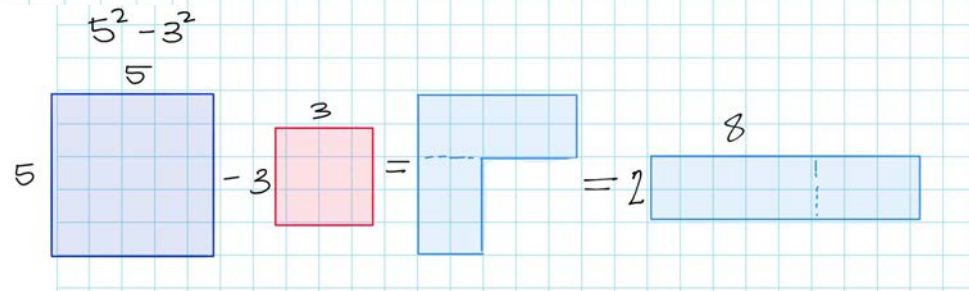
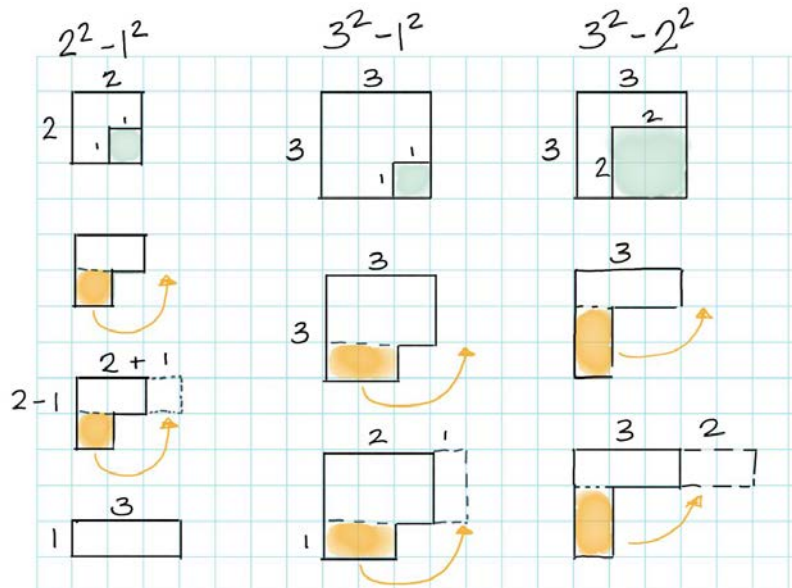
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Lower-level Demands	Higher-level Demands
Memorization	Procedures with Connections
Procedures without Connections	Doing Mathematics

# TASK

Evaluate  $21^2 - 19^2$

# Evaluate $21^2 - 19^2$





Let  $a$  and  $b$  be real numbers with  $a > b > 0$  and

$$\frac{a^3 - b^3}{(a - b)^3} = \frac{73}{3}.$$

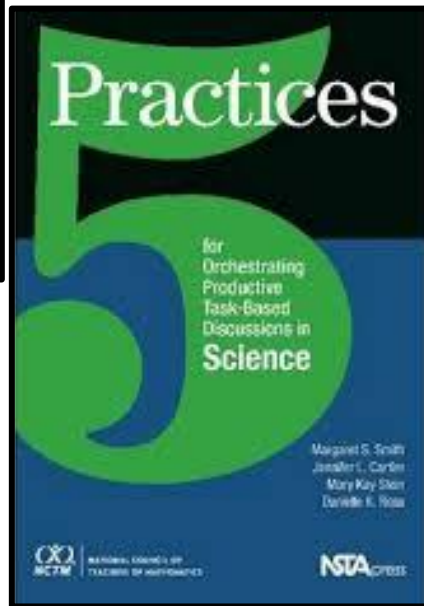
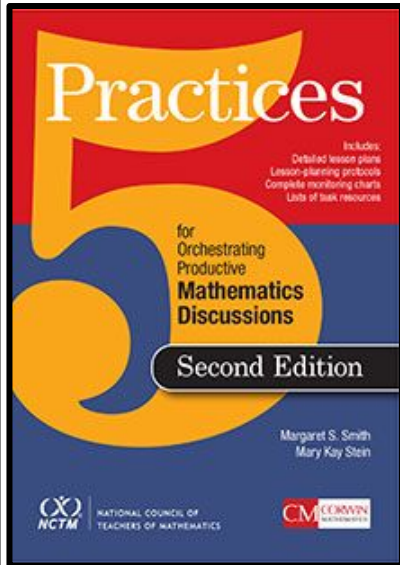
What is  $\frac{b}{a}$ ?

<http://bit.ly/t310918>

Lower-level Demands	Higher-level Demands
Memorization	Procedures with Connections
Procedures without Connections	Doing Mathematics

# 5 PRACTICES FOR ORCHESTRATING PRODUCTIVE MATHEMATICS DISCUSSIONS

(Determine Learning Goal) (Select Appropriate Task)



## 1. Anticipate

- Do the problem yourself
- What are students likely to produce?
- Which problems will most likely be the most useful in addressing the mathematics?

## 2. Monitor

- Listen, observe, identify key strategies
- Keep track of approaches
- Ask questions of students to get them back on track or to think more deeply

## 3. Select

- CRUCIAL STEP – what do you want to highlight?
- Purposefully select those that will advance mathematical ideas

## 4. Sequence

- In what order do you want to present the student work samples?
- Do you want the most common? Present misconceptions first?
- How will students share their work? Draw on board?
- Put under doc cam?

## 5. Connect

- Craft questions to make the mathematics visible.
- Compare and contrast 2 or 3 students' work – what are the mathematical relationships?
- What do parts of student's work represent in the original problem? The solution? Work done in the past?

# REFLECTION

I learned to pay attention to...

I learned to ask myself...

A new mathematical connection is...

# IMPLEMENT TASKS THAT PROMOTE REASONING AND PROBLEM SOLVING

Jill Gough, @jgough  
Jennifer Wilson, @jwilson828