Mathematical Investigations for Mathematical Methods and Specialist Mathematics

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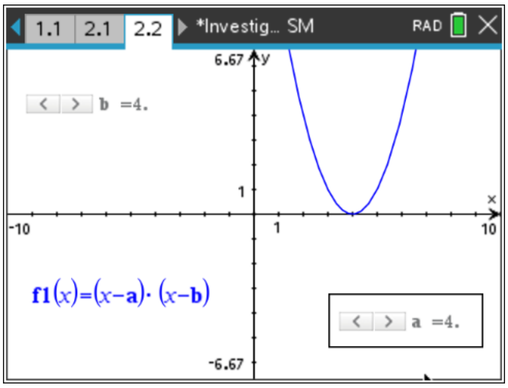
MM - Polynomial Investigation using CAS

For varying values of a, b, c and d

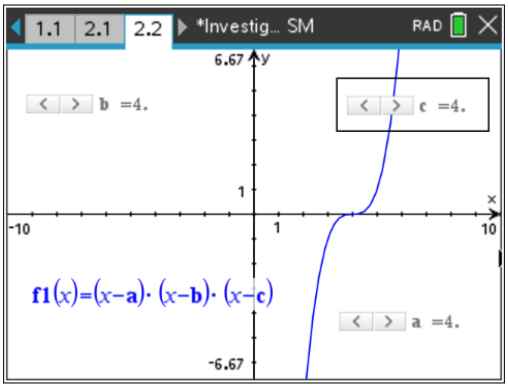
Sketch

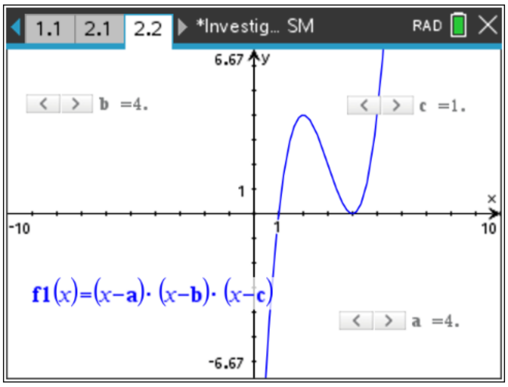
$y = x - a;$

$y = (x - a)(x - b);$



$y = (x - a)(x - b)(x - c);$





$y = (x - a)(x - b)(x - c)(x - d)$

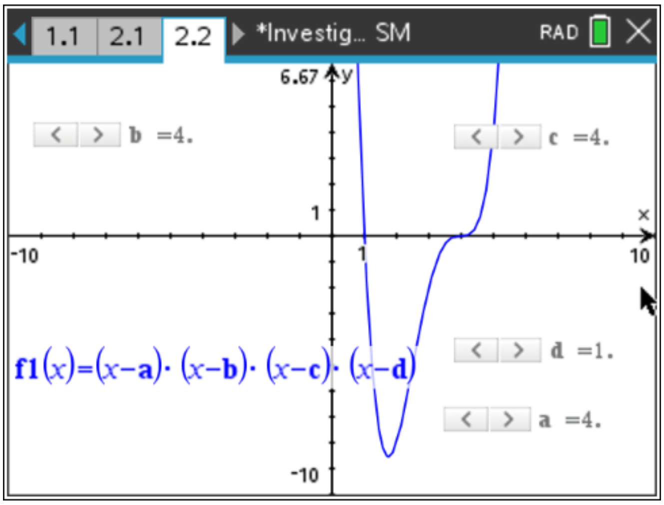
Now compare and contrast their features.

What feature/s do all graphs share (if any)?

What feature/s do some of the graphs share?

What happens to the graph of the quadratic, cubic and quartic if $a = b$?

What happens to the graph of the cubic and quartic if $a = b = c$?



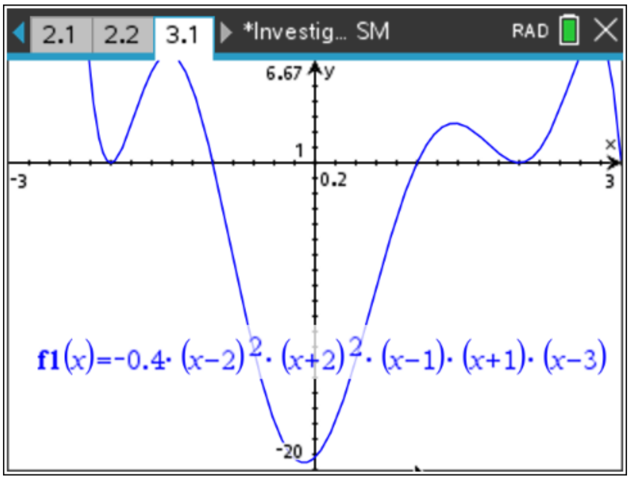
What happens to the graph of the quartic if $a = b = c = d$?

Create a function that would have x-intercepts at -2 and 5, where the intercept at 5 is also a turning point.

Create a function that would have x-intercepts at -1, 3 and 4, where the intercept at 3 is also a turning point.

Challenge or Exit Ticket

Create a function that has five x-intercepts, 2 of these being concave up turning points.



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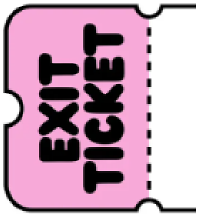
EXIT TICKET POLYNOMIAL GRAPHS

CAS



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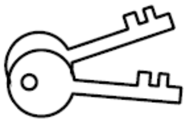

Key Points Exit Ticket

Write three key points from today's lesson below.

1 ______________________________

____

____

2 ______________________________

____

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3 ______________________________

____

____

NOTE to MYSELF

____

____

____

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____

____

SOLVE IT

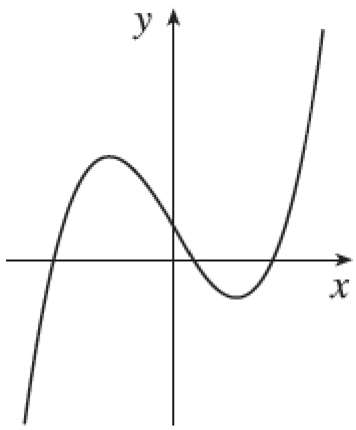
1. Create a function that would have x-intercepts at -2 and 5, where the intercept at 5 is also a turning point.

2. Create a function that would have x-intercepts at -1, 3 and 4, where the intercept at 3 is also a turning point.

3. Create a function that has five x-intercepts, 2 of these being concave up turning points.

MATHS METHODS GROUP WORK

The graph shows a sketch of the curve $y = f(x)$, where $f(x) = x^3 - 5x + 2$.



a) Use the fact that $x = 2$ is a root of $f(x) = 0$ to find the exact values of the other two roots of $f(x) = 0$.

b) Write down the roots of:

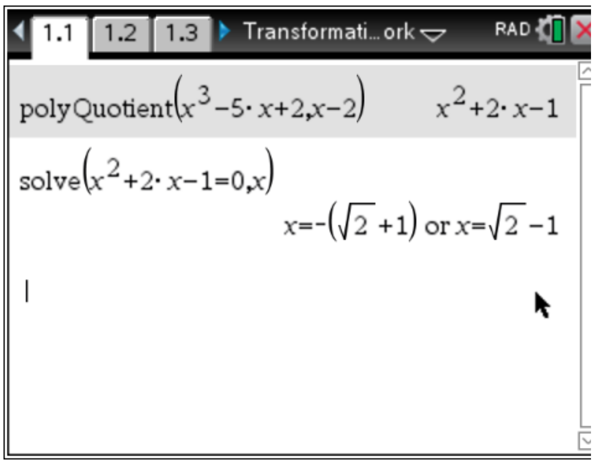
i) $f(x-3)$

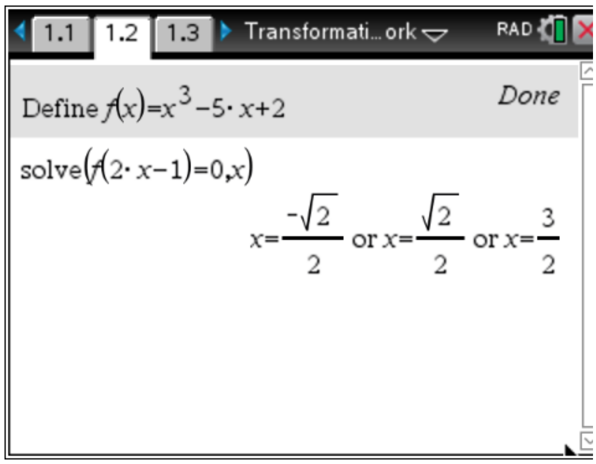
ii) $f(2x)$

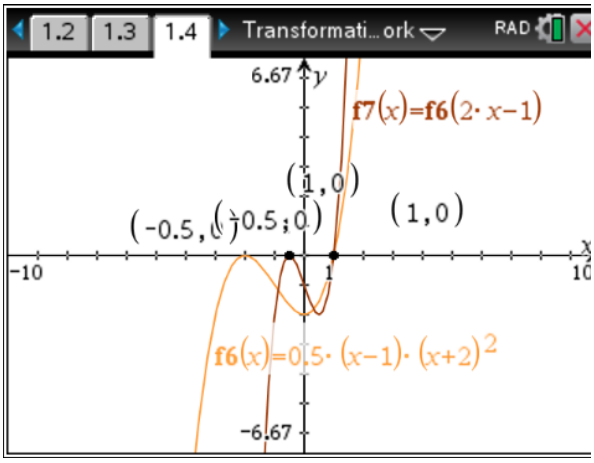
iii) $3f(-x)$.

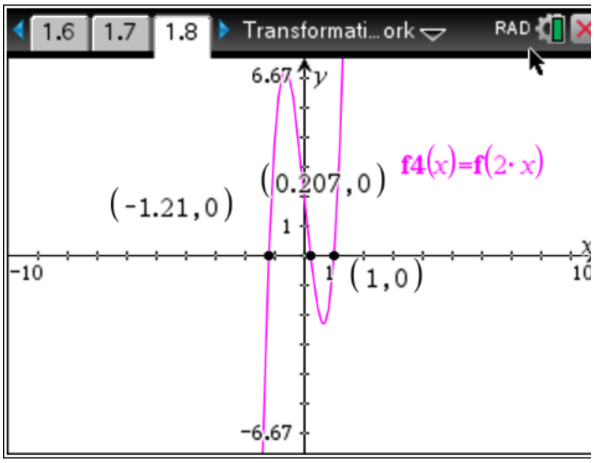
USING CAS:

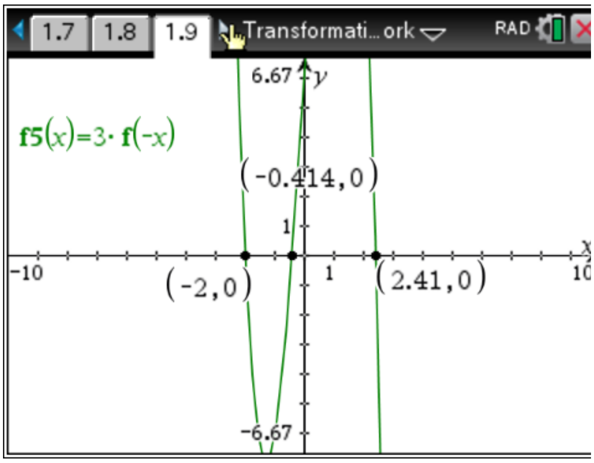
Combine algebraic manipulation and reasoning with CAS functionality to enhance understanding and check over the solutions. Using algebraic and graphing features of TI Nspire CAS.



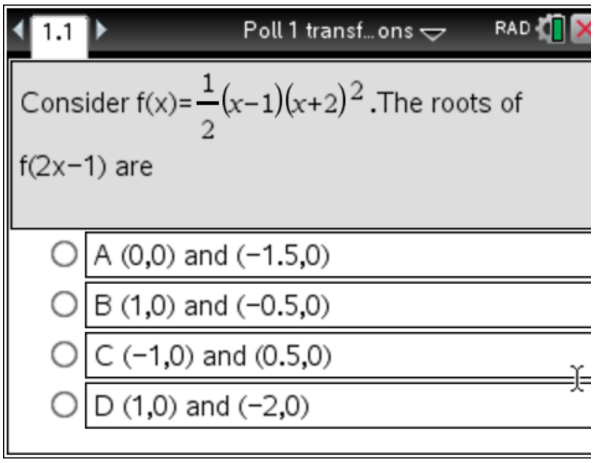


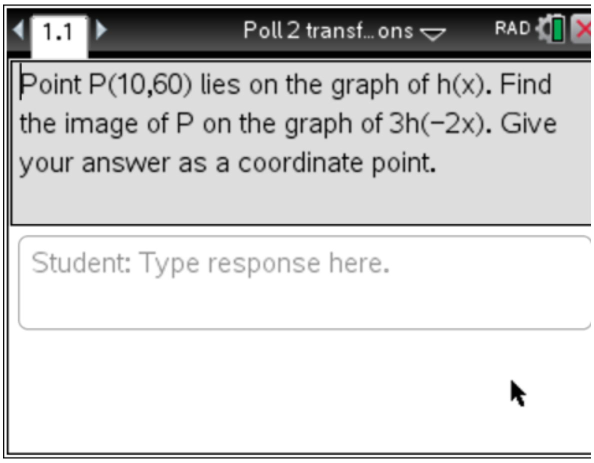




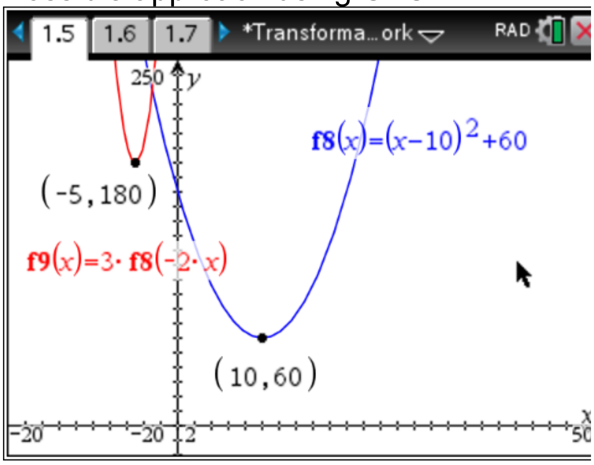


POLLS:





Possible approach using CAS:



Investigation: Seeing patterns in factors of $x^n - 1$

Question 1

a. Before using CAS, try to factorise each algebraic expression listed in the left column of this table:

Factorization by-hand	Verify your answer using the FACTOR command on CAS (write the result displayed by CAS)
$a^2 - b^2 =$	
$x^2 - 1 =$	
$a^3 - b^3 =$	
$x^3 - 1 =$	

Compare your results with the person next to you

1b. Expand the following expressions by-hand (do not use CAS).

$(x-1)(x+1) =$
$(x-1)(x^2+x+1) =$

1c. Without doing any algebra, anticipate the result of the following expansion:

$(x-1)(x^3+x^2+x+1) =$

1d. Verify your anticipated result above in 1c. by expanding by-hand, and then use CAS to check your answer.

Question 2

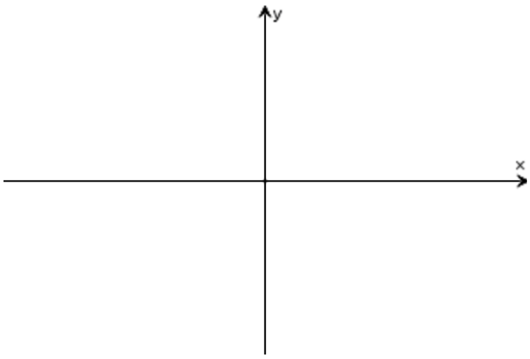
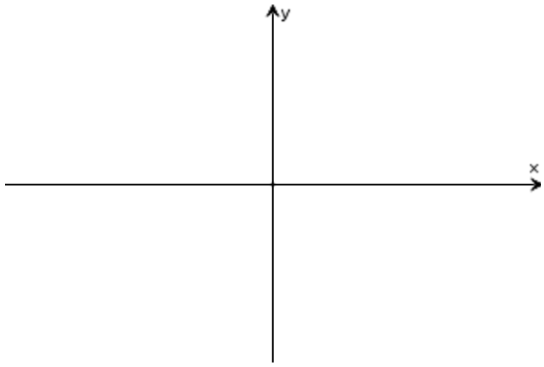
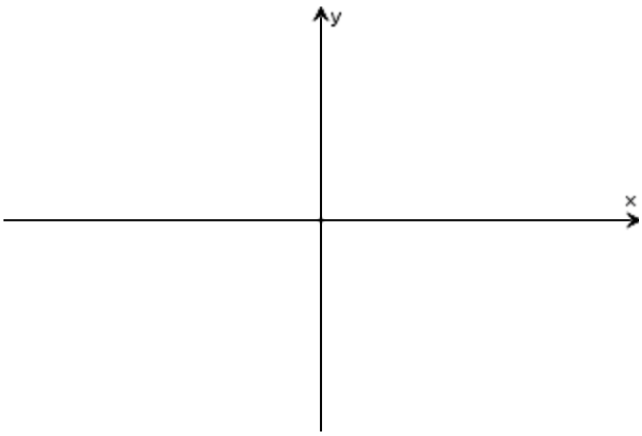
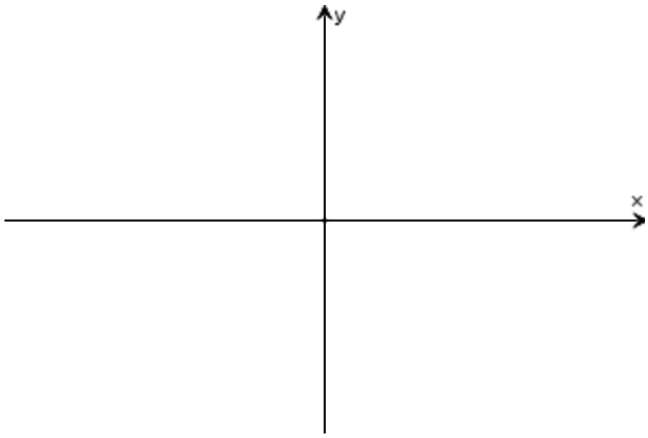
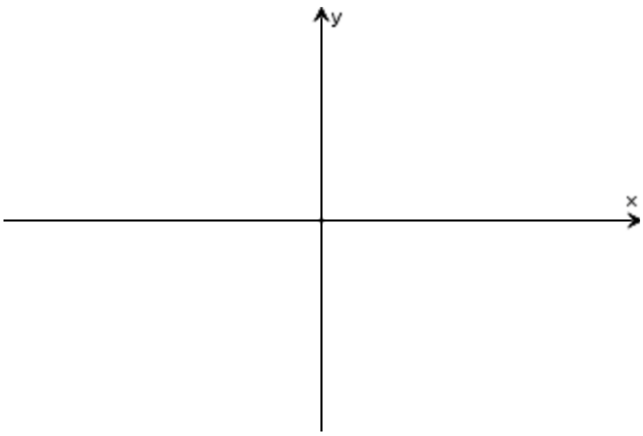
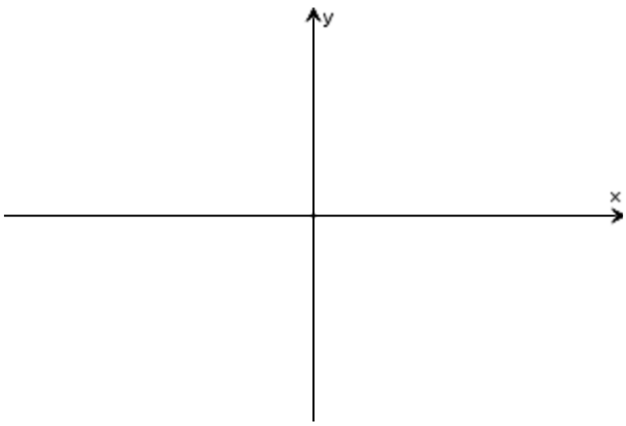
a. What do the following three expressions have in common? Also, how do they differ?

$(x-1)(x+1)$ $\quad$ $(x-1)(x^2+x+1)$ $\quad$ $(x-1)(x^3+x^2+x+1)$

b. Based on what you have seen so far, predict a factorization of the expression x^5-1.
Write your prediction, then check your answer using CAS.

Question 3

a. Sketch the following graphs and label all axial intercepts with their coordinates.

$y = x^2 - 1$	$y = x^3 - 1$
	
$y = x^4 - 1$	$y = x^5 - 1$
	
$y = x^6 - 1$	$y = x^7 - 1$
	

b. State any similarities and differences in your graphs from part a.

Similarities	Differences

c. Factorise the following expressions by hand, and then check your answers using CAS.

Expression	By hand	CAS output	Is the 'by-hand' factorization the same as the CAS factorization?
$x^2 - 1$			
$x^3 - 1$			
$x^4 - 1$			
$x^5 - 1$			
$x^6 - 1$			
$x^7 - 1$			

d. Comment on similarities and differences in your factorising results in part b.
Do you notice any patterns?
How many factors does $x^4 - 1$ have when you factorise by hand compared when you factorise by CAS?

e. Predict the factorisation of $x^9 - 1$. Write out your prediction and check your answer using CAS.

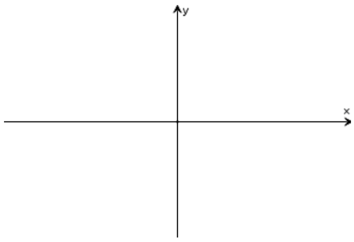
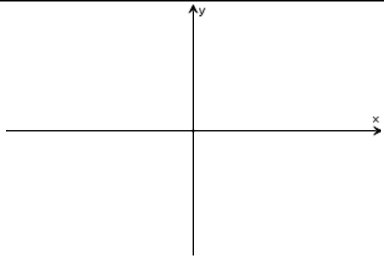
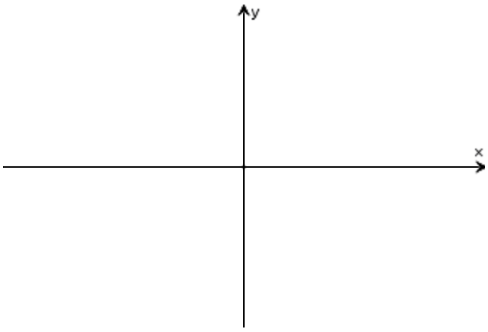
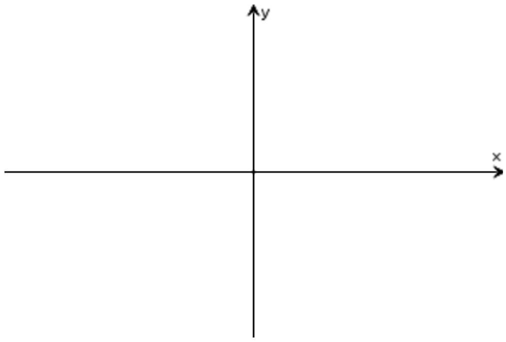
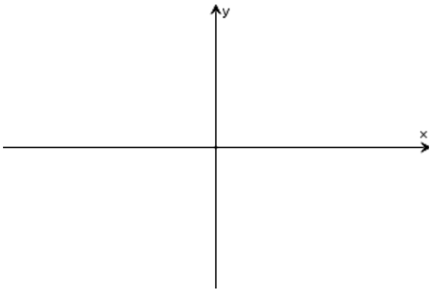
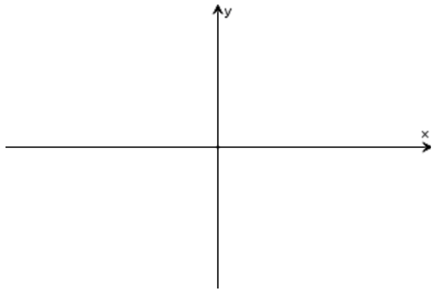
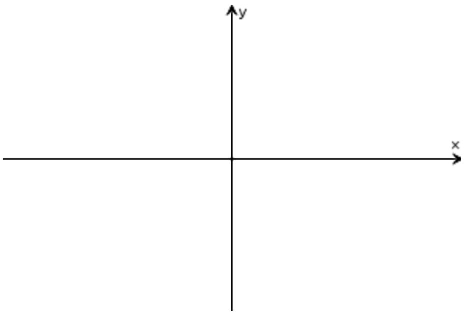
f. Predict the factorisation of $x^8 - 1$. Write out your prediction and check your answer using CAS.

Investigation: Product of Polynomials

Component 1

In this component you will consider graphs of the form $y = x^m(h - x)^n$, where h is non-zero real number and $m, n \in \{0, 1, 2, 3, 4\}$

1. Consider the case where $h = 2$.
   a. Sketch the following graphs, labelling all key features.

$m = 0, \quad n = 0 \quad y = x^0(2 - x)^0$	$m = 1, \quad n = 1 \quad y = x(2 - x)$
	
$m = 2, \quad n = 1 \quad y = x^2(2 - x)$	$m = 1, \quad n = 2 \quad y = x(2 - x)^2$
	
$m = 3, \quad n = 1 \quad y = x^3(2 - x)$	$m = 1, \quad n = 3 \quad y = x(2 - x)^3$
	
$m = 2, \quad n = 2 \quad y = x^2(2 - x)^2$	
	

Comment on any similarities/differences between your graphs.

Similarities	Differences

b. Discuss how m and n affect the behaviour/shape of the graph. Provide examples to support your ideas/conjectures by using a different value of h.

Point(s) of Inflection

There are two types of point of inflection that can occur on a graph.

<table>
<tr>
<td>Type 1: Stationary points of inflection

Think about graphs like $y = (x-1)^3 + 1$
This cubic is in point of inflection form</td>
<td>Type 2: Non-stationary points of inflection

These occur at the midpoint between two turning points, or the midpoint of a turning point and stationary point of inflection.

You are not expected to find these by-hand, but can use CAS to find them.</td>
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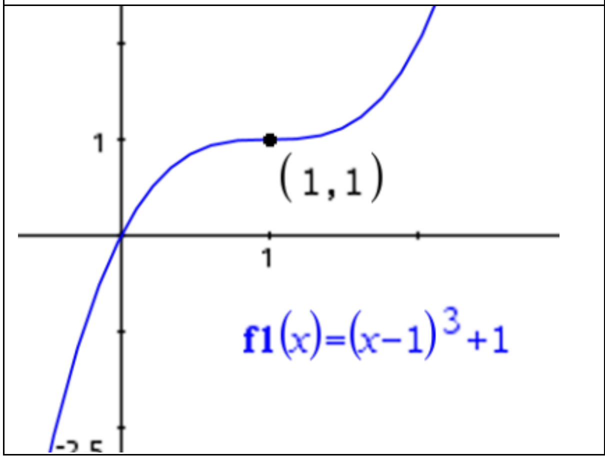
<td>
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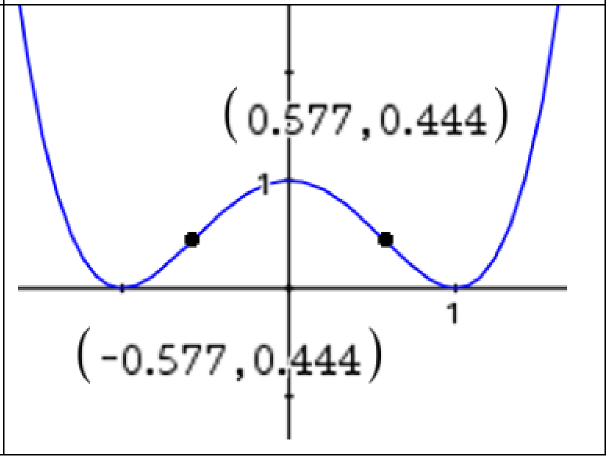
<td>
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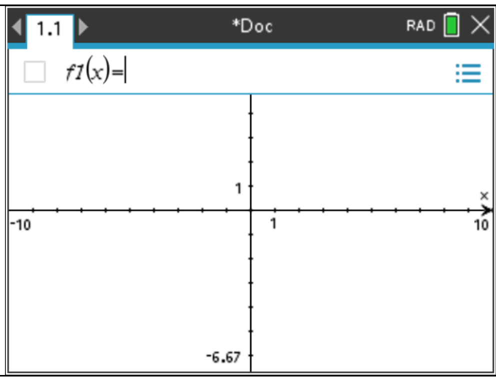
To find point(s) of inflection on CAS:

<table>
<tr>
<td>Go to a Graphs page</td>
<td>
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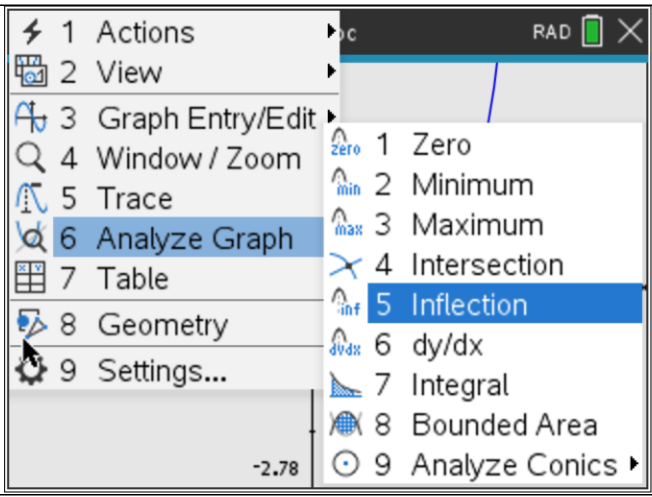
Press: • Menu • Analyse Graph • Inflection	

c. Find and label all of the points of inflection on all of your graphs in part a., correct to 1 decimal place.

d. Discuss how m and n affect the number and nature of any turning points/point of inflection. Provide examples to support your ideas/conjectures by using a different value of h.

2. What would happen to the graph and key features if h was negative? Investigate.

3. Create an equation of a graph of the form $y = x^m(h - x)^n$ that satisfies the following conditions:

Graph 1:

- Has an x-intercept and turning point at $x = 3$

Graph 2:

- Has at least 2 turning points
- Has a negative x-intercept

Graph 3:

- Has a stationary point of inflection
- Has a negative x-intercept

Graph 4:

- Has a stationary point of inflection
- Has a non-stationary point of inflection

Component 2

In this component you will consider graphs of the form $y = (x^m - a)(x^n - b)$, where a, b are non-zero real numbers and $m, n \in \{1, 2, 3\}$.

1. Consider the function $f(x) = (x - 1)(x^2 - 8)$
   a. Factorise $f(x)$ and state the values of x for which $f(x) = 0$
   b. Sketch the graph of $y = (x - 1)(x^2 - 8)$. Label all axial intercepts with coordinates in exact values, and any turning points and points of inflection with coordinates correct to 1 decimal place.

2. Consider the function $f(x) = (x - 1)(x^3 - 8)$
   a. Factorise $f(x)$ and state the values of x for which $f(x) = 0$
   b. Sketch the graph of $y = (x - 1)(x^3 - 8)$. Label all axial intercepts with coordinates in exact values, and any turning points and points of inflection with coordinates correct to 1 decimal place.

3. Comment on any similarities/differences between your graphs in Question 1 and Question 2.

Similarities	Differences

Consider the family of curves of the form $y = (x^m - 1)(x^n - 8)$ where $2 \leq m + n \leq 4$. Investigate how m and n affect:

- The location and number of axial intercepts
- The behaviour of the graph
- (The number of turning points/stationary points of inflection)

a. Consider the family of curves of the form $y = (x^m - 1)(x^n - a)$ where $2 \leq m + n \leq 4$ and a is a non-negative real number.
   Investigate how a, m and n affect:
   - The location and number of axial intercepts
   - The behaviour of the graph
   - (The number of turning points/stationary points of inflection)

End of Investigation

SM Roots of complex numbers investigation

Setting the stage: Fundamental Theorem of Algebra

Do you remember the Fundamental Theorem of Algebra?
If not, look it up to complete the following:

The Fundamental Theorem of Algebra (FTOA)
Every polynomial of degree ___Type equation here.____ has exactly ________ roots.

1. According to the FTOA, how many roots does $P(x) = x^2 + 4x + 3$ have?

   What are the roots of $P(x)$?

   According to the FTOA, how many roots does $f(x) = x^3 + 8$ have?

   How many roots can you find for $f(x)$? What are they?

   Finding all the roots

So, the big goal of this handout is: **how can we find the missing roots?** To do this, we are going to use a clever application of De Moivre's theorem.

2. Consider the equation $z^3 = -8$. We know there must be ______ roots for this equation, but with basic algebra we can only find one: $z =$ ________.

   a. Write -8 as a complex number in modulus-argument form.

   b. Using coterminal angles, write a general form for **all** complex numbers that are equivalent to 8.

   Can you see now how multiple solutions/roots might be generated?

   c. Using $z = r \operatorname{cis} \theta$, and your general complex form for -8, using complex numbers to write and solve separate equations involving r and θ.

   d. We know we are looking for exactly ______ roots, so we can let $k =$ ______, ______, ______ to find three different complex roots.

Therefore:

$k =$ ______ $\Rightarrow \theta = \frac{\pi+2\pi(\quad)}{3} =$ ________ $\Rightarrow z =$ ______ cis ______

$k =$ ______ $\Rightarrow \theta = \frac{\pi+2\pi(\quad)}{3} =$ ________ $\Rightarrow z =$ ______ cis ______

$k =$ ______ $\Rightarrow \theta = \frac{\pi+2\pi(\quad)}{3} =$ ________ $\Rightarrow z =$ ______ cis ______

Now, generalize the process above and fill out the box below:

If $z = r \operatorname{cis} \theta$ and $n \in \mathbb{Z}^*$, then z has n distinct n^{th} roots:

$$z_k = \underline{\qquad\qquad} \operatorname{cis}\left(\frac{\qquad\qquad}{\qquad}\right)$$

where $k = 0, 1, 2, \ldots$ ________

Symmetry of complex roots

3. Record your observations below.
   a. What do you notice about the **moduli** of all the roots?

      What do you notice about the **arguments** of the roots?

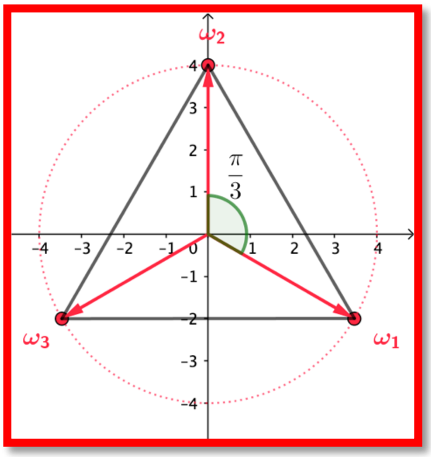
      Practice

4. Find all solutions to $z^5 = 1$.

The solutions to $z^n = 1$ are called the "n^{th} roots of unity".

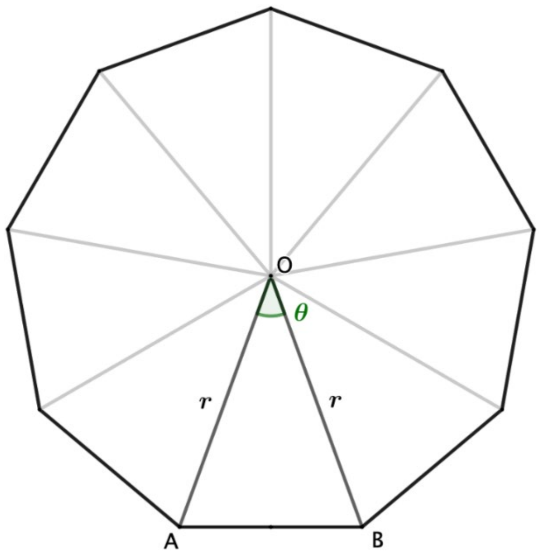
5. Given $z = -64i$:
   a. Find the three cube roots of z in Euler form.

   b. Given that the cube roots of z form a triangle when plotted in the Argand diagram, find the exact area of the triangle.



6. In this question, we will examine the polygons generated by the roots of a complex number.

a) What polygon will the roots of $z^4 = \omega$ create?
b) Find the area of the polygon create by the roots of the equation $z^4 = 2^{20}$.
c) What polygon will the roots of $z^5 = \omega$ create?
Any regular polygon with n sides can be divided into n congruent isosceles triangles. The diagram at right shows one such triangle.
(The diagram shows 9 sides, but you should consider a polygon with n sides.)



d) Give a reason why angle $\theta = \frac{2\pi}{n}$ for a polygon with n sides.
e) Show that the area of triangle AOB, for a polygon with n sides, is $A = \frac{1}{2}r^2 \sin\left(\frac{2\pi}{n}\right)$.
f) Hence find the area of the polygon created by the roots of $z^5 = 2^{20}$.
g) Hence find the area of the polygon created by the roots of $z^{10} = 2^{20}$.
h) Write a function $A(n)$ for the area of the polygon created by the roots of $z^n = 8$.

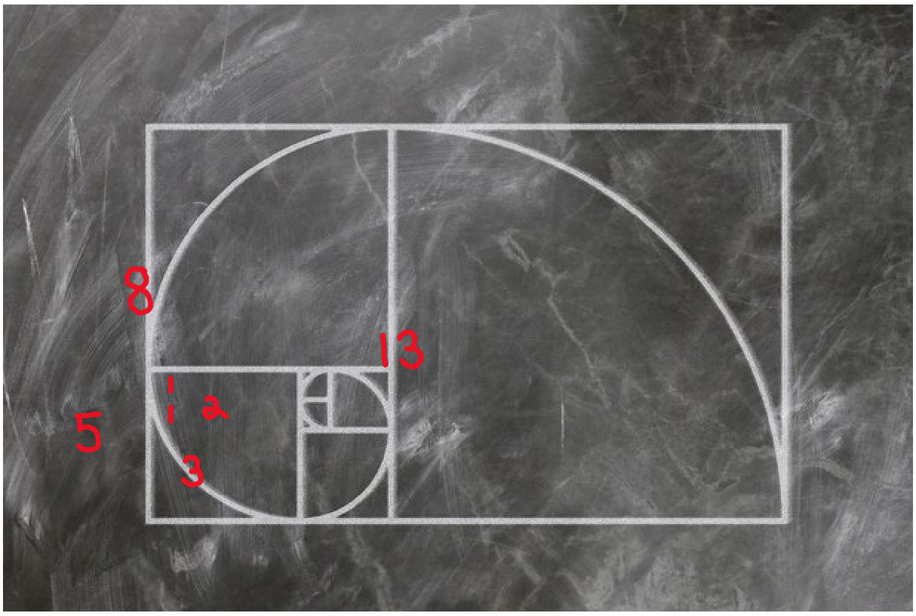
i) Conjecture the result of $\lim_{n\to\infty} A(n)$.

j) Give a geometric justification for your result.

VCE Unit 1 Specialist Mathematics

Fibonacci and Matrices Investigation

Technology Active



Section A – Recursive Formula

Question 1 [4 marks]

One of the most famous sequences is the Fibonacci sequence – 0, 1, 1, 2, 3, 5, 8, ...

In the sequence,

$F_0 = 0$

$F_1 = 1$

$F_2 = 1$

$F_3 = 2$ and so on

Most people work out the values of each term using the recursive formula:

To find a particular term, F_n, we add the previous two terms (F_{n-1} and F_{n-2}) together.

$$F_n = F_{n-1} + F_{n-2}$$

a) Verify this recursive formula works for the following Fibonacci numbers:

i) F_4

ii) F_5

iii) F_6

b) Find the value of F_{10}. Show all your working.

c) Why would it be difficult to find F_{50} using this method?

Section B – Using Matrices

Another method for calculating Fibonacci numbers makes use of matrices.

Let's define matrix $\boldsymbol{A}$ as $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$.

We'll set up a 2×1 matrix ($\boldsymbol{F}$) with the Fibonacci numbers $\boldsymbol{F_0}$ and $\boldsymbol{F_1}$ like this:

$$F = \begin{bmatrix} F_1 \\ F_0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Question 2 [6 marks]

a) Calculate the product of matrices $\boldsymbol{A}$ and $\boldsymbol{F}$.

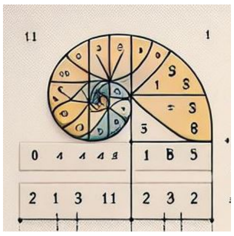
b) Multiply your result from part **a)** by matrix $\boldsymbol{A}$.
Show your working and interpret your result.

c) Multiply your result from part **b)** by matrix $\boldsymbol{A}$. Again, interpret your result.

d) Generalise your results to find a matrix expression that will allow you to calculate $\boldsymbol{F_n}$.

e) What advantages does this method have over the method shown in Section A?

Section C – The Golden Ratio



We can find the ratio between two successive terms of the Fibonacci sequence by taking $\frac{F_n}{F_{n-1}}$.

For instance, the ratio between F_3 and F_4 is $\frac{3}{2}$ which means that the value of F_4 is 1.5 times bigger than the value of F_3.

Question 3 [4 marks]

a) Calculate the ratio between F_n and F_{n-1} for $n = 2, 3, 4, 5, 6, 7$ and 8.
What do you notice?

b) Using the general Fibonacci term $\boldsymbol{F_n}$ found in **Section B**, find $\lim_{n\to\infty} \frac{F_{n+1}}{F_n}$.

You need to use the ratio of bigger numbers such as $\frac{F_{51}}{F_{50}}$ for example.

What do you notice about the value you found?

SM INVESTIGATING SERIES AND MATHEMATICAL INDUCTION

Please answer all questions showing full working and detailed explanations.

Tables and diagram may be helpful to see the patterns.

Question 1

The sequence of numbers is defined by $u_1 = 1\times1!,\ u_2 = 2\times2!, u_3 = 3\times3!,...$

Find the nth term of the sequence.

Question 2

Consider the sum $S_n = u_1 + u_2 + ... + u_n$.

Investigate S_n for different values of *n*.

Question 3

a) Conjecture an expression for S_n.
b) Verify that your conjecture works for the values of n investigated in Question 2.

Question 4

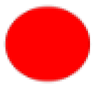
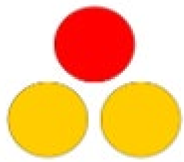
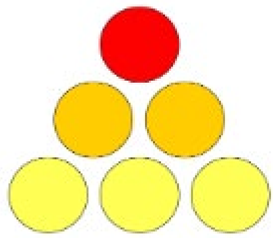
Use mathematical induction to prove your conjecture.

Polygonal Numbers

In mathematics, a polygonal number is a number represented by dots arranged in the shape of a regular polygon. For instance, square numbers are formed by dots arranged in a square, and triangular numbers form triangles.

Triangular Numbers

Consider the diagram below for the first three triangular numbers.

Number of dots per side	1	2	3
Shape			
Total number of dots	1	3	6

a. How many dots will you need to make the next triangular number?
b. **Find** the first 10 triangular numbers. **Summarize** your results in the table below.

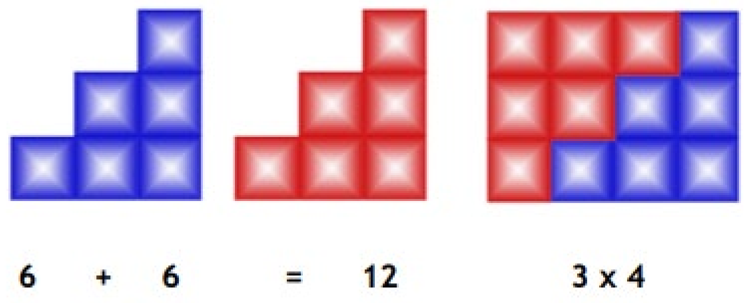
Triangular number	1	2	3	4	5	6	7	8	9	10
Number of dots	1	3	6							

c. Without doing any further drawings or calculation, **state your prediction** for the next two triangular numbers. **In words, describe** how many dots you are adding each time.
d. **Suggest** a recursive formula for the n^{th} triangular number, T_n. **Verify** your formula with known values, and your predicted values for T_6 and T_7. Include a table or diagram to **illustrate** how the number of dots added changes with each stage.

Take any triangular number.

- Make two copies of a triangular number
- Fit them together to make a rectangle

For example. The 3rd triangular number is 6.



e. Experiment with different triangular numbers. Record your observations in the table below.

Position	Triangle number	Double triangle number	Length of rectangle	Width of rectangle
1st triangular number	1			
2nd triangular number	3			
3rd triangular number	6	12	3	4
4th triangular number				
4th triangular number				
5th triangular number				
6th triangular number				
7th triangular number	28	56	7	8
8th triangular number				
9th triangular number				
10th triangular number				

f. Explain what is special about the length and width of rectangles made from two identical triangular numbers. Comment on any patterns you notice.

g. Predict the dimension of the rectangle that corresponds to two lots of the 250th triangular number. Verify your prediction.

h. Based on your results in f. and g., **derive** an explicit formula for the n^{th} triangular number. **Verify** your explicit formula using known results. Include a diagram to show how the formula relates to the triangular number pattern.

i. **Using** a method of your choice, **prove** your explicit formula for T_n is true for all $n \in N$.

j. Investigate what happens when you add consecutive triangular numbers. Experiment with different triangular numbers.

Triangle number	+	Triangle number	=	Answer
1	+	3	=	4
	+		=	
	+		=	
	+		=	
	+		=	

k. What do you **notice** about your results? **Comment** on anything special about your results. Include a diagram to **illustrate** your idea.

l. **Using** a method of your choice, **prove** the sum of two consecutive triangular numbers is a square number.

m. Hence, **prove** that the difference of two consecutive triangular numbers is the square root of the sum of two consecutive triangular numbers.

Extensions

Odds and Evens The first four triangular numbers are 1, 3, 6, 10. That is, (1) odd, (3) odd, (6) even, and (10) even. Investigate the pattern of odd and even triangular numbers.
Multiple Patterns

The table below shows the first eight triangular numbers which are multiples of three.

Triangular number	Multiples of 3
1	
3	Yes
6	Yes
1	
15	Yes
21	Yes
28	
36	Yes

Investigate the pattern in the multiples of three.
Extend your investigation to consider multiples of four, five, etc.

Pascal's Triangle

Continue this triangular arrangement of numbers.

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

Investigate for pattern in the arrangement. For example, one sloping line contains the counting numbers 1, 2, 3, 4, 5, …

Triangular sums

15 = 6+6+3
16= 6+10
17= 1+1+15
18= 3+15

Here 16 and 18 are written as the sum of two triangular numbers, and 15 and 17 are written as the sum of three triangular numbers.

Investigate other numbers which can be written as the sum of triangular numbers,

Square Numbers

36 is a triangular number and also a square number.

Are there any other numbers which are both triangular and square? Investigate.

Final Digits

The first five triangular numbers are 1, 3, 6, 10, 15.
Their digits are 1, 3, 6, 0, 5.

Investigate the pattern of final digits of triangular numbers.

Sum of triangular numbers → tetrahedral numbers

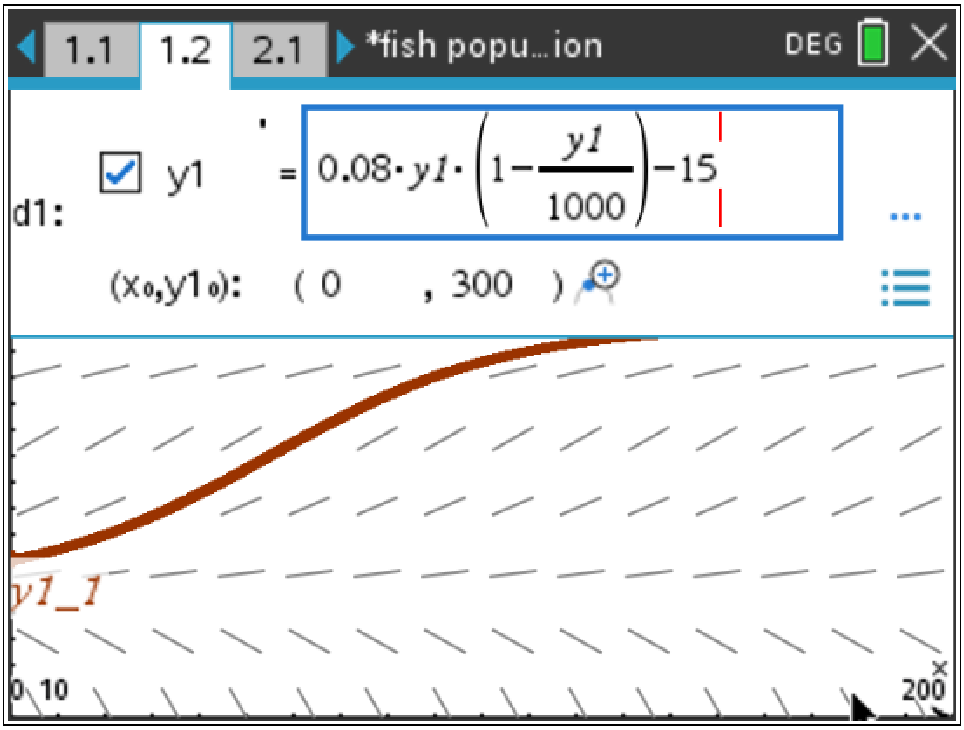
The sum of the first n triangular numbers is the nth tetrahedral number:

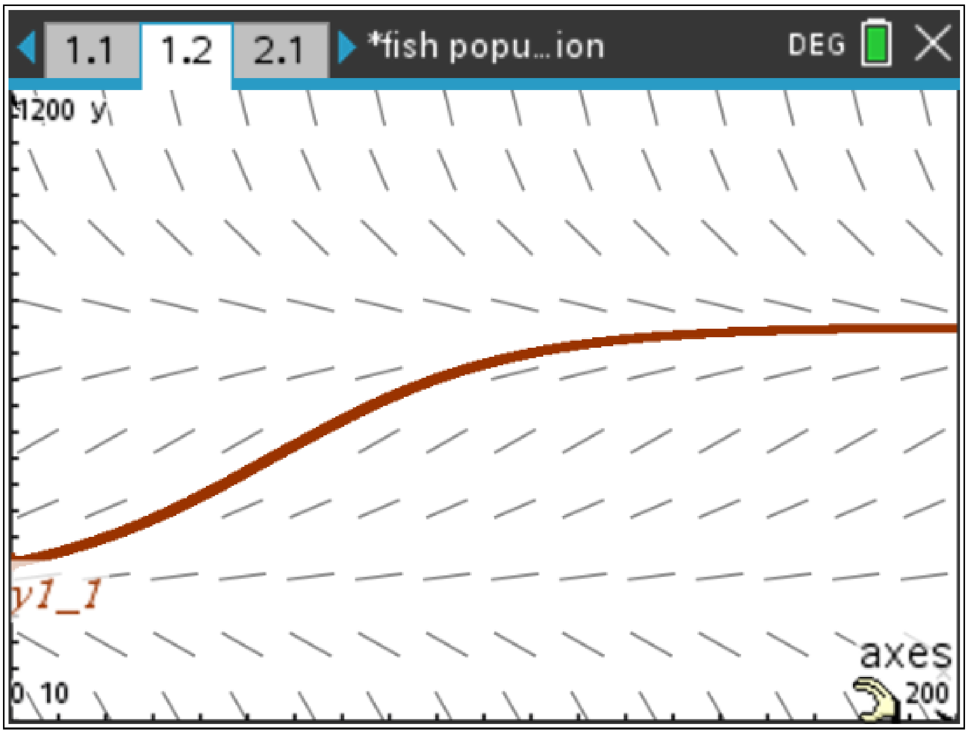
$$\sum_{k=1}^{n} T_k = \sum_{k=1}^{n} \frac{k(k+1)}{2} = \frac{n(n+1)(n+2)}{6}.$$

Modelling Task: Fish population slope fields (Adapted from Geese VCAA Sample Task)

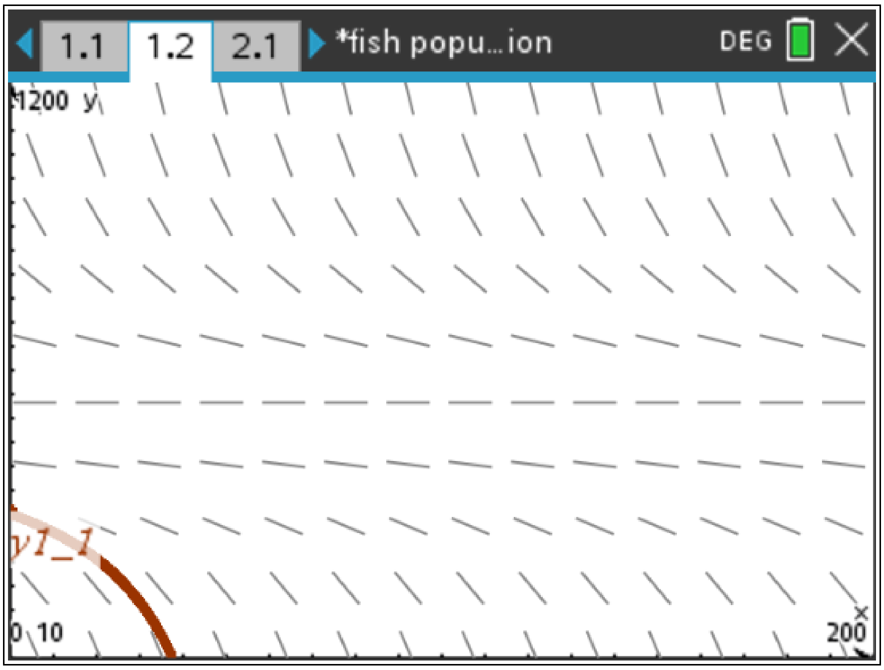
Some examples shown:

$P_0 = 300,\ c = 15$

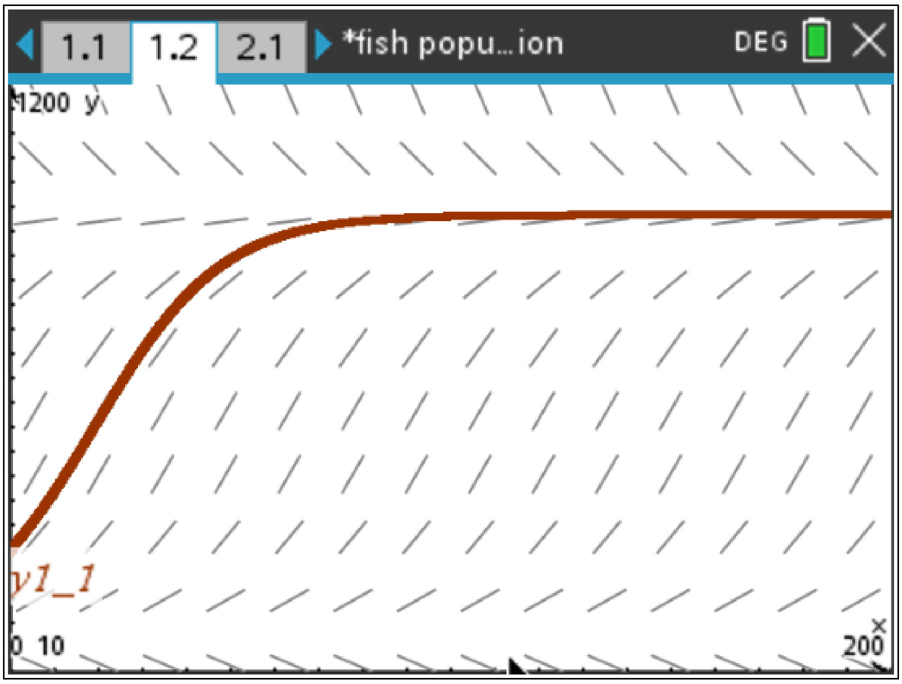




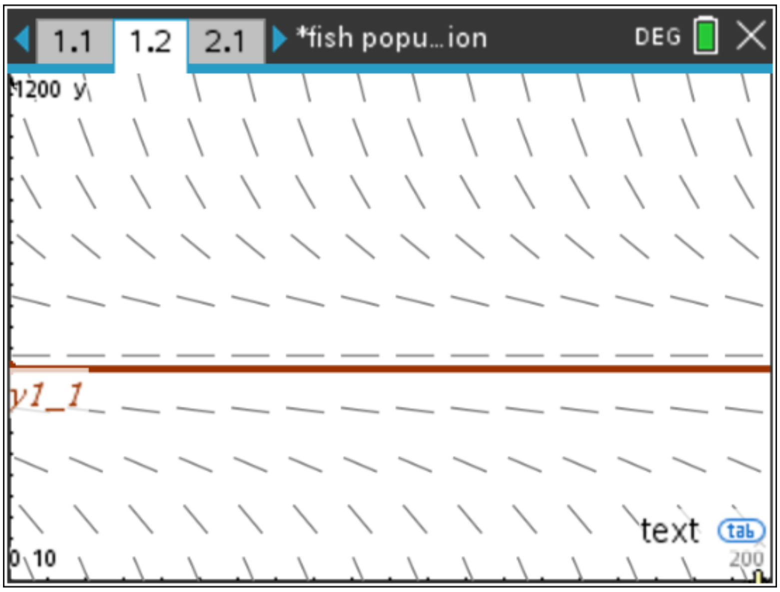
$P_0 = 300,\ c = 20$



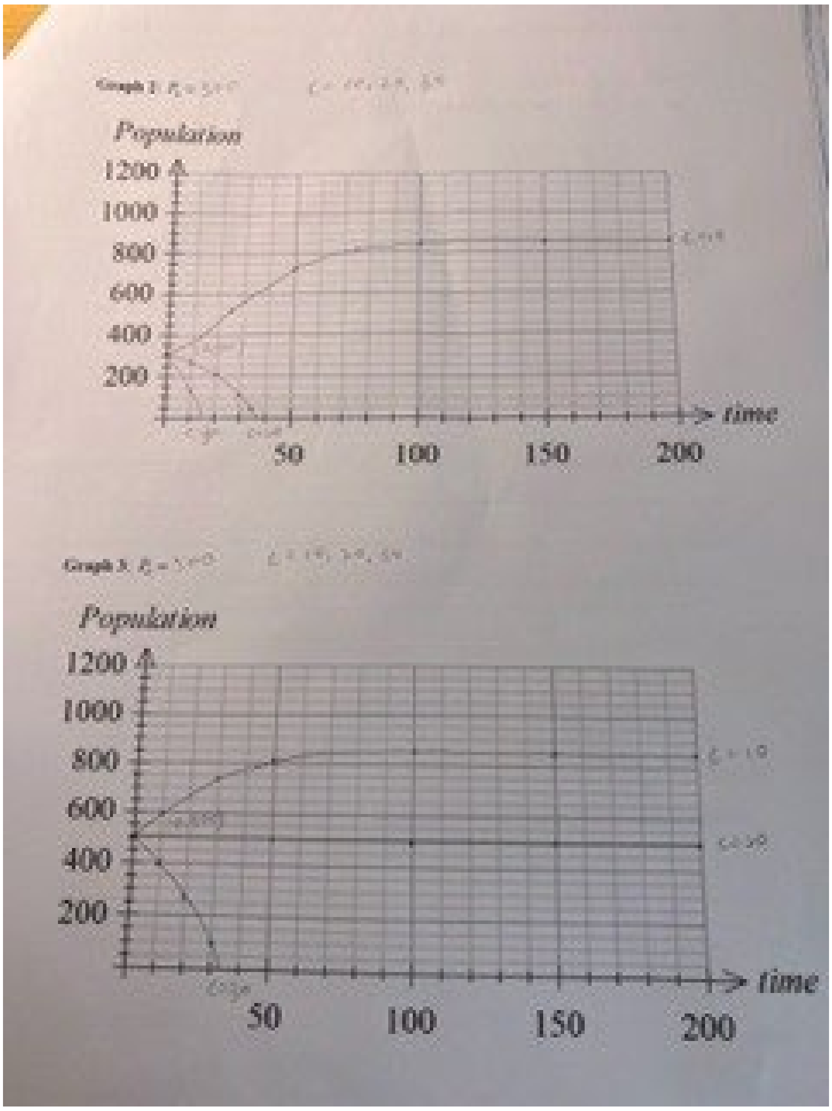
$P_0 = 250,\ c = 5$

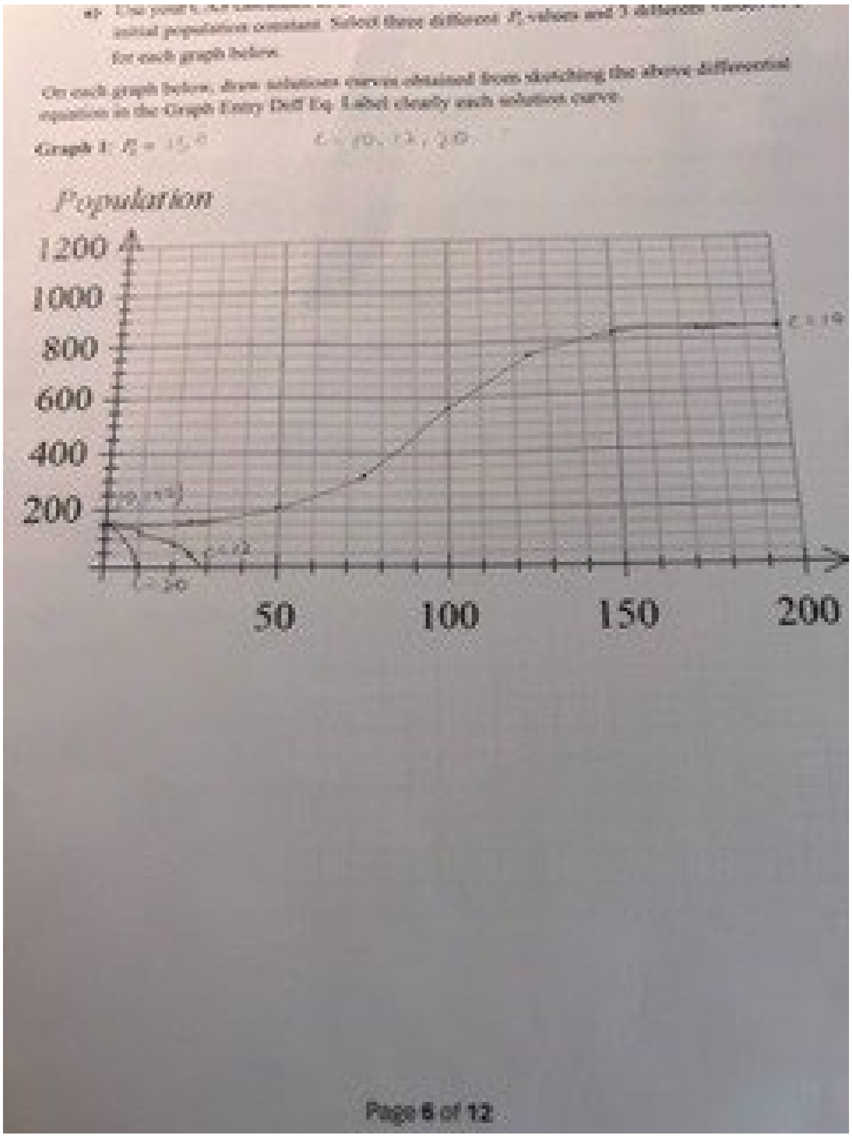


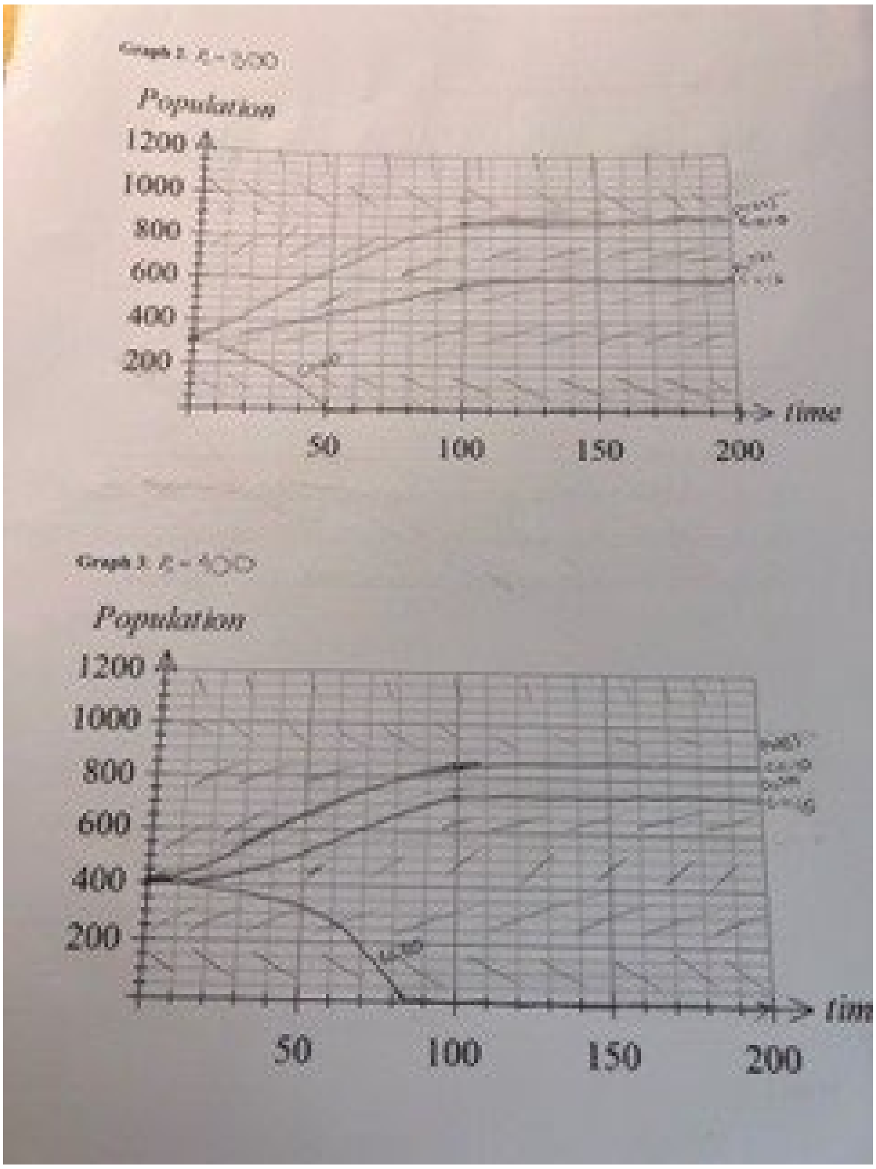
$P_0 = 500,\ c = 20$



Students' work examples:


Population
1200
1000
800
600
400
200
time
50
100
150
200
Population
1200
1000
800
600
400
200
time
50
100
150
200


initial population constant. Select three different
for each graph below.
On each graph below, draw solutions curves obtained from sketching the above differential equation in the Graph Entry Diff Eq. Label clearly each solutions curve.
Graph 1:
Population
1200
1000
800
600
400
200
50
100
150
200
Page 6 of 12



Graph 2: K = 300
Population
1200
1000
800
600
400
200
time
50
100
150
200
Graph 3: K = 400
Population
1200
1000
800
600
400
200
tim
50
100
150
200

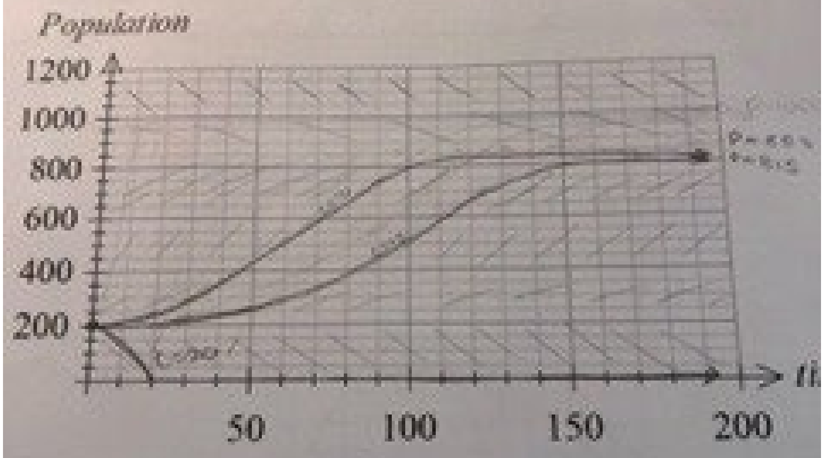
as a model for a fish population in a lake, where t is measured in weeks and c is the number of fish caught each week.

Assume the initial population in the range $P_0 \in [100, 500]$ and $c \in [10, 50]$.

a) Use your CAS calculator to draw direction fields for various values of c, keeping the initial population constant. Select three different P_0 values and 3 different values of c for each graph below.

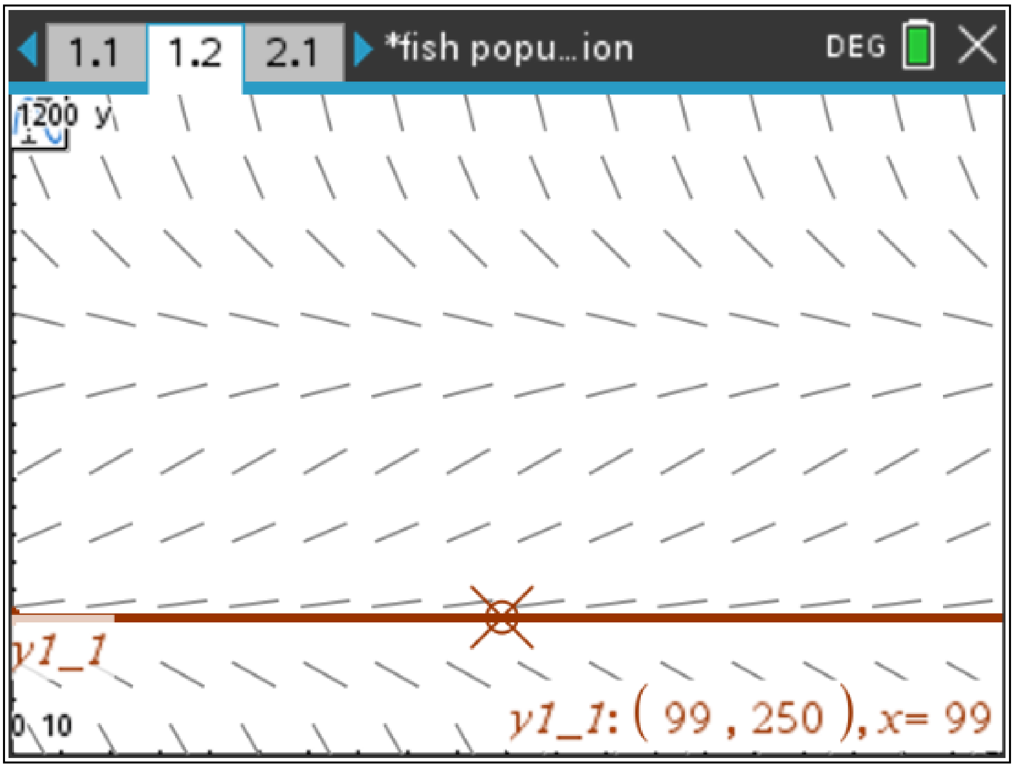
On each graph below, draw solutions curves obtained from sketching the above differential equation in the Graph Entry Diff Eq. Label clearly each solution curve.

Graph 1: $P_0 =$ 200



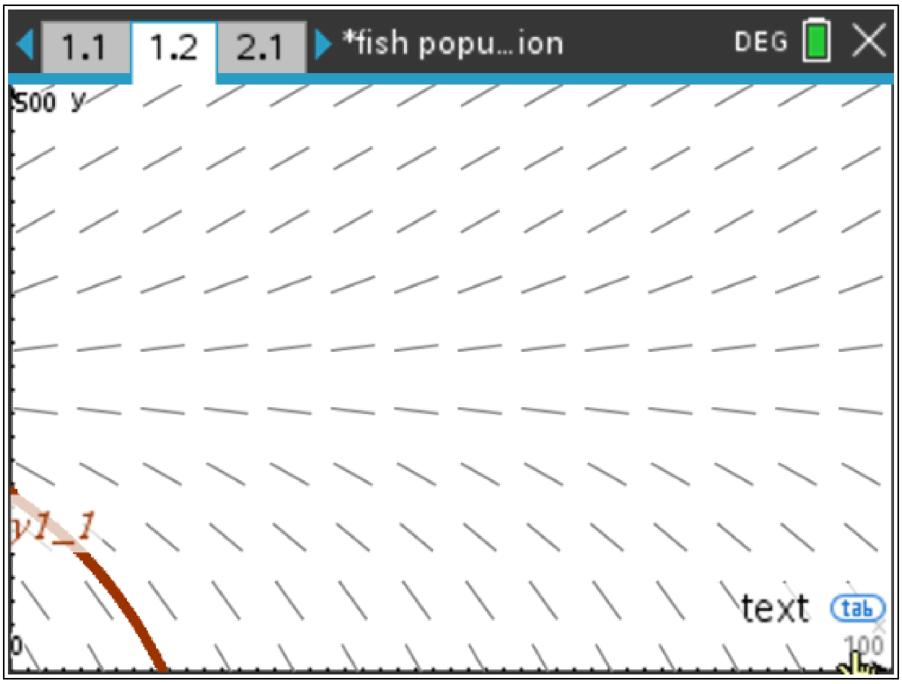
Question 3c

$P_0 = 250,\ c = 15$ the population stabilises at 250

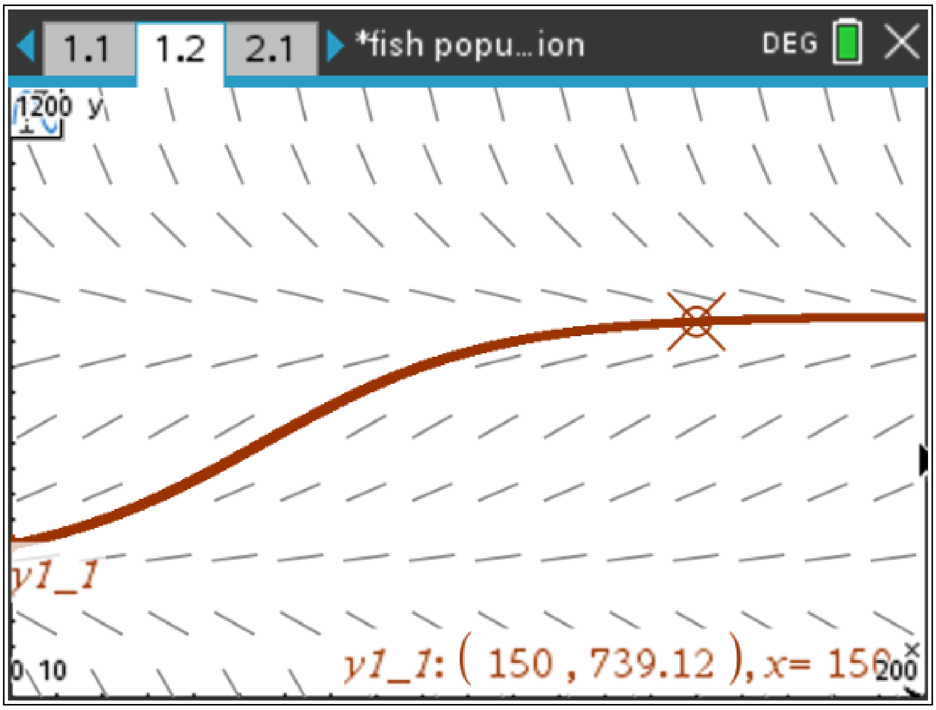


$P_0 = 150,\ c = 15$

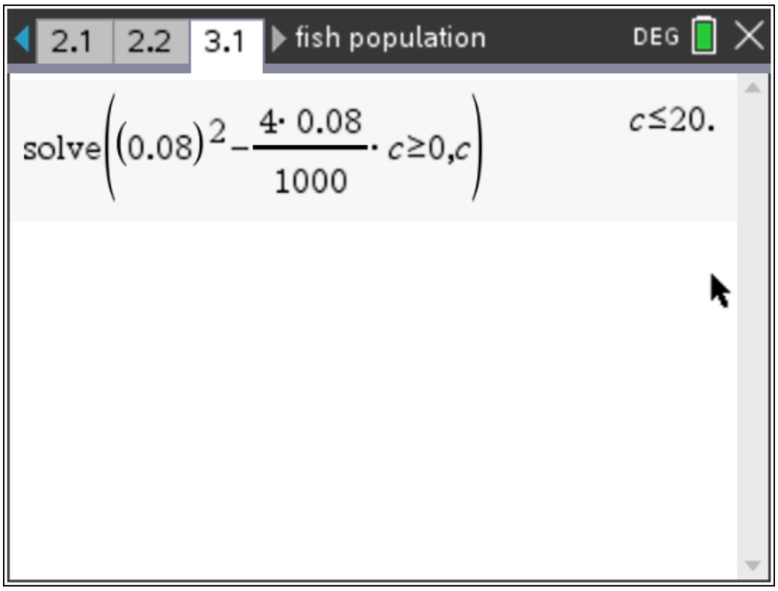
The fish population dies off



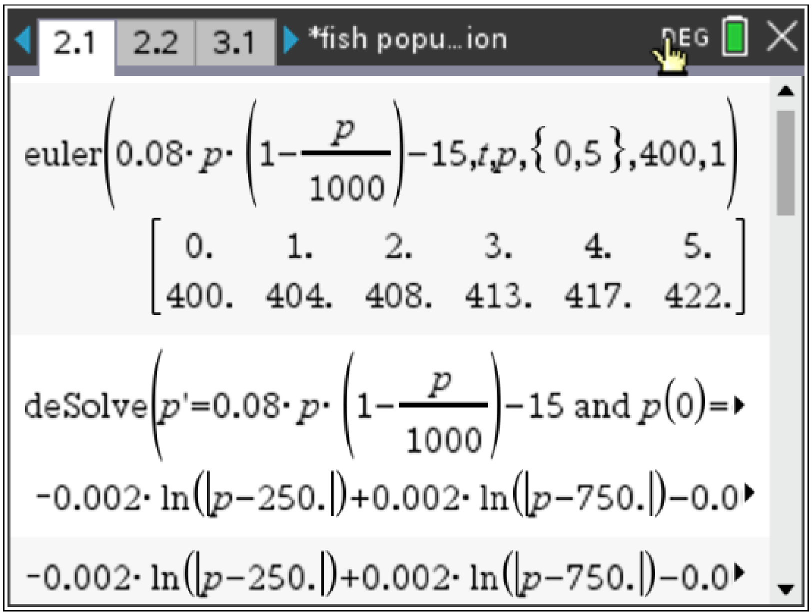
$P_0 = 300,\ c = 15$ the fish population reaches equilibrium

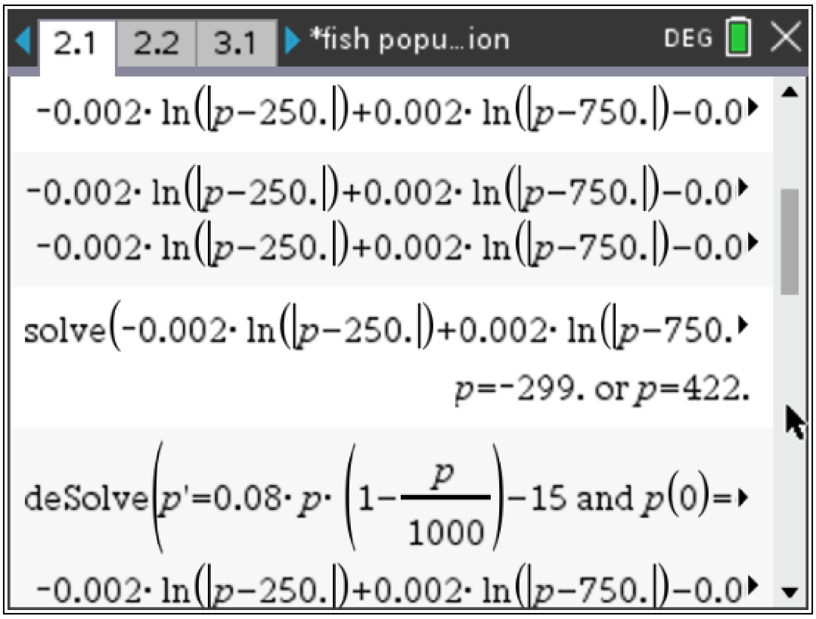


Q3f



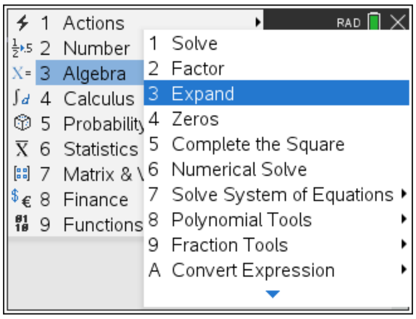
Q3h example

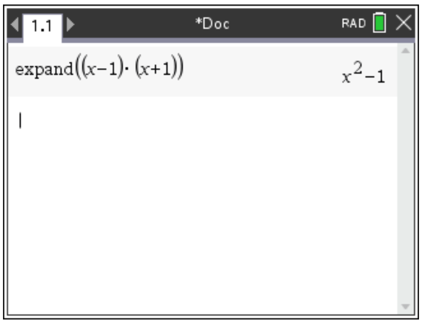

2.1 2.2 3.1 *fish popu...ion DEG
euler(0.08·p·(1−p/1000)−15,t,p,{0,5},400,1)
0. 1. 2. 3. 4. 5.
400. 404. 408. 413. 417. 422.
deSolve(p'=0.08·p·(1−p/1000)−15 and p(0)=
-0.002·ln(|p−250.|)+0.002·ln(|p−750.|)−0.0
-0.002·ln(|p−250.|)+0.002·ln(|p−750.|)−0.0


2.1 2.2 3.1 *fish popu...ion DEG
-0.002·ln(|p−250.|)+0.002·ln(|p−750.|)−0.0
-0.002·ln(|p−250.|)+0.002·ln(|p−750.|)−0.0
-0.002·ln(|p−250.|)+0.002·ln(|p−750.|)−0.0
solve(-0.002·ln(|p−250.|)+0.002·ln(|p−750.
p=-299. or p=422.
deSolve(p'=0.08·p·(1−p/1000)−15 and p(0)=
-0.002·ln(|p−250.|)+0.002·ln(|p−750.|)−0.0

Investigation: Seeing patterns in expanding $(x + a)^n$

To use the **expand** command, press [menu] , **3: Algebra, 3: Expand.**





To use the **expand** command, you input your expression.

expand(*expression*).

Question 1

a. Use CAS to expand the following expressions:

	Column 1		Column 2
$(x+1)^2$		$(x+a)^2$	
$(x+1)^3$		$(x+a)^3$	
$(x+1)^4$		$(x+a)^4$	

b. Without using CAS, anticipate the result of the expansion of $(x+1)^5$

c. Verify your anticipated result above in 1b. by using CAS to check your answer.

d. Without using CAS, anticipate the result of the expansion of $(x+2)^3$

e. Verify your anticipated result above in 1d. by using CAS to check your answer.

f. Without using CAS, anticipate the result of the expansion of $(2x+3)^3$

g. Verify your anticipated result above in 1f. by using CAS to check your answer.

Question 2

a. Write down the numbers of the coefficients obtained in column 1 in the form of a triangle.

The first row $(n = 2)$ has been done for you. Recall, n denotes the power of the bracket $(x + 1)^n$.

$(n = 0)$	1				
$(n = 1)$	1	1			
$(n = 2)$	1	2	1		
$(n = 3)$					
$(n = 4)$					

b. Can you predict what the numbers in row 5 $(n = 5)$ would be? Check your prediction using CAS.

c. What patterns do you notice in the triangle?

d. How do you think you can use the numbers in row 2 $(n = 2)$ to make the numbers in row 3 $(n = 3)$?

e. What are the similarities/differences between the coefficients in Column 1 and Column 2 in Question 1?

Question 3

a. Expand the following expressions and write your answers in the columns. Try to expand the brackets first 'by-hand' and the patterns you identified in Questions 2, then check your results using CAS.

	Column 1		Column 2
$(a+b)^2$		$(a-b)^2$	
$(a+b)^3$		$(a-b)^3$	
$(a+b)^4$		$(a-b)^4$	

b. What are the similarities/differences between the coefficients in Column 1 and Column 2?

c. Without using CAS, anticipate the result of the expansion of $(2x-1)^5$

d. Verify your anticipated result above in 3c. by using CAS to check your answer.

e. Complete the following table.

	Expansion	Number of terms in the expansion	Sum of the coefficients	Sum of the indices in each term
$(a+b)^2$				
$(a+b)^3$				
$(a+b)^4$				
$(a-b)^2$				
$(a-b)^3$				
$(a-b)^4$				

f. What do you notice about the number of terms in each of the expressions and the corresponding power of the bracket in the expression $(a+b)^n$? How does n relate to the number of terms in the expansion?

g. What do you notice about the sum of the coefficients in each of the expansions? Can you describe, or find a relationship, between the sum of the coefficients and the expression $(a \pm b)^n$? Provide an example to verify your idea/claim. (Hint: consider the case $(a - b)^n$ and the case $(a + b)^n$ separately).

h. What do you notice about the sum of the indices in each term and each of the expansions? Can you describe, or find a relationship, between the sum of the indices in each term and the expression $(a \pm b)^n$? Provide an example to verify your idea/claim.

Investigation: Making better boxes and barrels

Overview of the task and breakdown of components

- **Component 1**
    - Maximizing the volume of an open-top box (classic problem) with consideration of domain restrictions
    - Consideration of the amount of materials that have been used to create an open-top box
    - Comparing the volume and amount of materials used between a box made of a rectangular sheet of cardboard and a square sheet of cardboard
- **Component 2**
    - Maximizing the volume of a closed-top box with consideration of domain restrictions, and comparing with an open-top box
    - Optimising the volume by varying the length and width with a fixed height, and then optimising the volume by varying the height and fixing the width and length
    - Writing an algorithm using pseudocode to describe the process of finding the volume of a box with the given information and/or optimising the volume of the box
- **Component 3**
    - Using 2 methods to constructing a cylinder from a sheet of metal, and comparing the volume and amount of material wasted.
    - Writing an algorithm using pseudocode to describe the process of finding the volume of a cylinder with the given information, and the corresponding dimensions and materials wasted
    - Finding the dimensions of a cylinder that would maximise volume and minimise the cost of materials

Please refer to the pdf file

Complex Spirals Investigation

Teachers Teaching with Technology
Professional Development from Texas Instruments

Teacher Notes

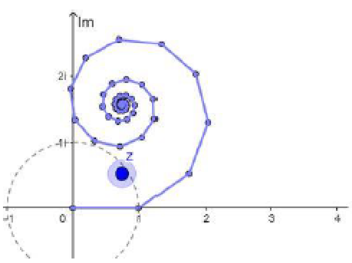
7 8 9 10 **11** **12**

TI-Nspire™ Activity Student 4 hours

Introduction

Complex numbers can be expressed in different forms. We can also create a geometric series consisting of the powers of complex numbers. When we graph the power series of complex numbers on the Argand diagram, we can observe various patterns. You will use your TI Nspire CX II CAS to investigate those patterns and establish the conditions under which they occur.

This document outlines reasoning behind the activity associated with the Complex Spirals Guided Exploration.



Please refer to the pdf file