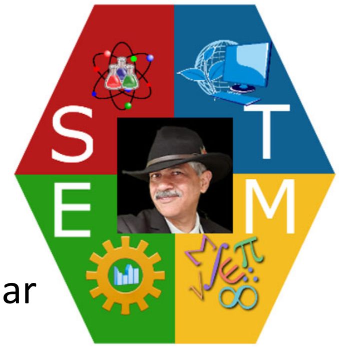
Sanjeev Meston

Strathcona Girls Grammar



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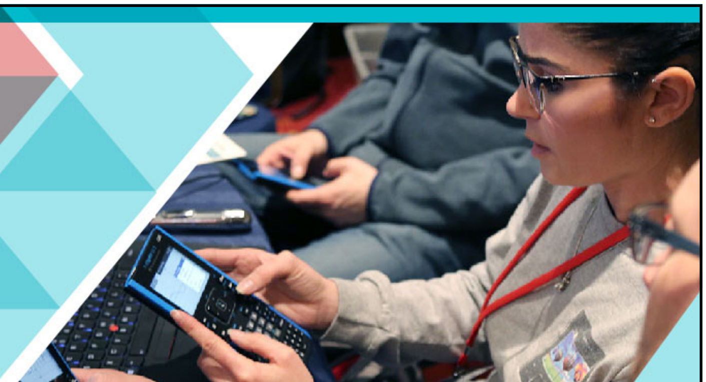
2025

Learn
Energise
Connect



Beyond the button: unlocking conceptual depth with CAS in Mathematical Methods

Sanjeev Meston

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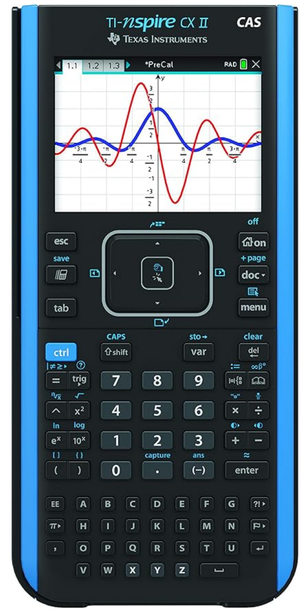
Unlocking Conceptual Depth with TI Nspire CX-II CAS In Mathematical Methods



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Widgets / User Defined Functions

1. Select all (CtrlA)
2. Under Actions: Evaluate Selection
3. Change coordinates

$\{3,17\}\to p \quad \{8,12\}\to q$

(3 17) (8 12)

slope: $^{-}1$

dist: $5\cdot\sqrt{2}$ approx 7.07

eq: $y=20-x$

eq2: $x+y-20=0$

$\{3,4\}\to p \quad \{8,{}^{-}12\}\to q$

(3 4) (8 $^{-}12$)

slope: $\frac{-16}{5}$

dist: $\sqrt{281}$ approx 16.76

eq: $y=\frac{68}{5}-\frac{16\cdot x}{5}$

eq2: $16\cdot x+5\cdot y-68=0$

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udfset1\coordg(3,4,8,-12)

"mp"	$x=\frac{11}{2}$ and $y=-4$
"dist"	$\sqrt{281}$
"slope : m"	$\frac{-16}{5}$
"y=mx+c"	$y=\frac{68}{5}-\frac{16\cdot x}{5}$
"Ax+By+C=0"	$16\cdot x+5\cdot y-68=0$
"angle in dd"	$\frac{2147}{20}$

$\{3,4\}\to p \quad \{8,-12\}\to q$

(3 4) (8 -12)

slope: $\frac{-16}{5}$

dist: $\sqrt{281}$ approx 16.76

eq: $y=\frac{68}{5}-\frac{16\cdot x}{5}$

eq2: $16\cdot x+5\cdot y-68=0$

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udfset1\avrcandavf()

func ln(3*x−1)+3
lv 5
uv 6

avrc$=\ln\left(\frac{17}{14}\right)$ or avrc =0.19

avf$=\frac{17\cdot\ln(17)}{3}-\frac{14\cdot\ln(14)}{3}+2$ or avf =5.74

Done

udfset1\avrcandavf()

func 5x^2−2x+11
lv 3
uv 6
avrc=43 or avrc =43.
avf=107 or avf =107.

Done

Average value and Average rate

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6

$\text{zeros}(5\cdot x^2-3\cdot x-11,x)$ $\left\{\frac{-(\sqrt{229}-3)}{10},\frac{\sqrt{229}+3}{10}\right\}$

$\text{zeros}(5\cdot x^2-3\cdot x-11,x)$ $\{-1.21327,1.81327\}$

udfset1\newtons()

init x val 3

Iterations 6

function $5\cdot x^2-3\cdot x-11$

derivative $10\cdot x-3$

[1 2.07407]
[2 1.83244]
[3 1.81339]
[4 1.81327]
[5 1.81327]
[6 1.81327]

Done

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7

udfset1\domandran()

function ln(5*x)−3

$\ln(5\cdot x)-3$

domain f(x) ▸ $0<x<\infty$

Inv of f(x)▸ $y=\frac{e^{x+3}}{5}$

Range of f(x) ▸ $-\infty<y<\infty$

if the domain of the original function is a union of two or more domains, check the range for each separately

Done

udfset1\domandran()

function 3*x^2−2*x

$3\cdot x^2-2\cdot x$

domain f(x) ▸ $-\infty<x<\infty$

Inv of f(x)▸ $y=\frac{\sqrt{3\cdot x+1}+1}{3}$ or $y=\frac{-(\sqrt{3\cdot x+1}-1)}{3}$

Range of f(x) ▸ $\frac{-1}{3}\leq y<\infty$

if the domain of the original function is a union of two or more domains, check the range for each separately

Done

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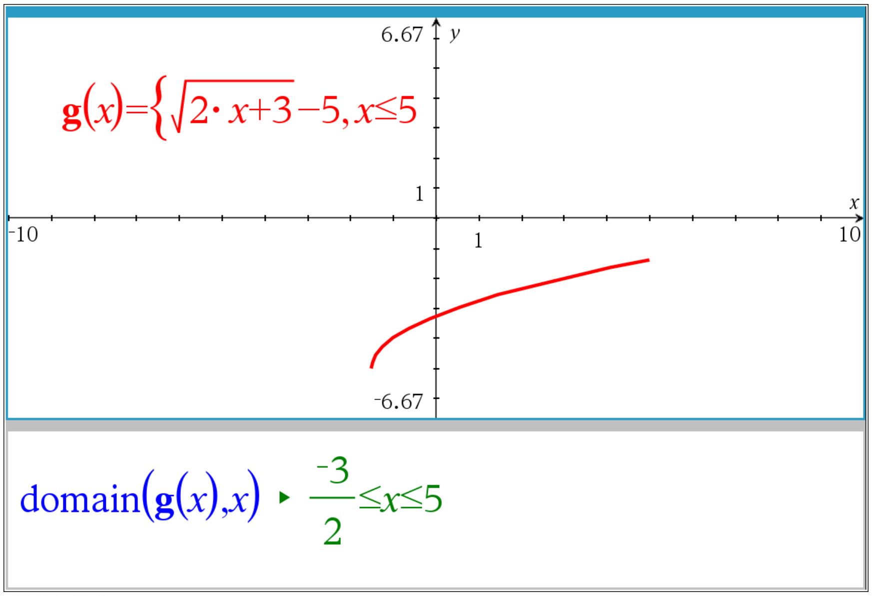
Domain of a Function



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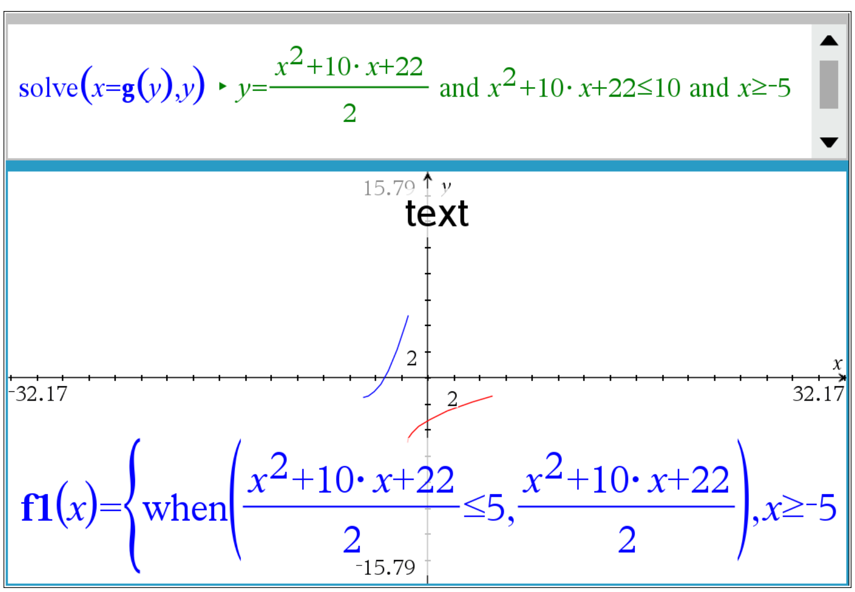
Domain of a Function and its Inverse



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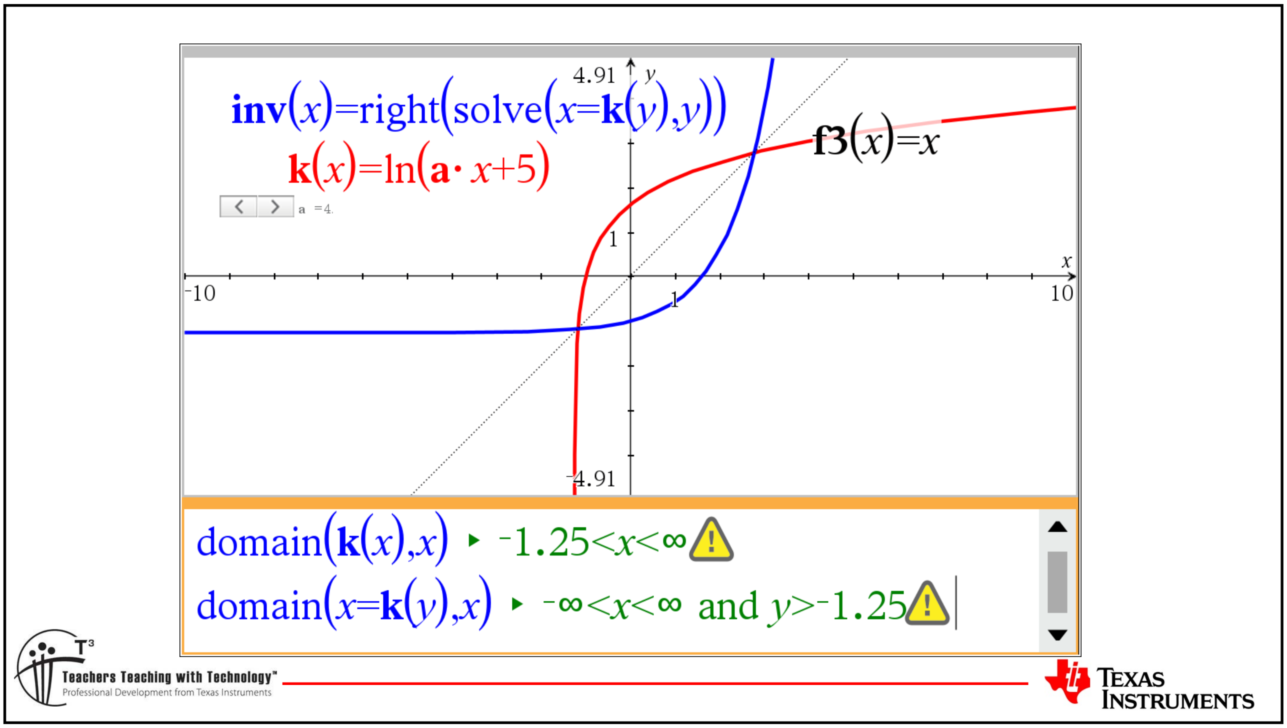

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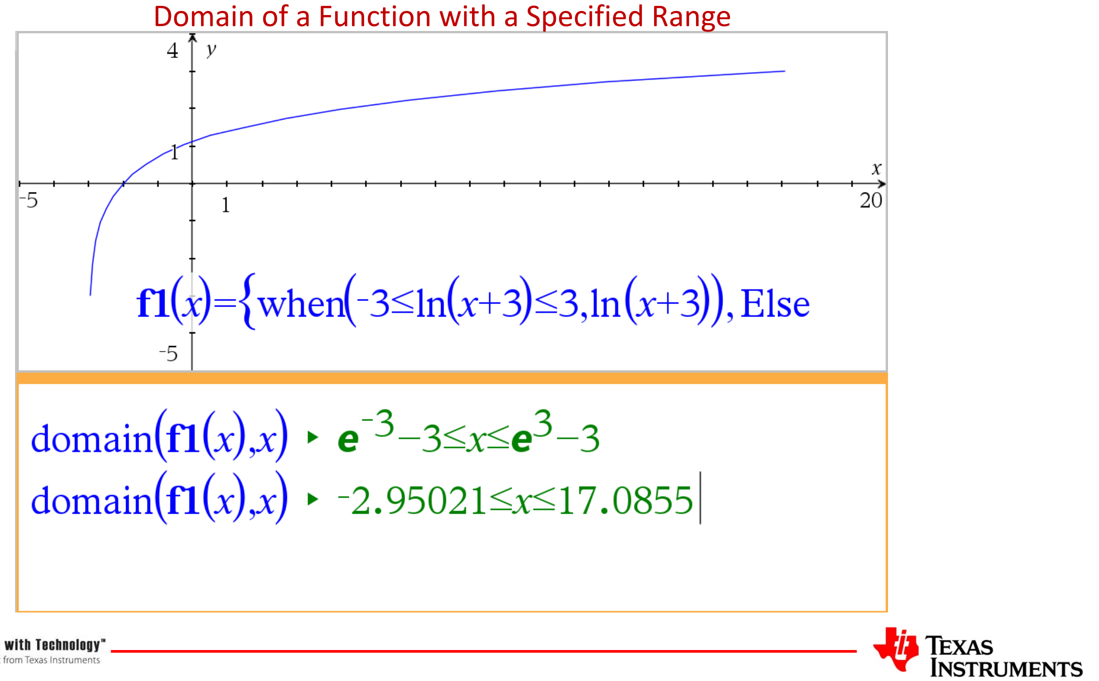
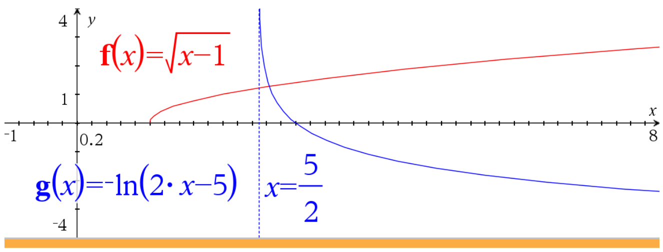
Domain of a Function with a Specified Range



12



	D	R
f	$x\ge 1$	$[0,\infty)$
g	$x>\frac{5}{2}$	$(-\infty,\infty)$

$f(g(x))$ $\quad R_g \not\subset D_g$ $\quad (-\infty,\infty)\cap[1,\infty)=[1,\infty)$

$g(f(x))$ $\quad R_f \not\subset D_f$ $\quad [0,\infty)\cap\left(\frac{5}{2},\infty\right)=\left(\frac{5}{2},\infty\right)$

$\text{domain}(f(x),x) \text{ and } \text{domain}(g(x),x) ▸ \frac{5}{2}<x<\infty$

$\text{domain}(f(x),x) \text{ or } \text{domain}(g(x),x) ▸ 1\le x<\infty$

$\text{domain}(f(x),x) \text{ nor } \text{domain}(g(x),x) ▸ x<1 \text{ or } x=\infty$

$\text{domain}(f(x)+g(x),x) ▸ \frac{5}{2}<x<\infty$

$\text{domain}(f(g(x)),x) ▸ \frac{5}{2}<x\le\frac{(5\cdot e+1)\cdot e^{-1}}{2}$

$\text{domain}(g(f(x)),x) ▸ \frac{29}{4}<x<\infty$

$-\infty<x<\infty \text{ and } 1\le x<\infty ▸ 1\le x<\infty$

$\text{domain}(\text{when}(1<g(x)<\infty,g(x)),x) ▸ 2.5<x<2.68394$

$\text{domain}(\text{when}(1<g(x)<\infty,g(x)),x) ▸ \frac{5}{2}<x<\frac{(5\cdot e+1)\cdot e^{-1}}{2}$

$0\le x<\infty \text{ and } \frac{5}{2}<x<\infty ▸ \frac{5}{2}<x<\infty$

$\text{domain}\left(\text{when}\left(\frac{5}{2}<f(x)<\infty,f(x)\right),x\right) ▸ \frac{29}{4}<x<\infty$

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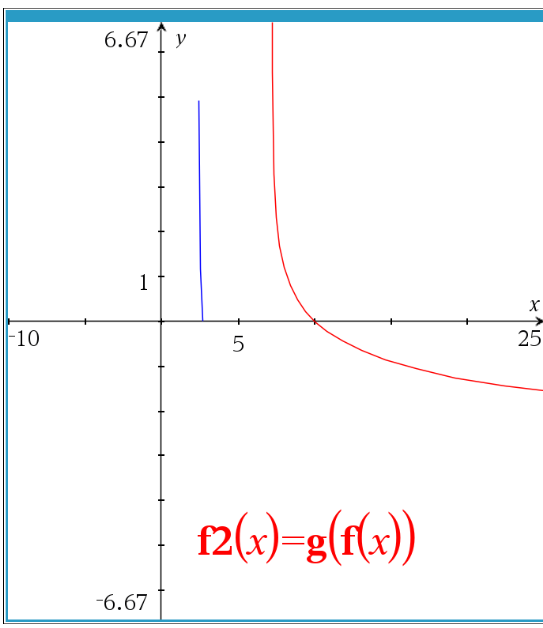

$f1(x) ▸ \sqrt{-\ln(2\cdot x-5)-1}$

$f2(x) ▸ -\ln(2\cdot\sqrt{x-1}-5)$

$\text{domain}(f1(x),x)$

$▸ \frac{5}{2}<x\le\frac{(5\cdot e+1)\cdot e^{-1}}{2}$

$\text{domain}(f1(x),x)$

$▸ 2.5<x\le 2.68394$

$\text{domain}(f2(x),x)$

$▸ \frac{29}{4}<x<\infty$

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Storing Functions and Variable and working with stored variables

$\mathbf{f}(a) \blacktriangleright 4\cdot a^3-7 \quad \mathbf{f}(\pi) \blacktriangleright 4\cdot \pi^3-7 \quad \mathbf{f}(^-2) \blacktriangleright {}^-39$

$x+2 \rightarrow \mathbf{g}(x) \blacktriangleright \textit{Done}$

$\mathbf{m}(x):=3\cdot x-2 \blacktriangleright \textit{Done}$

$\frac{\mathbf{m}(x)}{\mathbf{g}(x)} \blacktriangleright \frac{3\cdot x-2}{x+2} \quad \text{propFrac}\left(\frac{\mathbf{m}(x)}{\mathbf{g}(x)}\right) \blacktriangleright 3-\frac{8}{x+2}$

$\text{domain}\left(\frac{\mathbf{m}(x)}{\mathbf{g}(x)},x\right) \blacktriangleright x\neq {}^-2$

$\mathbf{b}:=\mathbf{e}^2 \blacktriangleright \mathbf{e}^2 \quad \mathbf{m}(\mathbf{b}) \blacktriangleright 3\cdot \mathbf{e}^2-2$

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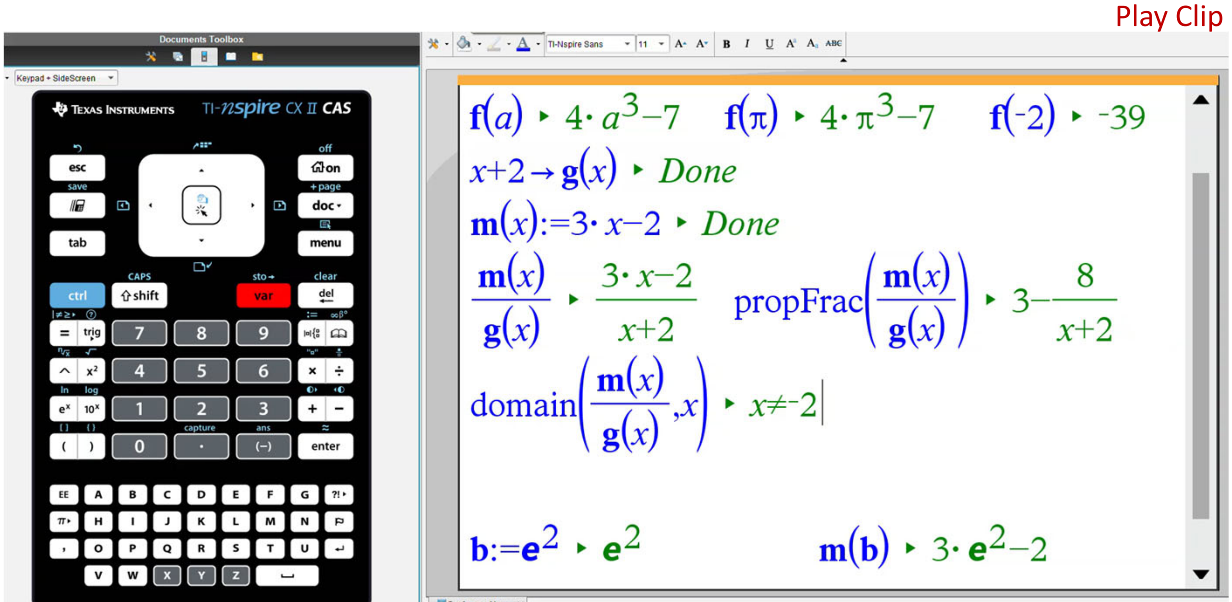
Play Clip



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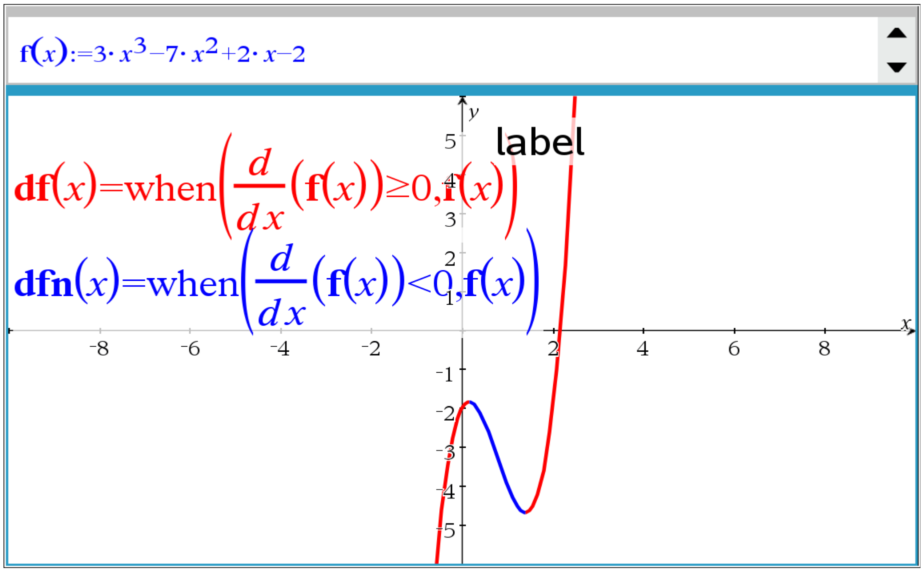
Sketching Graphs with conditions



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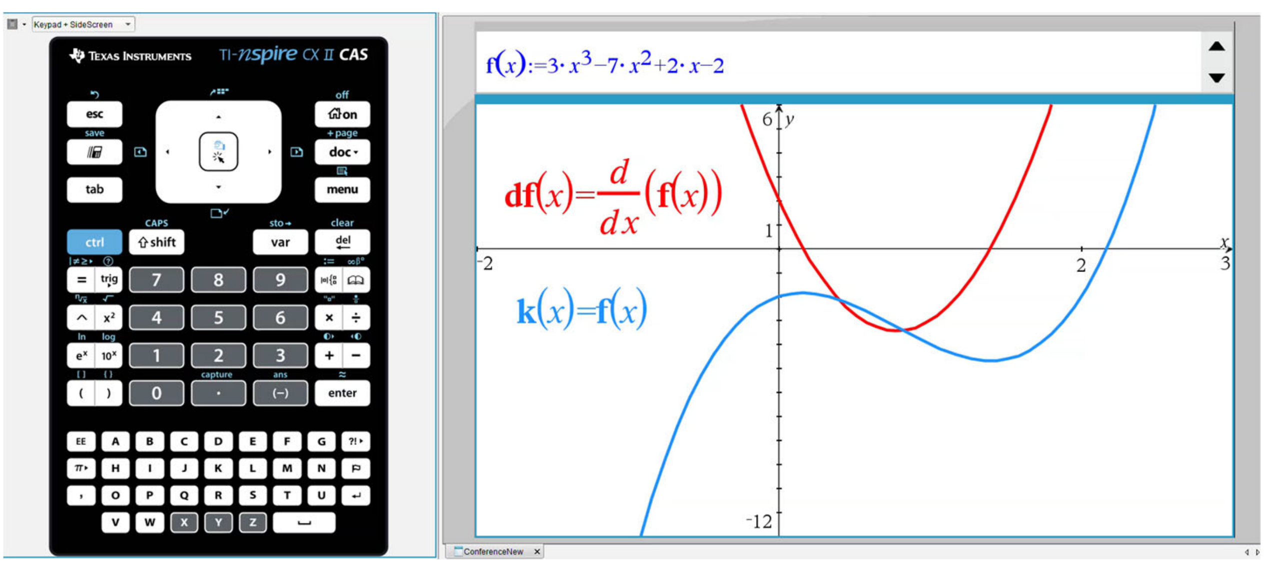

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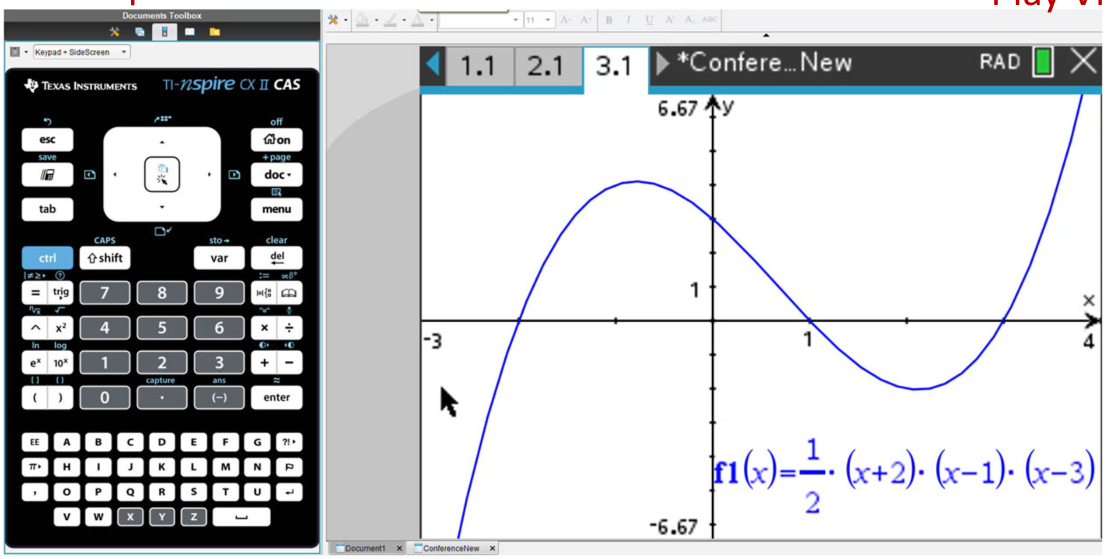
Data capture to understand derivative curves

Play Video



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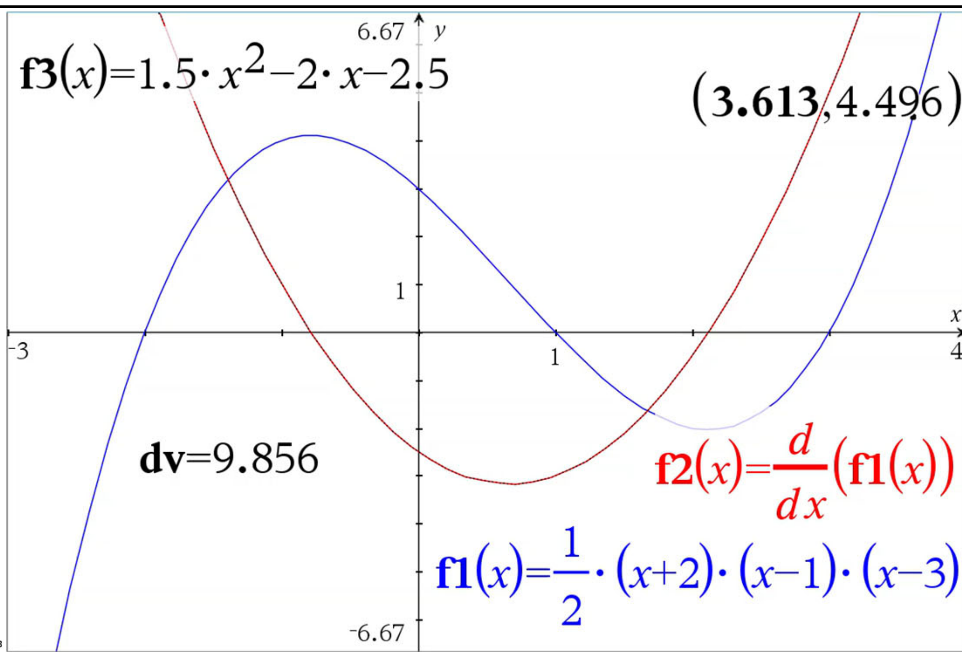

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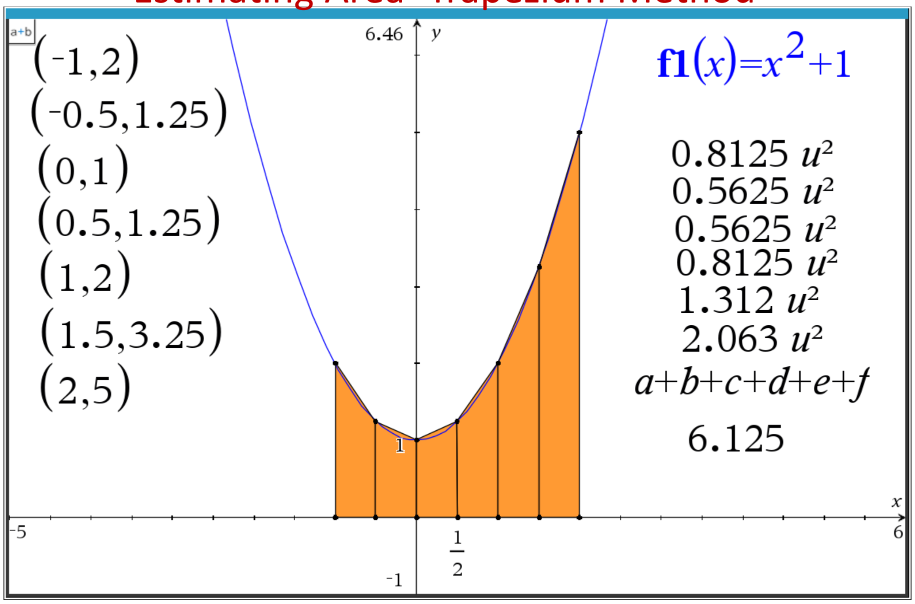
Estimating Area- Trapezium Method



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$$\text{seq}\left(-1+\frac{1}{2}\cdot n,n,0,6\right) \blacktriangleright \left\{-1,\frac{-1}{2},0,\frac{1}{2},1,\frac{3}{2},2\right\}$$

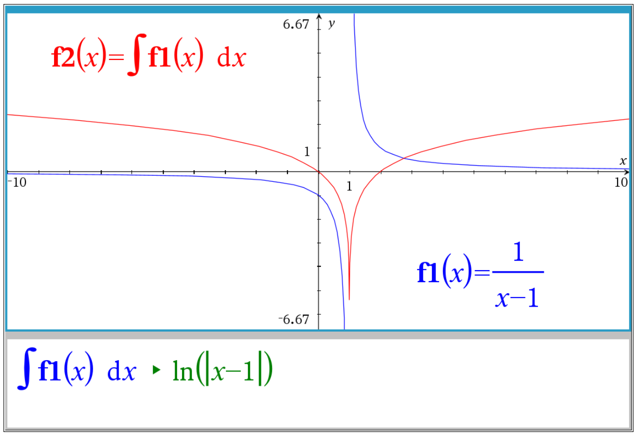
$$\text{seq}\left(x^2+1,x,-1,1.5,0.5\right)\to \textbf{seq1} \blacktriangleright \left\{2,\frac{5}{4},1,\frac{5}{4},2,\frac{13}{4}\right\}$$

$$\text{seq}\left(x^2+1,x,-0.5,2,0.5\right)\to \textbf{seq2} \blacktriangleright \left\{\frac{5}{4},1,\frac{5}{4},2,\frac{13}{4},5\right\}$$

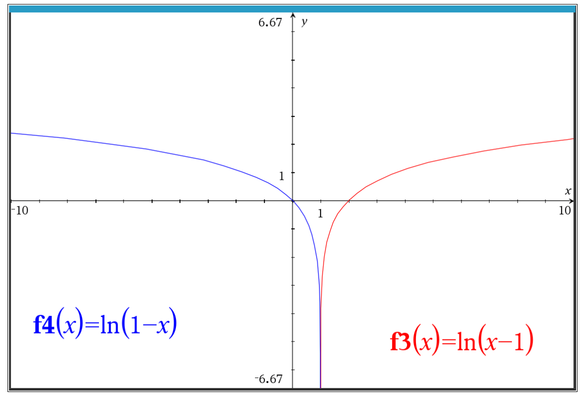
$$\textbf{seq1}+\textbf{seq2} \blacktriangleright \left\{\frac{13}{4},\frac{9}{4},\frac{9}{4},\frac{13}{4},\frac{21}{4},\frac{33}{4}\right\}$$

$$\frac{\frac{\text{sum}(\textbf{seq1}+\textbf{seq2})\cdot 1}{2}}{2} \blacktriangleright 6.125$$

22



Understanding Antiderivatives



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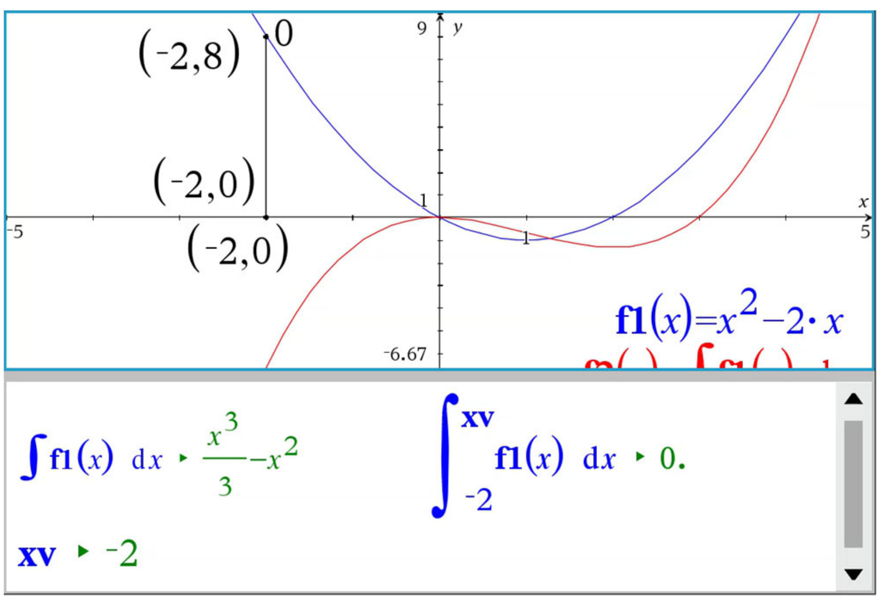

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Analysing Antiderivatives

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Newtons Method- Non programming method

evaluate $\sqrt[4]{11^3}$ using Newtons Method

$x=11^{\frac{3}{4}}$ ▸ $x=11^{\frac{3}{4}}$ or $x^{\frac{4}{3}}=11$ ▸ $x^{\frac{4}{3}}=11$

$x^{\frac{4}{3}}-11=0$

$\mathbf{f}(x):=x^{\frac{4}{3}}-11$ ▸ *Done*

$\mathbf{a}:=1$ ▸ 1

$\frac{d}{dx}(\mathbf{f}(x))\rightarrow \mathbf{g}(x)$ ▸ *Done*

$x-\frac{\mathbf{f}(x)}{\mathbf{g}(x)}\rightarrow \mathbf{n}(x)$ ▸ *Done*

$\mathbf{n}(\mathbf{a})$ ▸ 8.5

$\mathbf{n}(\mathbf{n}(\mathbf{a}))$ ▸ 6.1674777665

$\mathbf{n}(\mathbf{n}(\mathbf{n}(\mathbf{a})))$ ▸ 6.04054580569

$\mathbf{n}(\mathbf{n}(\mathbf{n}(\mathbf{n}(\mathbf{a}))))$ ▸ 6.04010535989

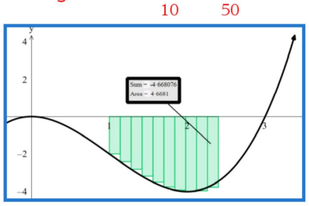
$\mathbf{n}(\mathbf{n}(\mathbf{n}(\mathbf{n}(\mathbf{n}(\mathbf{a})))))$ ▸ 6.04010535454

25

Find the approximate Area enclosed by the $f(x)=x^3-3x^2$ between x=1 and x=2.4 using **10 rectangles** (Left Edge)

$\mathbf{f}(x):=x^3-3\cdot x^2$ ▸ *Done* $\frac{24}{10}$ ▸ $\frac{12}{5}$

Rectangle width $=\frac{2.4-1}{10}$ ▸ $\frac{7}{50}$



▸

$\text{seq}\left(\mathbf{f}(x),x,1,2.4-\frac{7}{50},\frac{7}{50}\right)$

▸ $\left\{-2,\frac{-302157}{125000},\frac{-44032}{15625},\frac{-398239}{125000},\frac{-54756}{15625},\frac{-3757}{1000}\right\}$

$\frac{7}{50}\cdot \text{sum}\left(\text{seq}\left(\mathbf{f}(x),x,1,2.4-\frac{7}{50},\frac{7}{50}\right)\right)$ ▸ -4.66808

An Alternative option to using $\sum_{x=3}^{6}\left(x^2-3\cdot x\right)$ ▸ 32 (This is in step size of 1). **What if I need to do in step size of 0.5**

$\text{seq}\left(x^2-3\cdot x,x,3,6,0.5\right)$ ▸ $\left\{0,\frac{7}{4},4,\frac{27}{4},10,\frac{55}{4},18\right\}$

$\text{sum}\left(\text{seq}\left(x^2-3\cdot x,x,3,6,0.5\right)\right)$ ▸ **54.25**

$\text{sum}\left(\text{seq}\left(x^2-3\cdot x,x,3,6,1\right)\right)$ ▸ **32**

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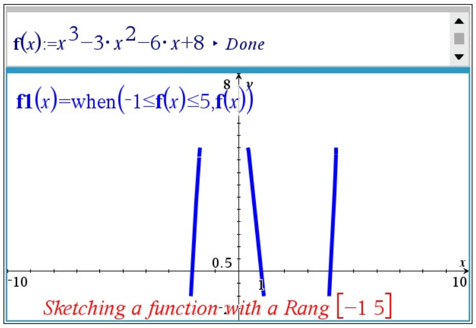

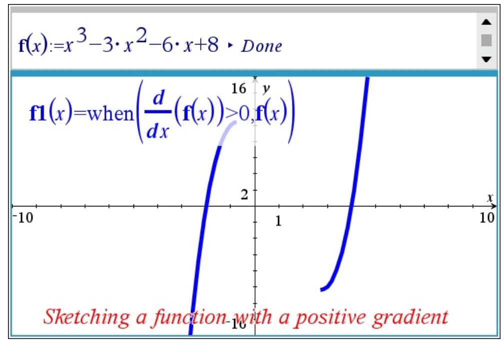

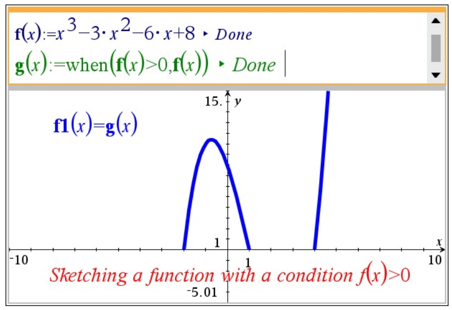

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Derivative at a Point, Equation of the Tangent at that point

$f(x):=e^{5\cdot x-2}$ ▸ *Done*

$\frac{d}{dx}(f(x))|x=4$ ▸ $5\cdot e^{18}$ Derivative at x=4

$\text{tangentLine}(f(x),x=4)$ ▸ $5\cdot e^{18}\cdot x-19\cdot e^{18}$ Tangent at x=4

Normal at x=4

$y=\text{normalLine}(f(x),x=4)$ ▸ $y=\frac{(5\cdot e^{36}+4)\cdot e^{-18}}{5}-\frac{e^{-18}\cdot x}{5}$

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1.1 *Doc RAD

Example 1: $f(x):=\sqrt{3\cdot x-2}$ ▸ *Done*

$\text{domain}(f(x),x)$ ▸ $\frac{2}{3}\leq x<\infty$

Inverse: $\text{solve}(x=f(y),y)$ ▸ $y=\frac{x^2+2}{3}$ and $x\geq 0$

this also gives the domain of the Inverse (this domain is also the Range of f(x) → y: **$y\geq 0$**

Example 2: $g(x):=\ln(3\cdot x-4)$ ▸ *Done*

$\text{domain}(g(x),x)$ ▸ $\frac{4}{3}<x<\infty$

Inverse: $\text{solve}(x=g(y),y)$ ▸ $y=\frac{e^x+4}{3}$

(Domain is x∊R) since the solve does not output a condition

the domain of the Inverse is x: **$x\geq 0$** (this is also the Range of f(x) y: **$y\geq 0$**

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Distance between 2 points

(x1,y1) and (x2,y2)

Example: (3,−2) and (4,7)

$a:=[3\ \ {}^-2]$ ▸ $[3\ \ {}^-2]$ $b:=[4\ \ 7]$ ▸ $[4\ \ 7]$

$\text{norm}(a-b)$ ▸ $\sqrt{82}$ Check: $\sqrt{(3-4)^2+({}^-2-7)^2}$ ▸ $\sqrt{82}$

Distance between 2 points in 3D

(x1,y1,z1) and (x2,y2,z2)

$m:=[3\ \ {}^-4\ \ 2]$ ▸ $[3\ \ {}^-4\ \ 2]$ $n:=[5\ \ {}^-2\ \ 6]$ ▸ $[5\ \ {}^-2\ \ 6]$

$\text{norm}(m-n)$ ▸ $2\cdot\sqrt{6}$

$\text{norm}(n-m)$ ▸ $2\cdot\sqrt{6}$

30

1.1 2.1 3.1 *Doc RAD

Solve: $2\cdot x+3\cdot y=-7$ and $4\cdot x-2\cdot y=3$

$a:=\begin{bmatrix}2&3\\4&-2\end{bmatrix}$ Coefficients of x and y terms

$c:=\begin{bmatrix}-7\\3\end{bmatrix}$ Constant Terms

$a\cdot\begin{bmatrix}x\\y\end{bmatrix}=c \blacktriangleright \begin{bmatrix}2\cdot x+3\cdot y=-7\\4\cdot x-2\cdot y=3\end{bmatrix}$

Method 1

$a\cdot a^{-1}\cdot\begin{bmatrix}x\\y\end{bmatrix}=a^{-1}\cdot c \blacktriangleright \begin{bmatrix}x=\frac{-5}{16}\\y=\frac{-17}{8}\end{bmatrix}$

Method 2

$\text{linSolve}\left(\begin{cases}2\cdot x+3\cdot y=-7\\4\cdot x-2\cdot y=3\end{cases},\{x,y\}\right) \blacktriangleright \left\{\frac{-5}{16},\frac{-17}{8}\right\}$

Method 3

$\text{rref}\left(\begin{bmatrix}2&3&-7\\4&-2&3\end{bmatrix}\right) \blacktriangleright \begin{bmatrix}1&0&\frac{-5}{16}\\0&1&\frac{-17}{8}\end{bmatrix}$

Method 4

$\text{simult}\left(\begin{bmatrix}2&3\\4&-2\end{bmatrix},\begin{bmatrix}-7\\3\end{bmatrix}\right) \blacktriangleright \begin{bmatrix}\frac{-5}{16}\\\frac{-17}{8}\end{bmatrix}$

You can also solve Graphically

Intersecting, Parallel or Colinear Lines

Question : for what value(s) of ***a*** will the 2 lines

$-2\cdot x-a\cdot y=-4 \rightarrow \mathbf{m}$ and $(a-1)\cdot x+6\cdot y=2\cdot(a-1) \rightarrow \mathbf{n}$

have

i. No Solution (parallel)

ii. Infinite Solutions (Colinear)

iii. Unique Solution (They Intersect)

$\text{zeros}\left(\det\left(\begin{bmatrix}-2&-a\\a-1&6\end{bmatrix}\right),a\right) \blacktriangleright \{-3,4\}$ ⚠ These values of ***a*** values make the gradient of both the lines the same

Check if parallel or colinear

$\text{rref}\left(\begin{bmatrix}-2&-a&-4\\a-1&6&2\cdot(a-1)\end{bmatrix}\right)|a=-3 \blacktriangleright \begin{bmatrix}1&\frac{-3}{2}&2\\0&0&0\end{bmatrix}$

$\frac{\mathbf{m}}{\mathbf{n}}|a=-3 \blacktriangleright \frac{1}{2}=\frac{1}{2}$ ⚠ Colinear (LHS=RHS)

$\text{rref}\left(\begin{bmatrix}-2&-a&-4\\a-1&6&2\cdot(a-1)\end{bmatrix}\right)|a=4 \blacktriangleright \begin{bmatrix}1&2&2\\0&0&0\end{bmatrix}$

$\frac{\mathbf{m}}{\mathbf{n}}|a=4 \blacktriangleright \frac{-2}{3}=\frac{-2}{3}$ ⚠ Colinear (LHS=RHS)

Intersecting, Parallel or Colinear Lines

Question : for what value(s) of ***a*** will the 2 lines

$a\cdot x-3\cdot y=5 \rightarrow \mathbf{m}$ and $3\cdot x-a\cdot y=8-a \rightarrow \mathbf{n}$ have

i. No Solution (parallel)

ii. Infinite Solutions (Colinear)

iii. Unique Solution (They Intersect)

$\text{zeros}\left(\det\left(\begin{bmatrix}a&-3\\3&-a\end{bmatrix}\right),a\right) \blacktriangleright \{-3,3\}$ ⚠ These values of ***a*** values make the gradient of both the lines the same

Check if parallel or colinear

Colinear if in rref( )one row has all zeros

$\text{rref}\left(\begin{bmatrix}a&-3&5\\3&-a&8-a\end{bmatrix}\right)|a=-3 \blacktriangleright \begin{bmatrix}1&1&0\\0&0&1\end{bmatrix}$

$\frac{\mathbf{m}}{\mathbf{n}}|a=-3 \blacktriangleright -1=\frac{5}{11}$ ⚠ Parallel (LHS≠RHS)

$\text{rref}\left(\begin{bmatrix}a&-3&5\\3&-a&8-a\end{bmatrix}\right)|a=3 \blacktriangleright \begin{bmatrix}1&-1&\frac{5}{3}\\0&0&0\end{bmatrix}$

Colinear if one of the rows is all zeros

$\frac{\mathbf{m}}{\mathbf{n}}|a=3 \blacktriangleright 1=1$ ⚠ Colinear (LHS=RHS)

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Finding Derivatives of varying orders

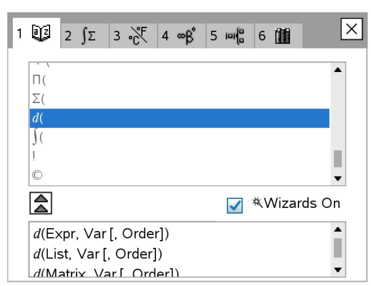
d(Expr, Var [, Order]) Enter this from the Catalogue

This is a CAS built−in command

$$\frac{d^1}{dx^1}(3\cdot x^7) \blacktriangleright 21\cdot x^6$$

$$\frac{d^5}{dx^5}(3\cdot x^7) \blacktriangleright 7560\cdot x^2$$

$$\frac{d^3}{dx^3}(\ln(x)) \blacktriangleright \frac{2}{x^3}$$ ⚠



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Fraction Tools for Rational Functions

$\text{propFrac}\left(\frac{4\cdot x+1}{2\cdot x-7}\right) ▸ \frac{15}{2\cdot x-7}+2$

$\text{comDenom}\left(\frac{15}{2\cdot x-7}+2\right) ▸ \frac{4\cdot x+1}{2\cdot x-7}$

$\text{getNum}\left(\frac{4\cdot x+1}{2\cdot x-7}\right) ▸ 4\cdot x+1$

$\text{getDenom}\left(\frac{4\cdot x+1}{2\cdot x-7}\right) ▸ 2\cdot x-7$

We can get the Quotient and remainder using the propfrac syntax

Method 2

$\text{polyQuotient}(4\cdot x+1,2\cdot x-7) ▸ 2$

$\text{polyRemainder}(4\cdot x+1,2\cdot x-7) ▸ 15$

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Polynomial Tools

$\text{polyCoeffs}(3\cdot x^3-5\cdot x+7) ▸ \{3,0,-5,7\}$

$\text{polyCoeffs}(3\cdot x^4-5\cdot x^2+7\cdot x) ▸ \{3,0,-5,7,0\}$

$\text{polyRoots}(3\cdot x^3-5\cdot x^2+7\cdot x+3,x) ▸ \left\{\frac{-1}{3}\right\}$

$\text{cPolyRoots}(3\cdot x^3-5\cdot x^2+7\cdot x+3,x) ▸ \left\{\frac{-1}{3},1-\sqrt{2}\cdot \boldsymbol{i},1+\sqrt{2}\cdot \boldsymbol{i}\right\}$

$\text{cZeros}(3\cdot x^3-5\cdot x^2+7\cdot x+3,x) ▸ \left\{1-\sqrt{2}\cdot \boldsymbol{i},1+\sqrt{2}\cdot \boldsymbol{i},\frac{-1}{3}\right\}$

$\text{polyGcd}(12\cdot x^3-6\cdot x^2,3\cdot x) ▸ 3\cdot x$

$\text{polyGcd}(12\cdot x^3-24\cdot x^2+12\cdot x+8,4\cdot x) ▸ 4$

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Combination nCr in Binomial Expansion

Question: Find the 6 th term in the expansion of $(3x-2y)^8$

On expansion we will get 9 terms (where the power of each x powered terms decreases from 8 to 0 and at the same time the power of each y powered term increases from 0 to 8

6 th term will have x^3 and y^5

on CAS $\mathrm{nCr}(8,3)\cdot(3\cdot x)^3\cdot(^-2\cdot y)^5 \blacktriangleright {}^-48384\cdot x^3\cdot y^5$

$\mathrm{expand}\left((3\cdot x-2\cdot y)^8\right)$

$\blacktriangleright 6561.\cdot x^8-34992.\cdot x^7\cdot y+81648.\cdot x^6\cdot y^2-108864.\cdot x^5\cdot y^3$
$+90720.\cdot x^4\cdot y^4\boldsymbol{-48384.\cdot x^3\cdot y^5}+16128.\cdot x^2\cdot y^6-3072.\cdot x\cdot y^7$
$+256.\cdot y^8$

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Find the sum of solutions of Sin(2x−pi/4) in a domain of [−2π,4π]

$\mathrm{zeros}\left(\sin\left(2\cdot x-\frac{\pi}{4}\right),x\right)|{}^-2\cdot\pi\le x\le 4\cdot\pi$

$\blacktriangleright \left\{\frac{^-15\cdot\pi}{8},\frac{^-11\cdot\pi}{8},\frac{^-7\cdot\pi}{8},\frac{^-3\cdot\pi}{8},\frac{\pi}{8},\frac{5\cdot\pi}{8},\frac{9\cdot\pi}{8},\frac{13\cdot\pi}{8},\frac{17\cdot\pi}{8},\frac{21\cdot\pi}{8},\frac{25\cdot\pi}{8}\right.$

$\mathrm{sum}\left(\mathrm{zeros}\left(\sin\left(2\cdot x-\frac{\pi}{4}\right),x\right)|{}^-2\cdot\pi\le x\le 4\cdot\pi\right) \blacktriangleright \frac{21\cdot\pi}{2}$

Find the sum of solutions of Cos(2x−pi/3)=$\frac{\sqrt{2}}{2}$ **in a domain of [−2π,4π]**

$\mathrm{zeros}\left(\cos\left(2\cdot x-\frac{\pi}{3}\right)-\frac{\sqrt{2}}{2},x\right)|{}^-2\cdot\pi\le x\le 3\cdot\pi$

$\blacktriangleright \left\{\frac{^-47\cdot\pi}{24},\frac{^-41\cdot\pi}{24},\frac{^-23\cdot\pi}{24},\frac{^-17\cdot\pi}{24},\frac{\pi}{24},\frac{7\cdot\pi}{24},\frac{25\cdot\pi}{24},\frac{31\cdot\pi}{24},\frac{49\cdot\pi}{24},\frac{55\cdot\pi}{24}\right.$

$\mathrm{sum}\left(\mathrm{zeros}\left(\cos\left(2\cdot x-\frac{\pi}{3}\right)-\frac{\sqrt{2}}{2},x\right)|{}^-2\cdot\pi\le x\le 3\cdot\pi\right) \blacktriangleright \frac{5\cdot\pi}{3}$

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$x := \{0,1,2,3,4\}$ → x-values

$prx := \left\{\frac{3}{20},\frac{1}{5},\frac{1}{4},\frac{4}{15},\frac{2}{15}\right\}$ → probabilities

$sum(prx) ▸ 1$ → sum of probabilities

$$\{x, prx, x\cdot prx\} ▸ \begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ \frac{3}{20} & \frac{1}{5} & \frac{1}{4} & \frac{4}{15} & \frac{2}{15} \\ 0 & \frac{1}{5} & \frac{1}{2} & \frac{4}{5} & \frac{8}{15} \end{bmatrix}$$ x x pr(x)

$sum(x\cdot prx) ▸ \boxed{2.03333}$ $\quad sum(x^2\cdot prx) ▸ 5.73333$

$sum(x^2\cdot prx)-(sum(x\cdot prx))^2 ▸ 1.59889$

$\sqrt{sum(x^2\cdot prx)-(sum(x\cdot prx))^2} ▸ 1.26447$

OneVar x,prx: stat.results

"Title"	"One-Variable Statistics"
"$\bar{x}$"	2.03333
"Σx"	2.03333
"Σx²"	5.73333
"sx := sn-1x"	undef
"σx := σnx"	1.26447
"n"	1.
"MinX"	0.
"Q₁X"	1.
"MedianX"	2.
"Q₃X"	3.
"MaxX"	4.
"SSX := Σ(x−x̄)²"	1.59889

$x := \{0,1,2,3,4\}$

Find E(X), Var(X),SD(X)
$3x+2$

$prx := \left\{\frac{3}{20},\frac{1}{5},\frac{1}{4},\frac{4}{15},\frac{2}{15}\right\}$

$sum(prx) ▸ 1$

$$\{3\cdot x+2, prx, (3\cdot x+2)\cdot prx\} ▸ \begin{bmatrix} 2 & 5 & 8 & 11 & 14 \\ \frac{3}{20} & \frac{1}{5} & \frac{1}{4} & \frac{4}{15} & \frac{2}{15} \\ \frac{3}{10} & 1 & 2 & \frac{44}{15} & \frac{28}{15} \end{bmatrix}$$

$sum((3\cdot x+2)\cdot prx) ▸ 8.1$ $\quad sum((3\cdot x+2)^2\cdot prx) ▸ 80.$

$sum((3\cdot x+2)^2\cdot prx)-(sum((3\cdot x+2)\cdot prx))^2 ▸ 14.39$

$\sqrt{sum((3\cdot x+2)^2\cdot prx)-(sum((3\cdot x+2)\cdot prx))^2} ▸ 3.79342$

OneVar 3·x+2,prx: stat.results

"Title"	"One-Variable Statistics"
"$\bar{x}$"	8.1
"Σx"	8.1
"Σx²"	80.
"sx := sn-1x"	undef
"σx := σnx"	3.79342
"n"	1.
"MinX"	2.
"Q₁X"	5.
"MedianX"	8.
"Q₃X"	11.
"MaxX"	14.
"SSX := Σ(x−x̄)²"	14.39

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Normal Probability Distribution

A company manufactures steel rods of length 1.2 m. The rods produced are normally distributed with a mean μ =1.2 and a (standard deviation) σ of 0.075m

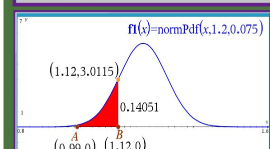
a. Find the function value on the graph when a rod selected at random will have a length of 1.120m

b. Find the probability that a rod selected at random will have a length of between 0.990m and 1.120 m

c. Find the probability Pr(0.990<X<1.120|1.00<X<1.122)

d. Find the minimum acceptable length of the rod, if 8% of the distrubution are considered undersized.

e. Find $c_1$and c_2 the acceptable lengths of the rod if 84% rods are of acceptable length and 8% are undersized and 8% are oversized



normPdf(1.12,1.2,0.075) ▸ 3.0115044
normCdf(0.99,1.12,1.2,0.075) ▸ 0.14050603

$X \sim N(1.2, 0.075^2)$
normCdf(0.99,1.12,1.2,0.075) ▸ 0.14050603

$0.99<x<1.12$ and $1.<x<1.122$ ▸ $1.<x<1.12$

$\frac{normCdf(1,1.12,1.2,0.075)}{normCdf(1,1.122,1.2,0.075)} ▸ 0.95796912$

invNorm(0.08,1.2,0.075) ▸ 1.0946196 m

zInterval 0.075,1.2,1,0.84: stat.results

"Title"	"z Interval"
"CLower"	1.0946196
"CUpper"	1.3053804
"$\bar{x}$"	1.2
"ME"	0.10538037
"n"	1.
"σ"	0.075

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Define $\mathbf{dis}(x1,x2,y1,y2)=\sqrt{(x2-x1)^2+(y2-y1)^2}$ ▸ *Done*

Define $\mathbf{slope}(x1,x2,y1,y2)=\frac{y2-y1}{x2-x1}$ ▸ *Done*

Define $\mathbf{mp}(x1,x2,y1,y2)=\left\{\frac{x2+x1}{2},\frac{y2+y1}{2}\right\}$ ▸ *Done*

Define $\mathbf{eq}(x1,x2,y1,y2)=\text{solve}(y-y1=\mathbf{slope}(x1,x2,y1,y2)\cdot(x-x1),y)$ ▸ *Done*

$\mathbf{mp}(4,3,2,^-7)$ ▸ $\{3.5,^-2.5\}$

$\mathbf{mp}(^-4,3,1,^-7)$ ▸ $\{^-0.5,^-3.\}$

$\mathbf{mp}(^-4,^-2,1,5)$ ▸ $\{^-3,3\}$

$\mathbf{mp}(4,^-2,2,5)$ ▸ $\{1.,3.5\}$

$\mathbf{dis}(3.5,^-0.5,^-2.5,^-3)$ ▸ 4.03113 $\mathbf{slope}(3.5,^-0.5,^-2.5,^-3)$ ▸ 0.125

$\mathbf{dis}(^-3,^-0.5,3,^-3)$ ▸ 6.5 $\mathbf{slope}(^-3,^-0.5,3,^-3)$ ▸ $^-2.4$

$\mathbf{dis}(^-3,1,3,3.5)$ ▸ 4.03113 $\mathbf{slope}(^-3,1,3,3.5)$ ▸ 0.125

$\mathbf{dis}(3.5,1,^-2.5,3.5)$ ▸ 6.5 $\mathbf{slope}(3.5,1,^-2.5,3.5)$ ▸ $^-2.4$

$\text{expand}(\mathbf{eq}(3.5,^-0.5,^-2.5,^-3))$ ▸ $y=0.125\cdot x-2.9375$

$\text{expand}(\mathbf{eq}(^-3,^-0.5,3,^-3))$ ▸ $y=^-2.4\cdot x-4.2$

$\text{expand}(\mathbf{eq}(^-3,1,3,3.5))$ ▸ $y=0.125\cdot x+3.375$

$\text{expand}(\mathbf{eq}(3.5,1,^-2.5,3.5))$ ▸ $y=5.9-2.4\cdot x$

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Question: The probability of an event happening is $p=\frac{3}{5}$, find the probabilty for the event to have at Pr(3≤X<5) (3 or 4 successes) if the event happened 5 times.

$1-p=\frac{2}{5}$ ▸ $1-p=\frac{2}{5}$

On CAS Method 1

$\text{nCr}(5,3)\cdot\left(\frac{3}{5}\right)^3\cdot\left(\frac{2}{5}\right)^2+\text{nCr}(5,4)\cdot\left(\frac{3}{5}\right)^4\cdot\left(\frac{2}{5}\right)^1$ ▸ $\frac{378}{625}$

$\frac{378}{625}$ ▸ 0.6048

On CAS Method 2 Built in Function BinomCdf

$\text{binomCdf}\left(5,\frac{3}{5},3,4\right)$ ▸ 0.6048

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Permutations and Combinations

Question: From 10 members, you would like to select a President, a Vice President and a Secretary (order matters) −hence Permutation

P(n,r) for this example: P(10,3)

On CAS: $\mathrm{nPr}(10,3) \blacktriangleright 720$ $\quad \dfrac{10!}{(10-3)!} \blacktriangleright 720$

Question: From 10 students, you would like to select a team of 3 members −This is a Combination (order does not matter)

C(n,r) for this example: C(10,3)

On CAS: $\mathrm{nCr}(10,3) \blacktriangleright 120$ $\quad \dfrac{10!}{(10-3)! \cdot 3!} \blacktriangleright 120$

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Given that $\boldsymbol{f(3x-4)=\dfrac{9x-10}{3(x-2)}}$

a. find $\boldsymbol{f(x)}$

$$f(3x-4)=\frac{9x-10}{3x-6}$$

$$(3x-4,y)\to(x',y')$$

$$\frac{x'+4}{3}=x \text{ and } =y$$

$$\frac{9x-10}{3x-6}\Rightarrow\frac{9\left(\frac{x'+4}{3}\right)-10}{3\left(\frac{x'+4}{3}\right)-6}$$

$$\frac{\left(\frac{9x'+36}{3}\right)-10}{\left(\frac{3x'+12}{3}\right)-6}$$

$$\frac{(3x'+12)-10}{(x'+4)-6}\Rightarrow\frac{3x'+2}{x'-2}$$

$$f(x)=\frac{3x+2}{x-2}$$

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1.1 *Doc RAD

$f(x):=\frac{3\cdot x+2}{x-2}$ ▸ *Done*

$f(3\cdot x-4)$ ▸ $\frac{9\cdot x-10}{3\cdot(x-2)}$

$\mathrm{propFrac}\left(\frac{3\cdot x+2}{x-2}\right)$ ▸ $\frac{8}{x-2}+3$

$g(u):=\frac{9\cdot u-10}{3\cdot(u-2)}$ ▸ *Done*

$g\left(\frac{u}{3}+\frac{4}{3}\right)$ ▸ $\frac{3\cdot u+2}{u-2}$

$g\left(\frac{u+4}{3}\right)$ ▸ $\frac{3\cdot u+2}{u-2}$

$g\left(\frac{x+4}{3}\right)$ ▸ $\frac{3\cdot x+2}{x-2}$

1.1 1.2 *Doc RAD

$f(x):=\frac{9\cdot x-10}{3\cdot(x-2)}$ ▸ *Done*

TRD instead of DRT

$f(x+4)$ ▸ $\frac{9\cdot x+26}{3\cdot(x+2)}$

$f\left(\frac{x+4}{3}\right)$ ▸ $\frac{3\cdot x+2}{x-2}$

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Given that $f(7x-\pi/3)=\sin(x)$

a. find $f(x)$

$(7x-\pi/3, y) \rightarrow (x', y')$

$\frac{x'+\pi/3}{7}=x$ and $=y$

$\sin(x) \Rightarrow \sin\left(\frac{x'+\pi/3}{7}\right)$

$$\sin\left(\frac{(3x'+\pi)}{21}\right)$$

$f(x):=\sin(x)$ ▸ *Done*

TRD instead of DRT

$f\left(x+\frac{\pi}{3}\right)$ ▸ $\sin\left(x+\frac{\pi}{3}\right)$

$f\left(\frac{x+\frac{\pi}{3}}{7}\right)$ ▸ $\sin\left(\frac{x}{7}+\frac{\pi}{21}\right)$

$\mathrm{comDenom}\left(\frac{x}{7}+\frac{\pi}{21}\right)$ ▸ $\frac{3\cdot x+\pi}{21}$

$g(x):=\sin\left(\frac{3\cdot x+\pi}{21}\right)$ ▸ *Done*

Check:

$g\left(7\cdot x-\frac{\pi}{3}\right)$ ▸ $\sin(x)$

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$f(x)=x^2+2$

$g(x)=2x+1$

$f(2x+1)=(2x+1)^2+2$

Question → if $f(2x+1)=(2x+1)^2+2$ Find $f(x)$

We can either reverse transformations or use the Substitution method to find $f(x)$

$t=2x+1$

$x=\frac{t-1}{2}$ or $\frac{1}{2}(t-1)$

Notice that this will work both for reversing transformations or for substitution.

$\left(2\left(\frac{t-1}{2}\right)+1\right)^2+2 \quad = t^2+2$

$f(x)=x^2+2$

$m(x):=(2\cdot x+1)^2+2 ▸ Done$

TRD

$m\left(\frac{x-1}{2}\right) ▸ x^2+2$

$\text{solve}(2\cdot x+1=n,x) ▸ x=\frac{n-1}{2}$

INSTRUMENTS

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$f(x+2)=x^2-3x+2$

find $f(x)$

Let $t=x+2$

Solve for x: $x=t-2$

$f(x)=(t-2)^2-3(t-2)+2$

$f(x)=t^2-4t+4-3t+6+2$

$f(x)=t^2-7t+12 \Rightarrow f(x)=x^2-7x+12$

Method-2 Reversing Transformations

$f(x+2)=(x-2)(x-1)$ Factorise

$f(x)=((x-2)-2)((x-1)-2)$

$f(x)=(x-4)(x-3) \Rightarrow f(x)=x^2-7x+12$

Verify

$g(x):=x^2-7\cdot x+12 ▸ Done$

$g(x+2) ▸ x^2-3\cdot x+2$

$t:=x+2 ▸ x+2$

$\text{solve}(t=a,x)|x=a ▸ x=a-2$

$x^2-3\cdot x+2|x=a-2 ▸ a^2-7\cdot a+12$

$a^2-7\cdot a+12|a=x ▸ x^2-7\cdot x+12$

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$$f\left(\frac{1}{x}\right)=x+\sqrt{1+x^2}$$

find $f(x)$: $x>0$

Let $t=\frac{1}{x}$

Solve for x : $x=\frac{1}{t}$

$$f(x)=\frac{1}{t}+\sqrt{1+\left(\frac{1}{t}\right)^2}$$

$$f(x)=\frac{1}{t}+\sqrt{\frac{t^2+1}{t^2}}$$

$$f(x)=\frac{1}{t}+\frac{\sqrt{t^2+1}}{\mp t}$$

$$f(x)=\frac{1}{t}+\frac{\sqrt{t^2+1}}{\operatorname{abs}(t)}$$

$$f(x)=\frac{1}{x}+\frac{\sqrt{x^2+1}}{\operatorname{abs}(x)}$$

$$\mathbf{g}(x):=\frac{\sqrt{x^2+1}}{|x|}+\frac{1}{x} \blacktriangleright \textit{Done}$$

$$\mathbf{g}\left(\frac{1}{x}\right) \blacktriangleright \sqrt{x^2+1}+x$$

$$\mathbf{t}:=\frac{1}{x} \blacktriangleright \frac{1}{x}$$

$$\operatorname{solve}(\mathbf{t}=a,x)|x=a \blacktriangleright x=\frac{1}{a}$$

$$x+\sqrt{1+x^2}\,|x=\frac{1}{a} \blacktriangleright \frac{\sqrt{a^2+1}}{|a|}+\frac{1}{a}$$

$$\frac{\sqrt{a^2+1}}{|a|}+\frac{1}{a}|a=x \blacktriangleright \frac{\sqrt{x^2+1}}{|x|}+\frac{1}{x}$$

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$$f\left(3x-\frac{\pi}{4}\right)=\sin(x)$$

find $f(x)$

$$f\left(3x-\frac{\pi}{4}+\frac{\pi}{4}\right)=\sin\left(x+\frac{\pi}{4}\right) \Rightarrow f(3x)=\sin\left(x+\frac{\pi}{4}\right)$$

$$f\left(\frac{3x}{3}\right)=\sin\left(\frac{x+\frac{\pi}{4}}{3}\right)$$

$$f(x)=\sin\left(\frac{1}{3}\left(x+\frac{\pi}{4}\right)\right)$$

$$f(x)=\sin\left(\frac{4x+\pi}{12}\right)$$

$$\mathbf{g}(x):=\sin\left(\frac{4\cdot x+\pi}{12}\right) \blacktriangleright \textit{Done}$$

Verify

$$\mathbf{g}\left(3\cdot x-\frac{\pi}{4}\right) \blacktriangleright \sin(x)$$

$$\mathbf{t}:=3\cdot x-\frac{\pi}{4} \blacktriangleright 3\cdot x-\frac{\pi}{4}$$

$$\operatorname{solve}(\mathbf{t}=a,x)|x=a \blacktriangleright x=\frac{4\cdot a+\pi}{12}$$

$$\sin(x)|x=\frac{4\cdot a+\pi}{12} \blacktriangleright \sin\left(\frac{4\cdot a+\pi}{12}\right)$$

$$\sin\left(\frac{4\cdot a+\pi}{12}\right)|a=x \blacktriangleright \sin\left(\frac{4\cdot x+\pi}{12}\right)$$

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b. write $f(x)$ in the form $a + \dfrac{b}{c \cdot x + d}$

Given $f(x) = \frac{3x+2}{x-2}$

$Denominator \quad x - 2 = 0 \ \rightarrow x$
$= 2 \quad (sub\ in\ Numerator) \quad 3(2) + 2$
$= 8 \quad (This\ is\ the\ remainder)$

$Quotient\colon \ \dfrac{3x}{3} = 3 \qquad \mathrm{Q} + \dfrac{R}{Divisor}$

$f(x) = 3 + \dfrac{8}{x-2} \quad \rightarrow \quad \mathrm{a} = 3, \quad \mathrm{b} = 8 \quad \mathrm{c} = 1 \text{ and } \mathrm{d} = -2$

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$\boxed{\text{propFrac}(\mathbf{f}(x)) \ \blacktriangleright \ \dfrac{8}{x-2}+3}$

write the values for a, b, c and d

$\mathrm{a} = 3, \quad \mathrm{b} = 8 \quad \mathrm{c} = 1 \text{ and } \mathrm{d} = -2$

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c . State the domain and range for $f(x)$

Domain and Range $x: x \in R \setminus \{2\}$ *and* $y: y \in R \setminus \{3\}$

$\boxed{\text{domain}(f(x),x) \blacktriangleright x \neq 2}$

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d. Sketch the graph for $f(x)$

X int: $\frac{3x+2}{x-2} = 0 \;\rightarrow\; 3x + 2 = 0$ or $x = -\frac{2}{3}$ $\left(-\frac{2}{3}, 0\right)$

Y int: $\frac{3(0)+2}{(0)-2} = \text{y} \;\rightarrow\; \text{y} = -1$ or $(0, -1)$

Dilation from x-axis is 8 units

H-**Asymptote** ***y*=3** and V-asymptote ***x* =2**

Domain and Range $x: x \in R \setminus \{2\}$ *and* $y: y \in R \setminus \{3\}$

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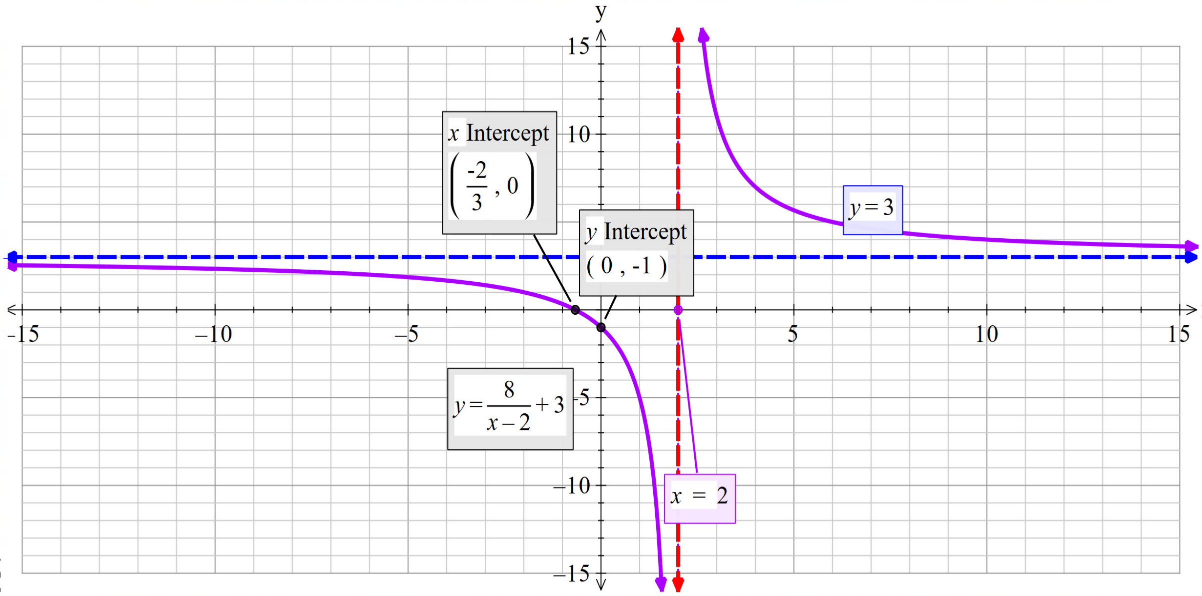

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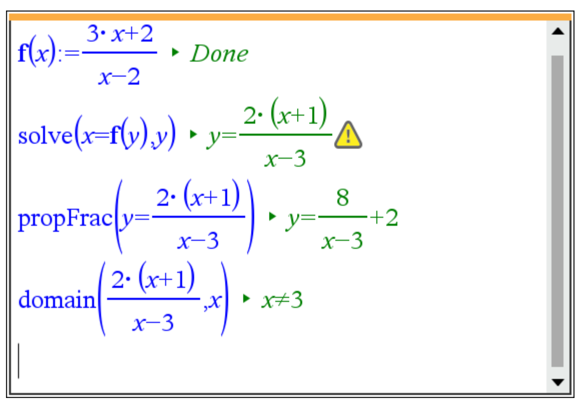
e. Write the rule for $f^{-1}(x)$



$$y = 3 + \frac{8}{x-2}$$

Inv: $\quad x = 3 + \frac{8}{y-2}$

$$x - 3 = \frac{8}{y-2} \quad \rightarrow \quad y - 2 = \frac{8}{x-3} \quad \rightarrow \quad y = \frac{8}{x-3} + 2$$

$$f^{-1}(x) = \frac{8}{x-3} + 2 \quad ; \quad x : x \in R \setminus \{3\}$$

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f. State the coordinates for the points where $f(x) = f^{-1}(x)$

$$Equate\ \ y = \frac{8}{x-3} + 2 \quad and\ \ y = x \quad \rightarrow \quad x - 2 = \frac{8}{x-3}$$

$$(x-2)(x-3) = 8 \quad \rightarrow \quad (x-2)(x-3) - 8 = 8$$

$$x^2 - 5x + 6 - 8 = 0 \quad \rightarrow \quad x^2 - 5x - 2 = 0$$

$$\text{zeros}\left(f(x) - \left(\frac{8}{x-3}+2\right), x\right) \blacktriangleright \left\{\frac{-(\sqrt{33}-5)}{2}, \frac{\sqrt{33}+5}{2}\right\}$$

$$f\left(\frac{-(\sqrt{33}-5)}{2}\right) \blacktriangleright \frac{-(\sqrt{33}-5)}{2}$$

$$f\left(\frac{\sqrt{33}+5}{2}\right) \blacktriangleright \frac{\sqrt{33}+5}{2}$$

By Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad x = \frac{5 \pm \sqrt{25+8}}{2}$$

$$x = \frac{5-\sqrt{33}}{2} \quad or \quad x = \frac{5+\sqrt{33}}{2}$$

Points of Intersection are

$$\left(\frac{5-\sqrt{33}}{2}, \frac{5-\sqrt{33}}{2}\right) \quad and \quad \left(\frac{5+\sqrt{33}}{2}, \frac{5+\sqrt{33}}{2}\right)$$

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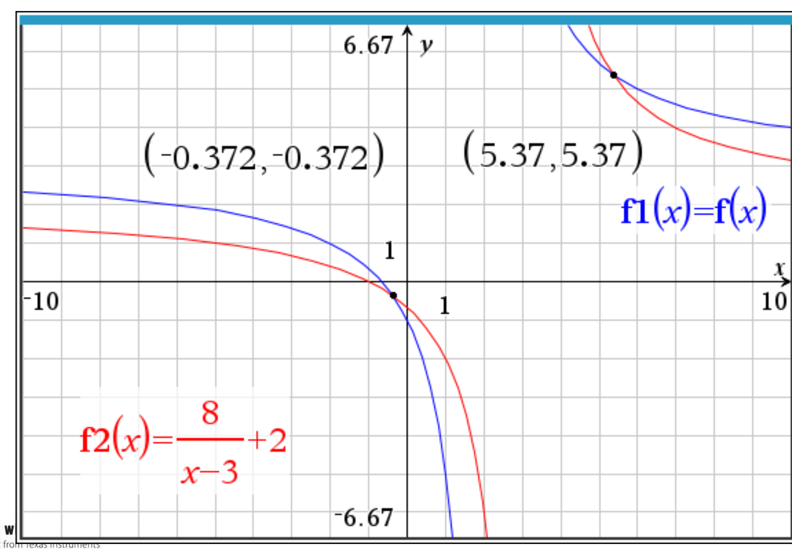
g. sketch the graph for
$f(x)$ and $f^{-1}(x)$ and represent the point of Intersection



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h. Find the the derivative for f(x)
and hence write the equation of the tangent line at $x = 1$

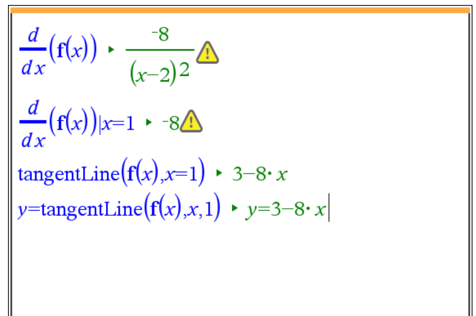
$$f(x) = \frac{3x+2}{x-2}$$

By Quotient Rule $\quad f'(x) = \dfrac{q(x) * p'(x) - p(x) * q'(x)}{\big(q(x)\big)^2}$

$3x + 2 \rightarrow \mathrm{p(x)}$ **and** $x - 2 \rightarrow \mathrm{q(x)}$

$$f'(x) = \frac{(x-2)*3-(3x+2)*1}{(x-2)^2}$$

$$f'(x) = \frac{\mathbf{3x-6-3x-2}}{(x-2)^2} \quad \textbf{or} \quad f'(x) = \frac{\mathbf{-8}}{(x-2)^2}$$



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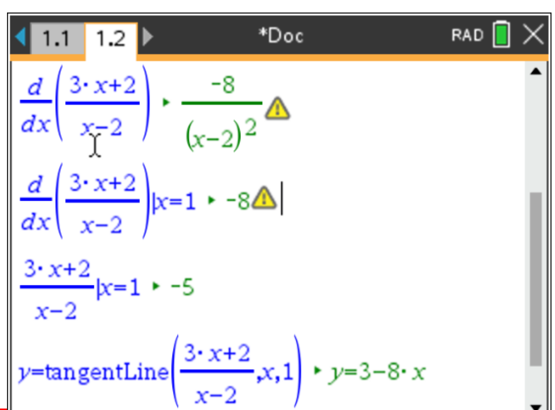
$f'(1) = \mathbf{-8}$ **and** $f(1) = \frac{\mathbf{5}}{\mathbf{-1}}$ $\;$ ***or*** $\;\boldsymbol{f(1) = -5}$

Tangent:

$\boldsymbol{y + 5 = -8(x-1)} \quad \rightarrow \quad \boldsymbol{y = -8x + 8 - 5}$

or $\;\boldsymbol{y = -8x + 3}$



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i. Write the equation and coordinates for the tangent line

to the curve that is also parallel to the tangent at $x = 1$ (CAS)

$$-8 = \frac{-8}{(x-2)^2} \qquad \rightarrow (x-2)^2 = \mathbf{1}$$

$$(x-2)^2 = \mathbf{1} \qquad \boldsymbol{x = 1 \; or \; x = 3}$$

$$f(3) = \mathbf{11}$$

$$\boldsymbol{y - 11 = -8(x-3)} \qquad \boldsymbol{y = -8x + 24 + 11}$$

$$\boldsymbol{or \quad y = -8x + 35}$$

$\text{solve}\left(\frac{-8}{(x-2)^2}=-8,x\right) \blacktriangleright x=1 \text{ or } x=3$ ⚠

$y=\text{tangentLine}\left(\frac{3\cdot x+2}{x-2},x,3\right) \blacktriangleright y=35-8\cdot x$

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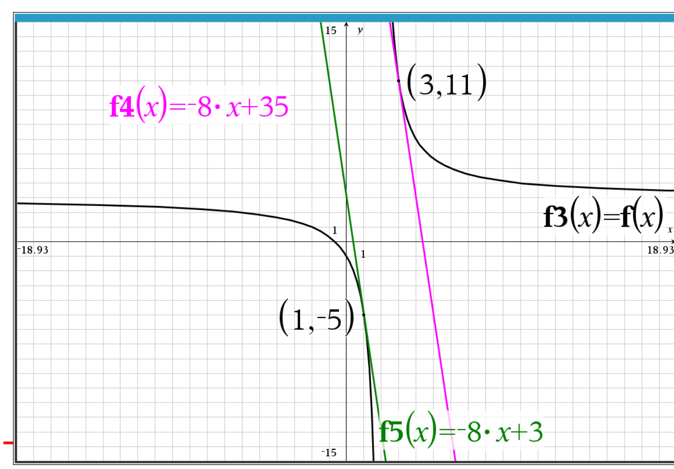
$\text{solve}\left(\frac{-8}{(x-2)^2}=-8,x\right) \blacktriangleright x=1 \text{ or } x=3$ ⚠

$y=\text{tangentLine}\left(\frac{3\cdot x+2}{x-2},x,3\right) \blacktriangleright y=35-8\cdot x$



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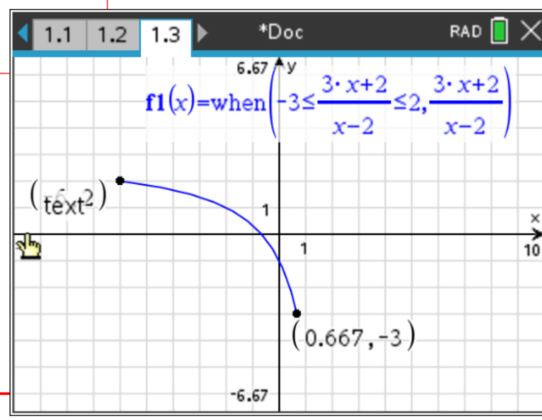
j. State the domain for $f(x)$ where $-3 \le f(x) \le 2$ (CAS)

$$\text{domain}\left(\text{when}\left(-3\le\frac{3\cdot x+2}{x-2}\le 2,\frac{3\cdot x+2}{x-2}\right),x\right)$$

$$\blacktriangleright\ -6\le x\le\frac{2}{3}$$



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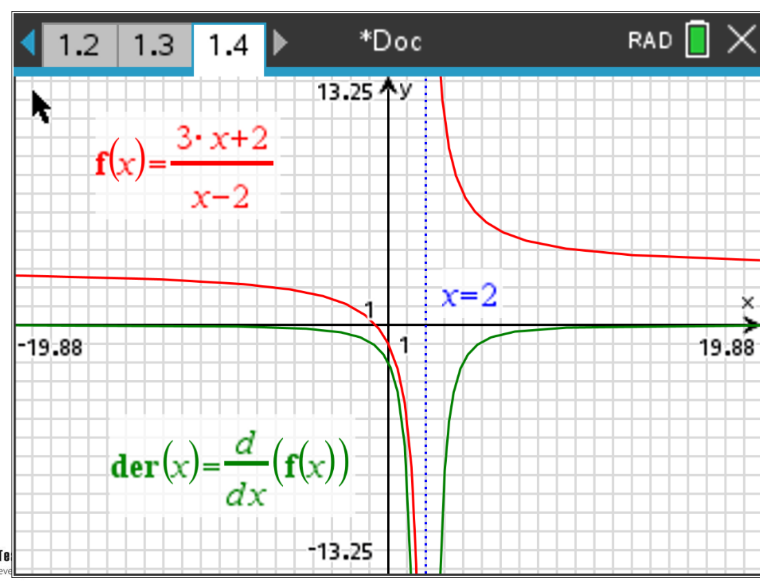
k. sketch the derivative curve for $f(x)$



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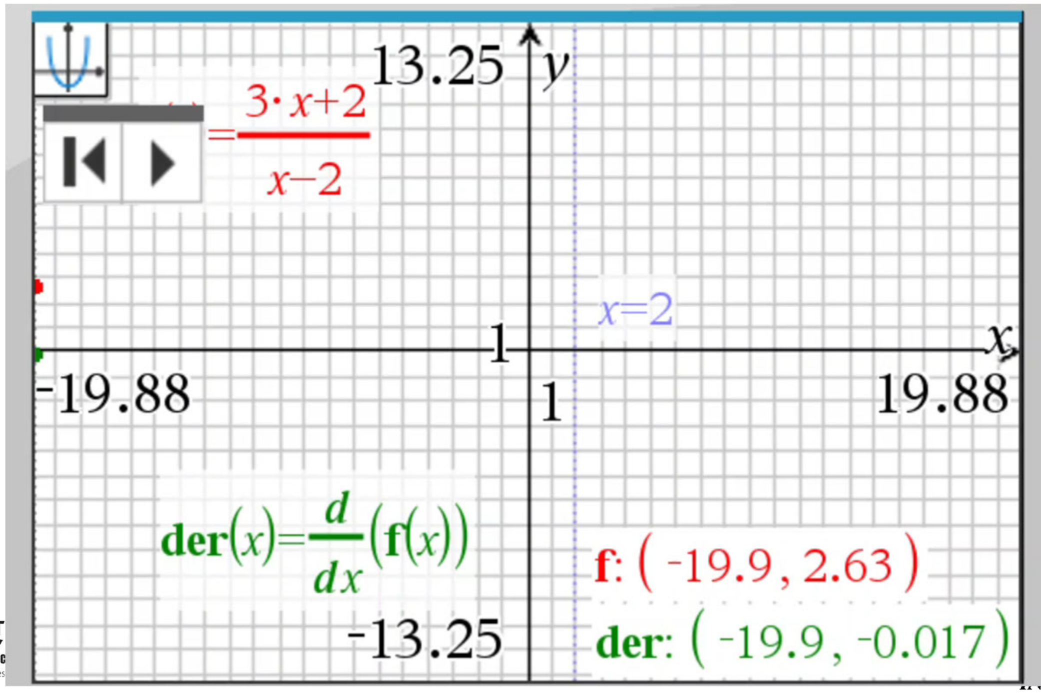

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I. Find the coordinates of the point b such that the average rate of change for $f(x)$ in the interval $[3, b] = -2$ CAS

$$\frac{f(b) - f(3)}{b - 3} = -2$$

$$\frac{f(b) - 11}{b - 2} = -2$$

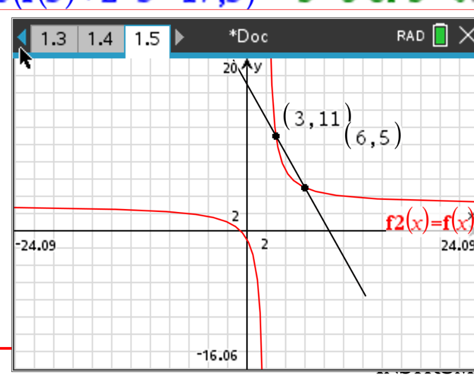
$$f(b) + 2b = 13$$

$$b = 6$$

f(3) ▸ 11

solve$\left(\frac{f(b)-f(3)}{b-3}=-2,b\right)$ ▸ b=6

solve(f(b)+2· b=17,b) ▸ b=3 or b=6



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m. Find the average value of the function $f(x)$ when $x \in [-2,1]$

Average value = $\frac{1}{b-a}\int_a^b f(x)dx$

$$\frac{1}{1-(-2)}\int_{-2}^{1}\frac{3x+2}{x-2}dx$$

$$\frac{1}{3}\int_{-2}^{1}\frac{8}{x-2}+3dx$$

$$\frac{8}{3}\int_{-2}^{1}\frac{1}{x-2}dx+\frac{3}{3}\int_{-2}^{1}1dx$$

$$\frac{8}{3}[\ln(|x-2|)]_{-2}^{1}+[x]_{-2}^{1}$$

$$\frac{8}{3}[\ln|1-2|-\ln|-2-2|]+[1-(-2)]$$

$$\frac{8}{3}[\ln(1)-2\ln 2)]+3]$$

$$\frac{8}{3}[0-2\ln 2)]+3 \quad \rightarrow \quad 3-\frac{16\ln(2)}{3}$$

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$$\frac{1}{1--2}\cdot\int_{-2}^{1}\left(3+\frac{8}{x-2}\right)dx \blacktriangleright \frac{^{-}(16\cdot\ln(2)-9)}{3}$$

$$\text{expand}\left(\frac{^{-}(16\cdot\ln(2)-9)}{3}\right) \blacktriangleright 3-\frac{16\cdot\ln(2)}{3}$$

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Question 1 (1 mark)

A discrete random variable X has a binomial distribution with a probability of success of $p = 0.1$ for n trials, where $n > 2$.

If the probability of obtaining at least two successes after n trials is at least 0.5, then the smallest possible value of n is

A. 15
B. 16
C. 17
D. 18
E. 19

invBinomN(0.5,0.1,1)	17
invBinomN(0.5,0.1,1,1)	$\begin{bmatrix} 16 & 0.514727829141 \\ 17 & 0.481785246693 \end{bmatrix}$

Question 1

$X \overset{d}{=} \mathrm{Bi}(n = ?, p = 0.1)$

$\Pr(X \geq 2) \geq 0.5$

$1 - [\Pr(X = 0) + \Pr(X = 1)] \geq 0.5$

$0.9^n + n \times 0.1 \times 0.9^{n-1} = 0.5,\ n = 16.44$

$n = 17$

The correct answer is **C**.

TEXAS INSTRUMENTS

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Confidence interval for P

An approximate confidence interval for a population proportion is given by $(\hat{p} - z\sigma_{\hat{p}}, \hat{p} + z\sigma_{\hat{p}})$, where

$$\sigma_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}.$$

Margin of error

The distance between the endpoints of the confidence interval and the sample estimate i
the margin of error, M.

Worked example 10 considered a 95% confidence interval of $(\hat{p} - 1.96\sigma_{\hat{p}}, \hat{p} + 1.96\sigma_{\hat{p}})$.
case the margin of error would be $M = 1.96\sigma_{\hat{p}}$.

Margin of error

For a **95%** level of confidence,

$$M = 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}.$$

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TEXAS INSTRUMENTS

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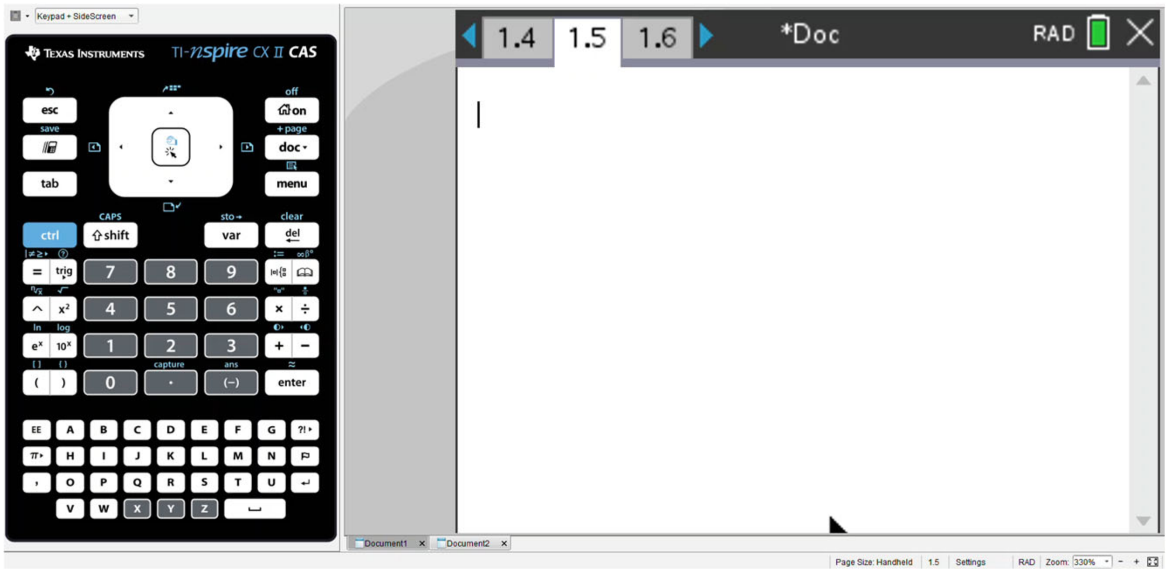

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1.4 1.5 1.6 *Doc RAD

zInterval 1,0,1,0.9: *stat.results*

"Title"	"z Interval"
"CLower"	-1.64485362591
"CUpper"	1.64485362591
"$\bar{x}$"	0.
"ME"	1.64485362591
"n"	1.
"σ"	1.

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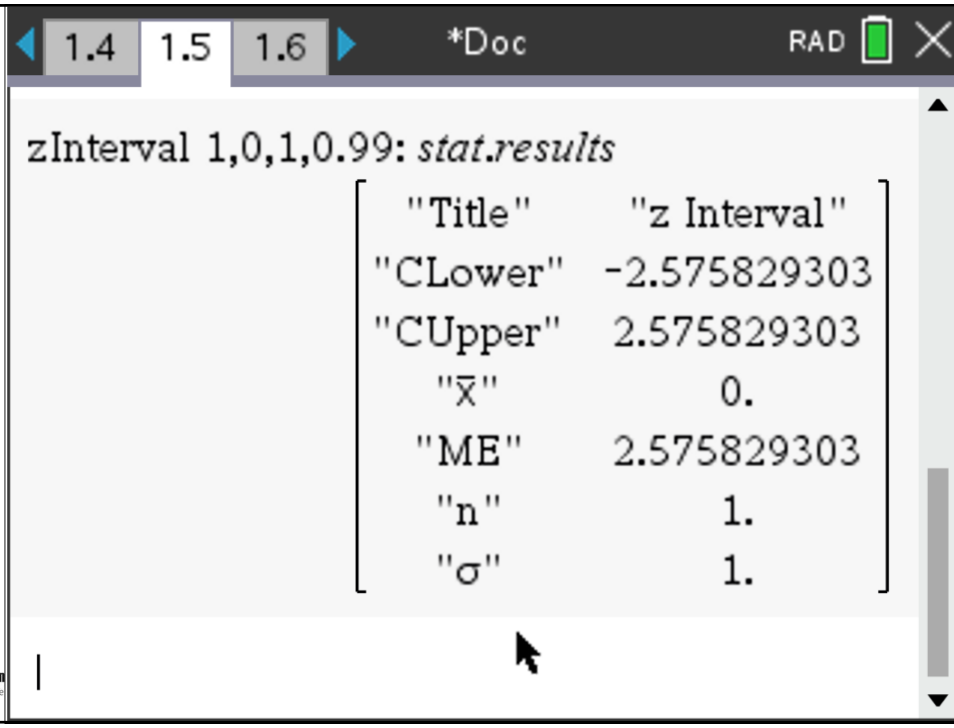

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In a random sample of 72 people, 55 have blue eyes.
Generate a 95 % CI Interval using this sample to test the population mean

$\hat{p} = \frac{55}{72} \qquad n = 72$

$(\hat{p} - z \cdot \sigma_{\hat{p}}, \hat{p} + z \cdot \sigma_{\hat{p}})$

$(\hat{p} - ME, \hat{p} + ME)$

$ME = z \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \rightarrow ME = 1.96 \cdot \sqrt{\frac{\frac{55}{72} \cdot \frac{17}{72}}{72}}$

$\boxed{\begin{array}{l} ME \approx 0.098098 \\ \hat{p} \approx 0.76389 \end{array}}$

$(0.76389 - 0.098098, 0.76389 + 0.098098)$

$\boxed{(0.6658, 0.8620)}$

zInterval_1Prop 55,72,0.95: *stat.results*

"Title"	"1−Prop z Interval"
"CLower"	0.665792018512
"CUpper"	0.861985759265
"p̂"	0.763888888889
"ME"	0.098096870376
"n"	72.

$1.96 \cdot \frac{\sqrt{\frac{55}{72} \cdot \frac{17}{72}}}{\sqrt{72}}$ 0.09809867289

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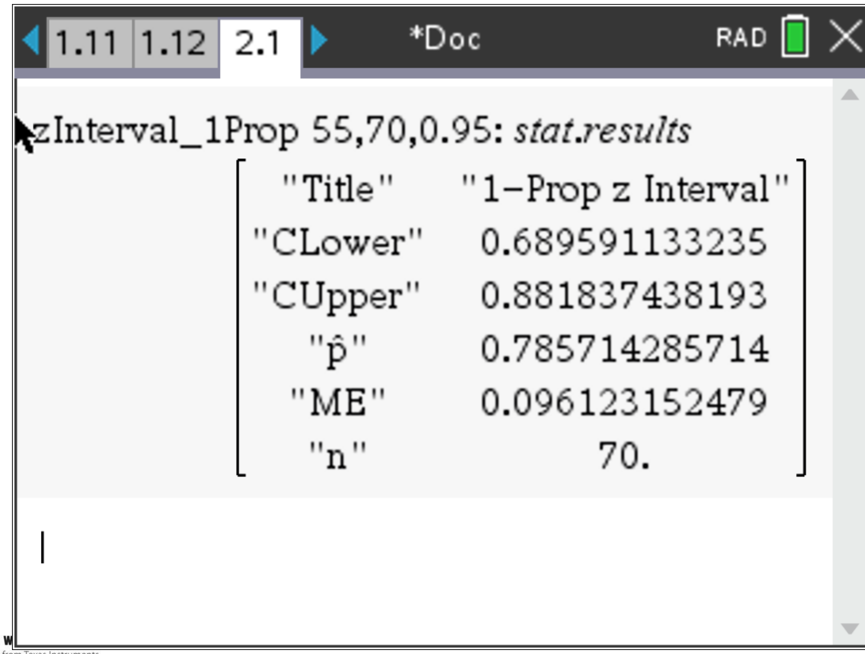

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This means that 95 % of the samples with size 72 when taken from a population will have a sample mean that lies in the Interval **(0.6668 - 0.8612)**

There will approximately be 5% samples where the sample mean is likely to be less on greater than the interval

Approximately 5% will lie outside the Interval

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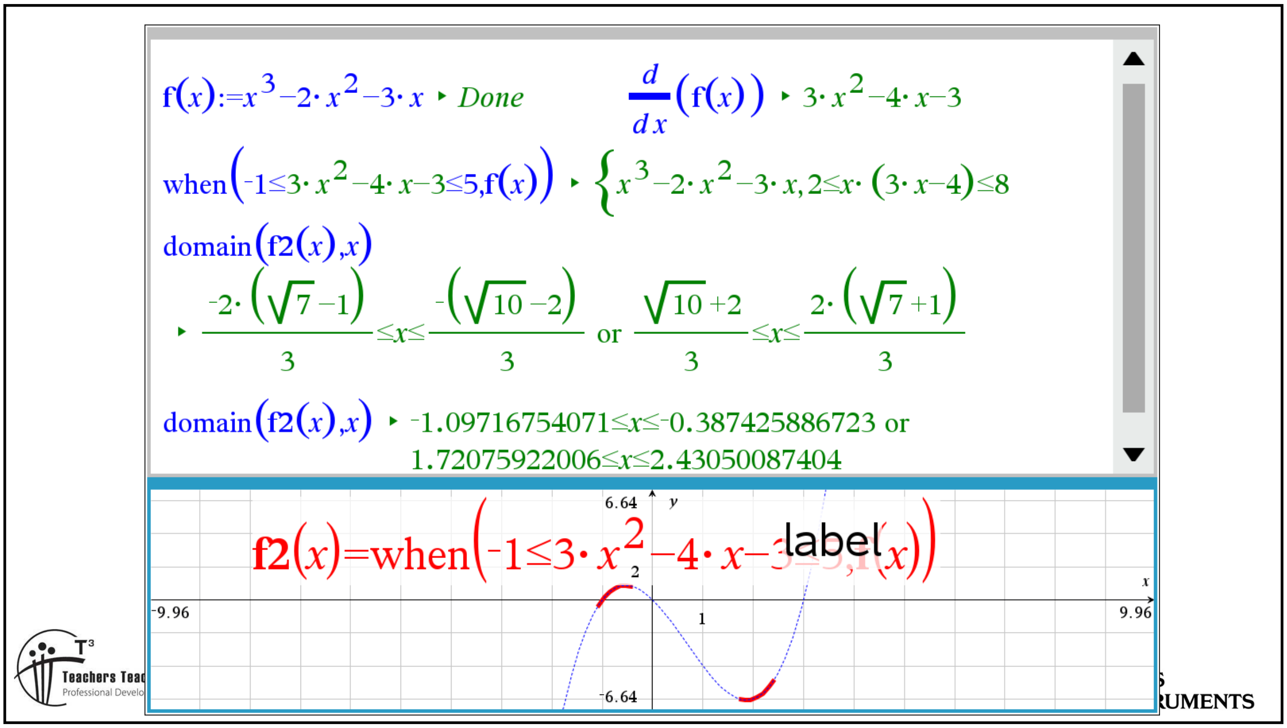
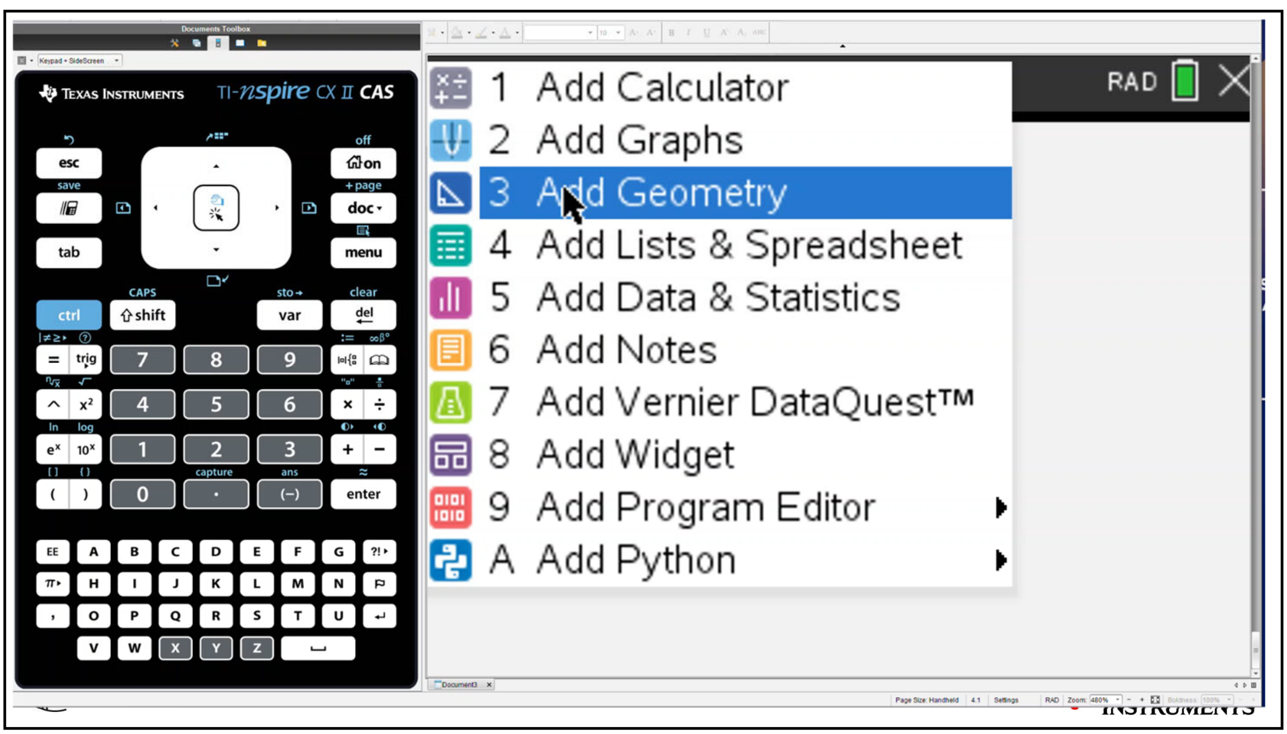

f(x):=x^3−2·x^2−3·x ▸ Done
d/dx(f(x)) ▸ 3·x^2−4·x−3
when(-1≤3·x^2−4·x−3≤5,f(x)) ▸ {x^3−2·x^2−3·x, 2≤x·(3·x−4)≤8
domain(f2(x),x)
▸ -2·(√7−1)/3 ≤x≤ -(√10−2)/3 or (√10+2)/3 ≤x≤ 2·(√7+1)/3
domain(f2(x),x) ▸ -1.09716754071≤x≤-0.387425886723 or 1.72075922006≤x≤2.43050087404
f2(x)=when(-1≤3·x^2−4·x−3≤5,f(x))
label
6.64
-6.64
-9.96
9.96
y
x
1
2

75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1 Add Calculator
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5 Add Data & Statistics
6 Add Notes
7 Add Vernier DataQuest™
8 Add Widget
9 Add Program Editor
A Add Python

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