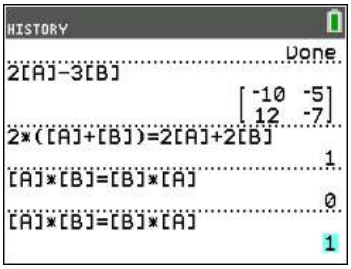
LEC QLD Session 1G: Specialist Mathematics – Complex Numbers and Matrices in Units 1 and 2 (Simpson Room)

Unit 1 Topic 5 Matrices

Verifying matrix properties

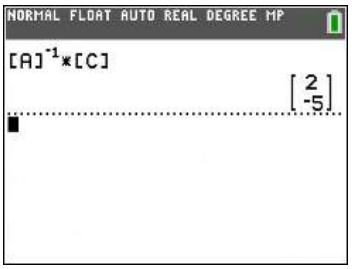


Solving systems of equations

Solve the following system of linear equations

$$-2x+3y=-19$$
$$5x-2y=20$$

(a) Using matrix inverse method
(b) Using the PolySmlt2 App.
(c) Using the Reduced Row Echelon Form (rref) command.



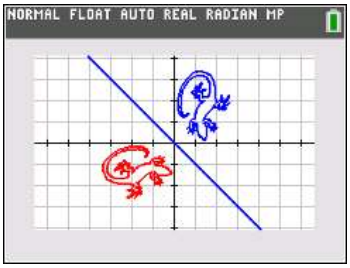
Note: How will your calculator handle solving this system?

$$-2x+3y=-19$$
$$4x-6y=38$$

Unit 2 Topic 5 Matrices and Transformations

Representing transformations

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$



Dilations & Reflections

```
PROGRAM:TRANSMAT
:{0,1,1,0,0}→L1
:{0,0,1,1,0}→L2
:Plot1(xyLine,L1,L2,·)
:Prompt A,B,C,D
:A*L1+B*L2→L3
:C*L1+D*L2→L4
:Plot2(xyLine,L3,L4,·)
:DispGraph
:Pause
```

Rotation

```
PROGRAM:ROTMAT
:{0,1,1,0,0}→L1
:{0,0,1,1,0}→L2
:Plot1(xyLine,L1,L2,·)
:Prompt θ
:cos(θ)→A
:-sin(θ)→B
:sin(θ)→C
:cos(θ)→D
:A*L1+B*L2→L3
:C*L1+D*L2→L4
:Plot2(xyLine,L3,L4,·)
:DispGraph
:Pause
```

Reflection in y = mx

```
PROGRAM:REFLMAT
:{0,1,1,0,0}→L1
:{0,0,1,1,0}→L2
:Plot1(xyLine,L1,L2,·)
:Prompt M
:tan⁻¹(M)→θ
:cos(2θ)→A
:sin(2θ)→B
:sin(2θ)→C
:-cos(2θ)→D
:A*L1+B*L2→L3
:C*L1+D*L2→L4
:Plot2(xyLine,L3,L4,·)
:"M*X"→Y1
:DispGraph
:Pause
```

Unit 2 Topic 1 Complex numbers

Exploring roots of a quadratic polynomial over $\mathbb{Q}, \mathbb{R}$ and $\mathbb{C}$

Consider the following quadratic polynomials:

- $p_1(x) = x^2 - 1$
- $p_2(x) = x^2 + 1$
- $p_3(x) = x^2 - 4x + 4$
- $p_4(x) = x^2 - 4x - 5$
- $p_5(x) = x^2 - 4x + 1$
- $p_6(x) = x^2 - 4x + 13$

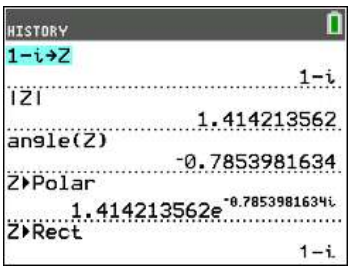
(a) For each case, find the roots of the polynomial. Comment on the results in each case.

(b) For each case, find the discriminant of the polynomial and describe any apparent connections between the discriminant of the polynomial and its real and complex roots.

(c) Some of the polynomials have no real roots and their complex roots include the complex number $\boldsymbol{i}$. In these cases, multiply the pair of roots for each polynomial and suggest a definition for the value of $\boldsymbol{i}$.

Unit 2 Topic 2 Complex arithmetic and algebra

Cartesian and polar form



Solving real quadratic equations with complex conjugates

Verify that $\dfrac{3 - i\sqrt{31}}{2}$ is a root of the quadratic polynomial $p(z) = z^2 - 3z + 10$.

Hence express $p(z)$ as the product of linear factors over $\mathbb{C}$.