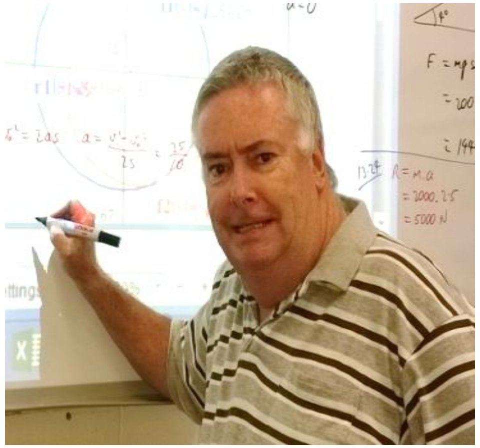
TI-30X Plus Mathprint

1D: Consumer Arithmetic, Loans, Investments & Annuities

Brian Lannen



Brian Lannen

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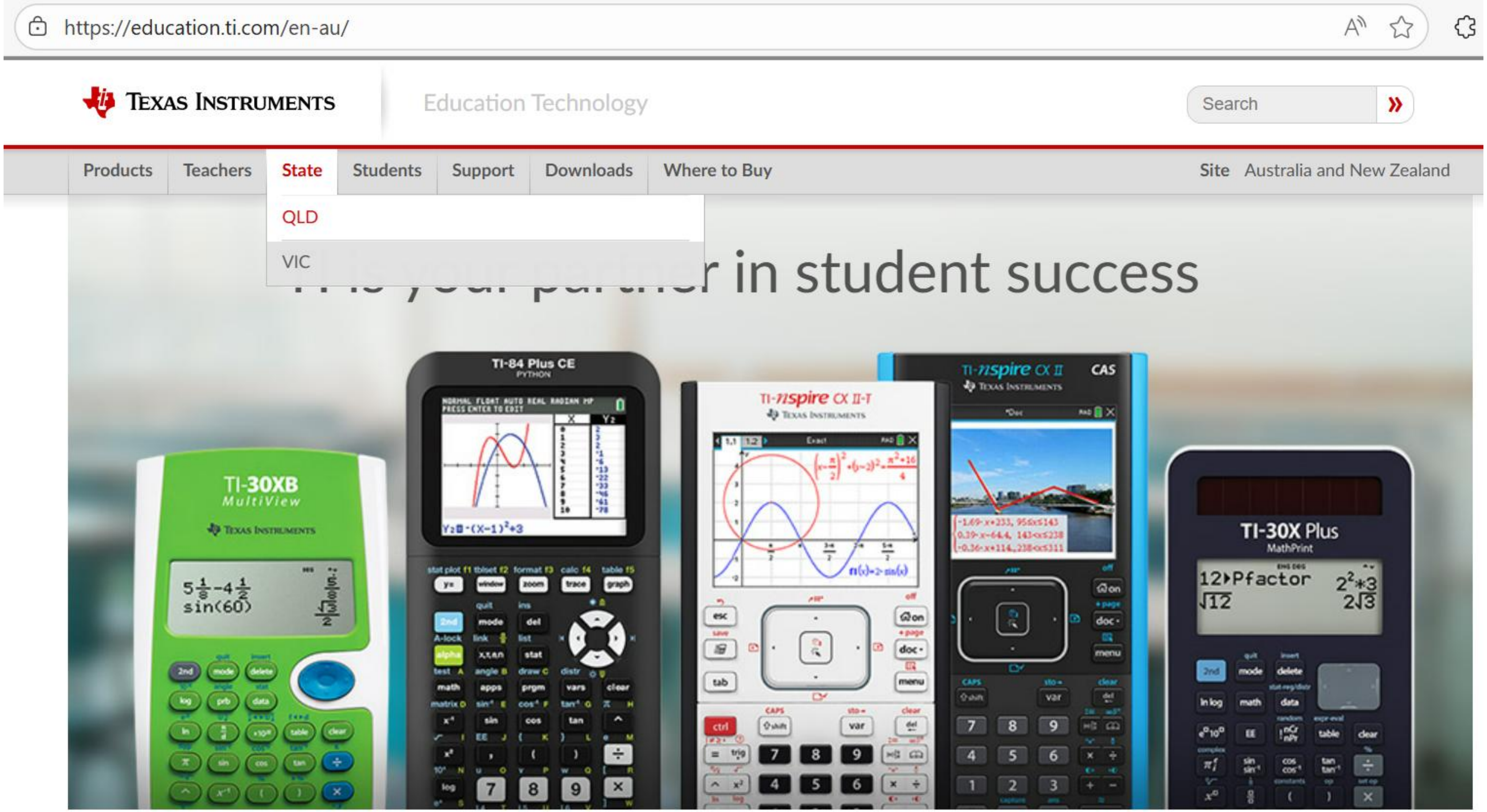
Consumer Arithmetic, Loans, Investments & Annuities (TI-30X Plus Mathprint)

Participants in this session will work through a range of activities from the new QCE General Mathematics Teacher Resource Book for TI-30XPlus scientific calculator. Focus areas will be on:

- Use of a spreadsheet to display example computations when multiple or repeated computations are required.
- Use of the compound interest formula to model a compound interest loan or investment.
- Effective annual rate of interest.
- Practical problems involving compound interest loans.



TI calculators and resources are built with the classroom in mind —

Selected activities from the Teacher Resource Book

Activity Name	Page in TRB	Unit	Syllabus reference
Task 1: Battery break-even	4	1	*Use a spreadsheet to display example computations when multiple or repeated computations are required.*
Task 16: Growing interest	55	4	*Use the compound interest formula to model a compound interest loan or investment.*
Task 17: That's effective!	58	4	*Calculate the effective annual rate of interest.*
Task 18: Mortgage matters	61	4	*Solve practical problems involving compound interest loans.*

Task 1: Battery break-even

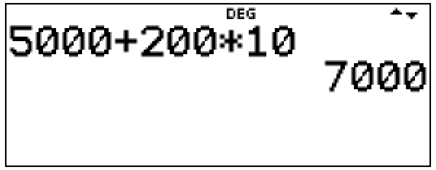
Topic 1: Consumer arithmetic > Applications of rates, percentages and use of spreadsheets

Focus: Use a spreadsheet to display example computations when multiple or repeated computations are required.

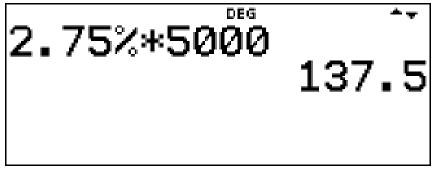
Brian has solar panels on his house roof and is considering adding a battery to the system to further reduce his power bills. He has seen an offer for a 9.6 kWh battery costing $5000 upfront, plus $200/year for 10 years. Brian presently has $5000 in an investment that returns 2.75% p.a. simple interest. From examining his power bills, he is confident that on most days his rooftop solar panels will be able to fully charge the battery. He has also calculated that he will use all the stored power each night with about 4 kWh being used in peak tariff time and the remainder in non-peak time.

He is presently being charged 0.433 $/kWh in peak time and 0.224 $/kWh in non-peak. He receives a solar feed-in credit of 0.04 $/kWh.

(a) With an upfront payment of $5000 plus $200/year for 10 years, how much in total will Brian pay for the battery?



(b) After paying the initial $5000, Brian would no longer earn the 2.75% p.a. simple interest. How much interest (to the nearest cent) will he then miss out on each year?



(c) Develop an expression in terms of years that represents the cumulative amount paid for the battery with $5000 upfront plus the loss of interest each year [your answer from part **(b)**], plus the $200/year for 10 years.

Answer: Cumulative amount paid = $\$(5000 + 337.5 \times n)$.

Task 1: Battery break-even

Topic 1: Consumer arithmetic > Applications of rates, percentages and use of spreadsheets

Focus: Use a spreadsheet to display example computations when multiple or repeated computations are required.

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He is presently being charged 0.433 $/kWh in peak time and 0.224 $/kWh in non-peak. He receives a solar feed-in credit of 0.04 $/kWh.

(d) How much feed-in credit to the nearest tenth of a cent does Brian miss out on each day to charge the 9.6 kWh battery?

```
0.04*9.6          0.384
```

Answer: $0.384 (or 38.4 cents)

(e) Using the average daily estimates of 4 kWh of battery power used in peak time (saving 0.433 $/kW and the rest in non-peak time (saving 0.224 $/kWh) and subtracting the lost feed-in tariff [your answer from part **(d)**], how much is the daily saving to Brian's power bill to the nearest tenth of a cent?

Enter as shown.

$$4 \times 0.433 + 5.6 \times 0.224 - 0.384$$

Answer: savings = $2.6024/day

```
◂.6*0.224-0.384
                2.6024
```

(f) Use your result from part **(e)** to develop an expression in terms of years that reflects the total cumulative savings achieved by using the battery.

Approximate yearly savings = daily savings × 365.

Answer: Cumulative savings = $\$(949.876 \times n)$

Task 1: Battery break-even

Topic 1: Consumer arithmetic > Applications of rates, percentages and use of spreadsheets

Focus: Use a spreadsheet to display example computations when multiple or repeated computations are required.

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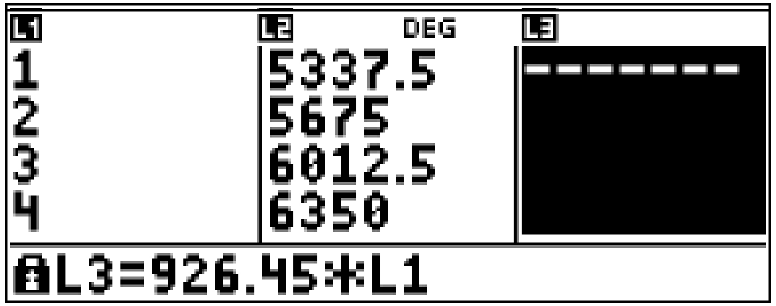
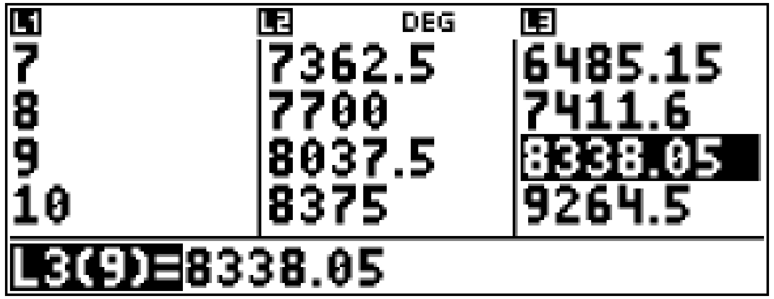
He is presently being charged 0.433 $/kWh in peak time and 0.224 $/kWh in non-peak. He receives a solar feed-in credit of 0.04 $/kWh.

(g) Use the expressions from parts **(c)** and **(f)** to construct a spreadsheet or lists and use this to determine the "break even" point (in number of years) for the battery investment.





Answer: The battery will have paid for itself after 9 years.

Task 16: Growing interest

Topic 1: Loans, investments and annuities 1 > Compound interest loans and investments

Focus: Use the compound interest formula to model a compound interest loan or investment.

Daisy, a hard-working mathematics student, was the lucky winner of a scholarship lump sum payment of $2000. Of course, she was absolutely delighted but not really needing to use the money straight away, she thought she would put it into an investment account so that she could withdraw it later to help her with her studies. When she went to her bank, Daisy saw that there were two different investment options available:

- *Simple interest paid at 3.1% p.a.*
- *Compound interest at 2.8% p.a.*

INVESTMENT ACCOUNT	INVESTMENT ACCOUNT
3.1% per annum	2.8% per annum
(simple interest paid annually)	(compound interest paid annually)

In this task we consider which investment option Daisy should choose, and whether there are any other factors that could affect her decision.

(a) How much interest will the simple interest plan pay per year and what will be the balance of the investment after 1 year?

```
2000*3.1%        62
ans+2000       2062
```

(b) How much interest will the compound interest plan pay in the first year and what will be the balance of the investment after that 1 year?

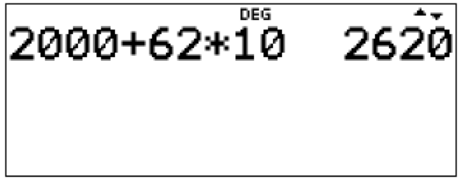
```
2000*0.028       56
ans+2000       2056
```

Task 16: Growing interest

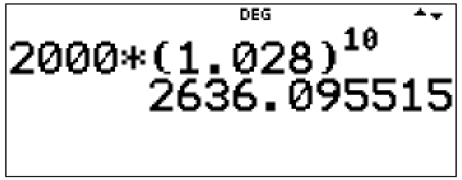
Topic 1: Loans, investments and annuities 1 > Compound interest loans and investments

Focus: Use the compound interest formula to model a compound interest loan or investment.

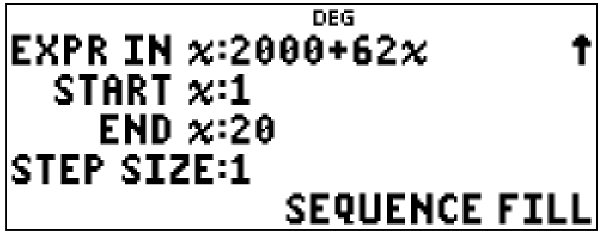
(c) How much would the investment be worth after 10 years of the simple interest plan?

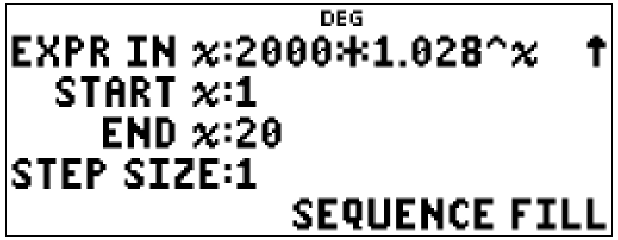


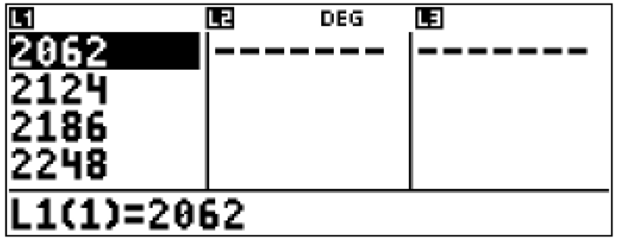
(d) To the nearest cent, how much would the investment be worth after 10 years of the compound interest plan?

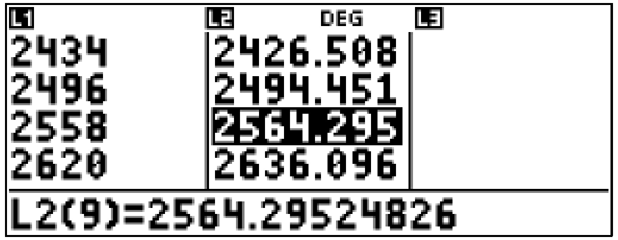


(e) During what year would both investment plans each accumulate to the same value?









Scroll down the lists until the value in L2 is greater than that in L1. This is the year during which the balance of the compound interest plan will draw level with and then exceed the simple interest plan.

Answer: Savings will be the same during the 9th year.

Task 16: Growing interest

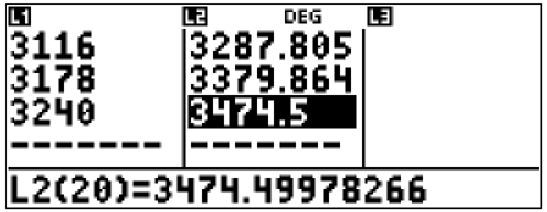
Topic 1: Loans, investments and annuities 1 > Compound interest loans and investments

Focus: Use the compound interest formula to model a compound interest loan or investment.

(f) If Daisy kept the investment for 20 years, which would be the better investment plan and by how much (rounded to nearest cent)?

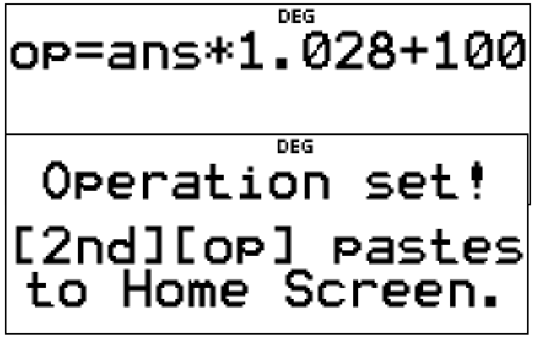
Scroll down the lists to year 20 and note the values in each list. After 20 years the value of the compound interest investment will have grown to $3474.50 and the simple interest plan to $3240.

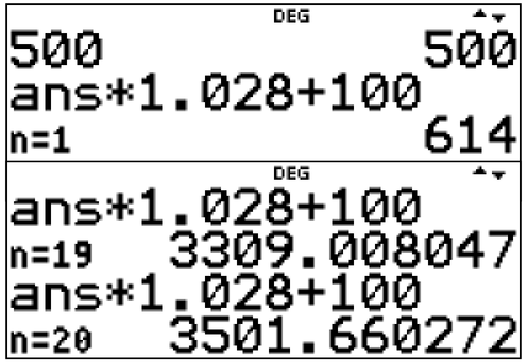
Answer: After 20 years, the compound interest plan will be better by $234.50 (rounded to nearest cent).



(g) Daisy's friend, Emily, also wanted to build an investment for 20 years into the future but wasn't fortunate enough to win a similar scholarship. Instead, she did have $500 in savings which she decided to deposit into the compound interest plan on her birthday and then make an additional deposit of $100 on her birthday every year for the next 20 years. How much will Emily's investment grow to after 20 years (rounded to nearest cent)?

- Press [2nd][×] to enter the operation setup screen.
- Press [2nd][(−)] to paste **ans** to the start of the operation, then [×] 1.028 [+] 100 and arrow down twice. The operation is now set.
- Clear the home screen. Type 500 and press [enter].
- Press [2nd][)] to see the account balance after 1 year. Note the **n=1** counter at the bottom left of screen.
- Repeat pressing [2nd][)] to see the account balance until the counter displays **n=20**. This now shows the account balance after 20 years.





Answer: The account balance will grow to $3501.66 after 20 years.

Texas Instruments

Task 17: That's effective!

Topic 1: Loans, investments and annuities 1 > Compound interest loans and investments

Focus: Calculate the effective annual rate of interest.

(a) Calculate the effective annual rate of interest for the following nominal interest rates and compounding periods:

(i) 4.2% p.a. compounding daily

(ii) 4.6% p.a. compounding weekly

(iii) 5.1% p.a. compounding monthly

(iv) 5.4% p.a. compounding quarterly

(v) 5.7% p.a. compounding six-monthly

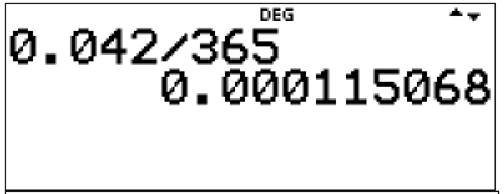
(i) Use the effective interest rate formula

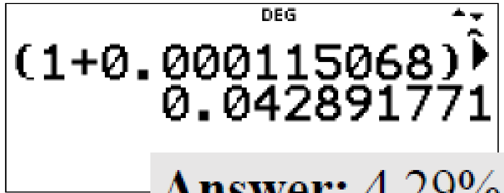
$i_{effective} = (1+i)^k - 1$ where i is interest rate per compounding period and k is number of compounding periods per year.

Divide the nominal annual interest rate by 100 to convert it to a decimal and then divide it by 365 to find the daily interest rate.

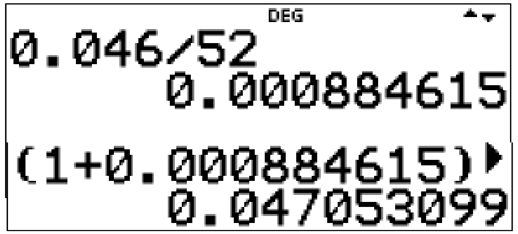
Use this value for i in the effective interest rate formula and 365 for k. Calculate as shown.

Multiply this result by 100 to find the effective annual interest rate.

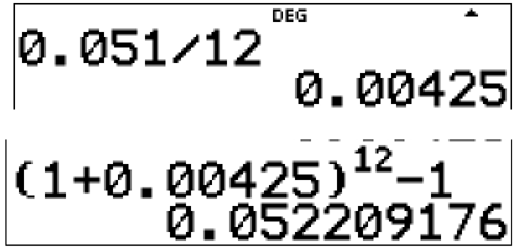




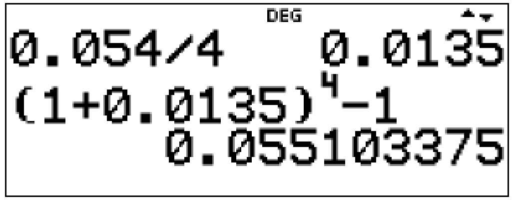
Answer: 4.29%



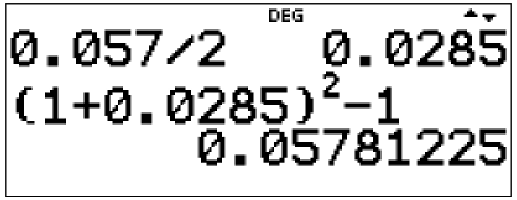
Answer: 4.71%



Answer: 5.22%



Answer: 5.51%



Answer: 5.78% (rounded to 2 decimal places).

Texas Instruments

Task 17: That's effective!

Topic 1: Loans, investments and annuities 1 > Compound interest loans and investments
Focus: Calculate the effective annual rate of interest.

Sharni is considering a car loan of \$40 000. Her bank is offering two different interest rate options:

Option A: 7.7% p.a. compounding quarterly *Option B*: 7.6% p.a. compounding weekly

(b) Calculate the effective annual interest rate for each of these options.

```
0.077/4      0.01925
(1+0.01925)^4-1
         0.079252046
```

```
          0.001461538
(1+0.001461538)▸
          0.078902684
```

Use $i_{nominal} = 7.7\%$ and $k = 4$ to find the effective interest rate for *Option A* and $i_{nominal} = 7.6\%$ and $k = 52$ to find the effective interest rate for *Option B*.

Answer: *Option A*: $i_{effective} = 7.93\%$.
Option B: $i_{effective} = 7.89\%$.
(rounded to 2 decimal places)

(c) How much interest would Sharni pay in the first year for each of these options?

Enter calculations as shown.

Answer:
Option A: loan repayments for first year = \$3170.08
Option B: loan repayments for first year = \$3156.11

```
0.079252046*400▸
          3170.08184
0.078902684*400▸
          3156.10736
```

(d) Which option should she choose and how much would that option save her in the first year compared to the other option?

Answer:
Sharni should choose *Option B* with a saving of \$13.97.

```
0.078902684*400▸
          3156.10736
3170.08184-3156▸
            13.97448
```

Task 17: That's effective!

Topic 1: Loans, investments and annuities 1 > Compound interest loans and investments

Focus: Calculate the effective annual rate of interest.

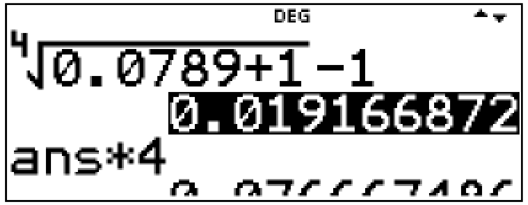
(e) The bank has decided to adjust the per annum rate of the quarterly option so that it matches the same effective interest rate of the weekly option. Find the required rate of the quarterly option.

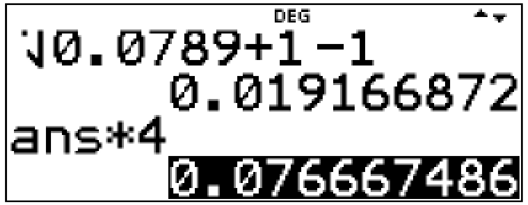
(e) The effective interest rate formula can be rearranged algebraically to make $i_{nominal}$ the subject.

$$i_{nominal} = \sqrt[k]{i_{effective} + 1} - 1$$

- Type [2nd] [$x^{\square}$] to access the n^{th} root template.
- Complete as shown to find the quarterly nominal interest rate.
- Multiply this result by 4 to find the annual nominal interest rate (decimalized) that will give an effective quarterly interest rate = 7.89% (the same as the effective interest rate for Option B, weekly compounding)
- Multiply this result by 100 to convert to a percentage.

Answer: 7.67% (rounded to 2 decimal places)





Task 17: That's effective!

Topic 1: Loans, investments and annuities 1 > Compound interest loans and investments

Focus: Calculate the effective annual rate of interest.

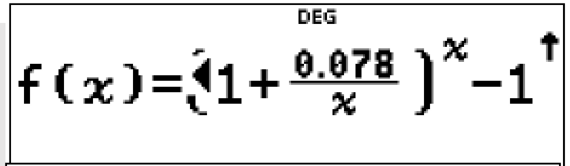
(f) A different bank is advertising loans at 7.8% p.a. Set up a function where the number of compounding periods per year is the variable and use this function to determine the effective interest rates for compounding periods of six-monthly, quarterly, monthly and daily.

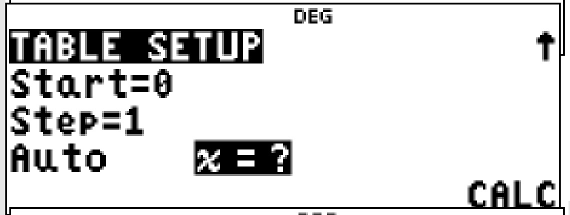
(f) Enter commands as follows:

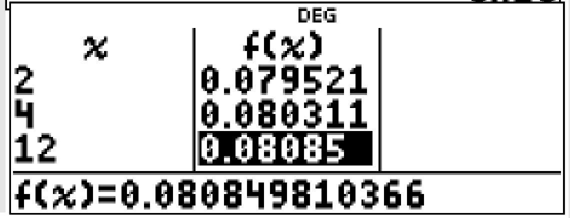
- Define $f(x)$ for the table feature as shown.

$$f(x) = \left(1 + \frac{0.078}{x}\right)^x - 1$$

- Leave $g(x)$ blank and arrow down to set up the table as shown making the selection ***x* = ?**
- The effective interest rates for the different compounding periods can now be found by entering different values for x (2 for 6-monthly, 4 for quarterly, 12 for monthly and 365 for daily).







Answer:

- Effective interest rate for compounding every
  - six months = 7.95%.
  - three months = 8.03%.
  - one month = 8.09%.
  - one day = 8.11%.

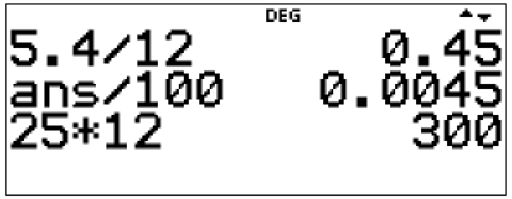
- Alternatively the ***f*(** command can be accessed from the **table** menu and different variables for x can be used to complete the command.

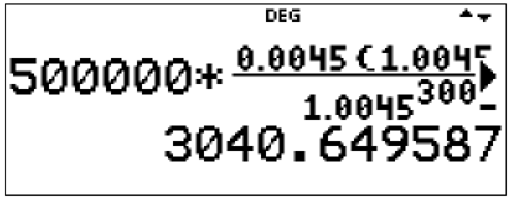
Task 18: Mortgage matters

Topic 1: Loans, investments and annuities 1 > Compound interest loans and investments
Focus: Solve practical problems involving compound interest loans.

A newly married couple Chris and Chandra are planning to borrow $500 000 in a 25-year mortgage to purchase their first home together. The bank's current variable interest rate is 5.4% p.a. compounding monthly.
They can afford to make monthly payments for this now but are concerned by the impact that rising interest rates may have on their monthly repayments.
Answer the following, rounding your results to the nearest cent.

(a) Based on current figures, how much is the monthly repayment for this loan?





Answer: $3040.65 (rounded to the nearest cent)

(a) Use the mortgage repayment formula:

$$M = P \times \frac{i(1+i)^n}{(1+i)^n - 1}$$

where M = mortgage repayments per month
P = principal loan amount
i = monthly interest rate
n = number of months

- Note that i = the monthly interest rate as a decimal. (Divide the per annum rate by 12 to make it monthly and then divide by 100 to make it decimal.)
- Note that number of months = 25 × 12 = 300.
  Use these values in the mortgage repayment formula.

$$500000 \times \frac{0.0045 \times (1.0045)^{300}}{(1.0045)^{300} - 1}$$

Task 18: Mortgage matters

Topic 1: Loans, investments and annuities 1 > Compound interest loans and investments

Focus: Solve practical problems involving compound interest loans.

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They can afford to make monthly payments for this now but are concerned by the impact that rising interest rates may have on their monthly repayments.

(b) Construct a table of values to illustrate how the monthly repayments are affected by changing interest rates from 3% p.a. to 8% p.a. at intervals of 0.1% (= 10 basis points).

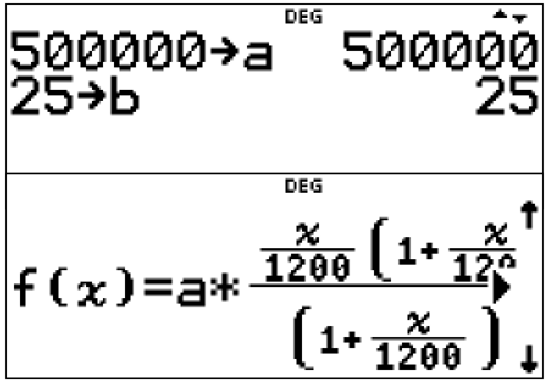
- Important values such as the loan principal and the term of the mortgage (in years) can be stored [sto→] as variables in the calculator as shown.

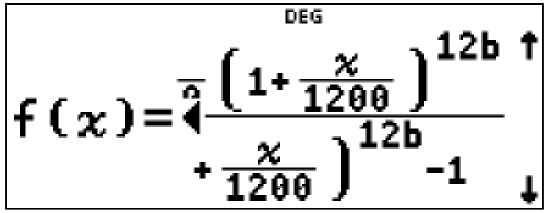
Note: The a,b variables are accessed by pressing the [x^{yzt}_{abcd}] key. Press 5 times for a, and 6 times for b.

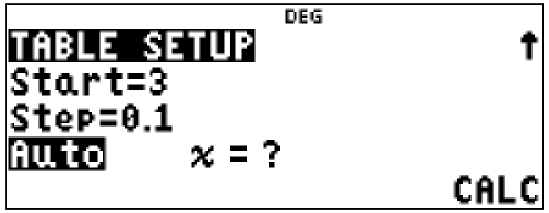
- Define $f(x)$ for the table feature as shown.

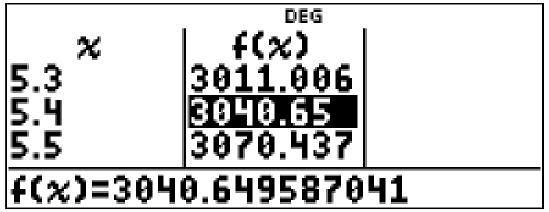
$$f(x) = a \times \frac{\frac{x}{1200}\left(1+\frac{x}{1200}\right)^{12b}}{\left(1+\frac{x}{1200}\right)^{12b} - 1}$$

- Leave $g(x)$ blank and arrow down to set up the table as shown.
- Press [enter] to see the table and scroll up and down to see repayment amounts that correspond to the different interest rates.









Task 18: Mortgage matters

Topic 1: Loans, investments and annuities 1 > Compound interest loans and investments
Focus: Solve practical problems involving compound interest loans.

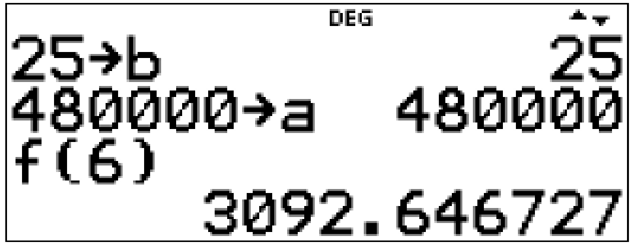
The couple estimate that at the very most they can only afford monthly loan repayments of $3100.
However, interest rates have increased to 6% p.a. The couple have options of either negotiating with the bank to increase the lifetime of the mortgage to 28 years or to purchase a less expensive home with a $480 000 mortgage.

(c) How much would the monthly payments be (at 6% p.a.) if they renegotiated the mortgage lifetime to 28 years? Is this solution within their budget?

(c) Enter commands as follows:
- Update the stored value of b (number of years) to 28.
- Press the table key and select the $f($ command.
- Complete the command to show $f(6)$.

Answer: New payment is $3075.62 (rounded to the nearest cent). Yes, this is within the budget.

(d) What would the monthly repayments be if they needed to keep the mortgage at 25 years, but instead purchased the other home with $480 000 mortgage? Is this solution within their budget?



(d) Enter commands as follows:
- Return the stored value of b (number of years) to 25.
- Update the stored value of a to 480 000
- Press the table key and select the $f($ command.
- Complete the command to show $f(6)$.

Answer: New payment is $3092.65 (rounded to the nearest cent). Yes, this is within the budget.