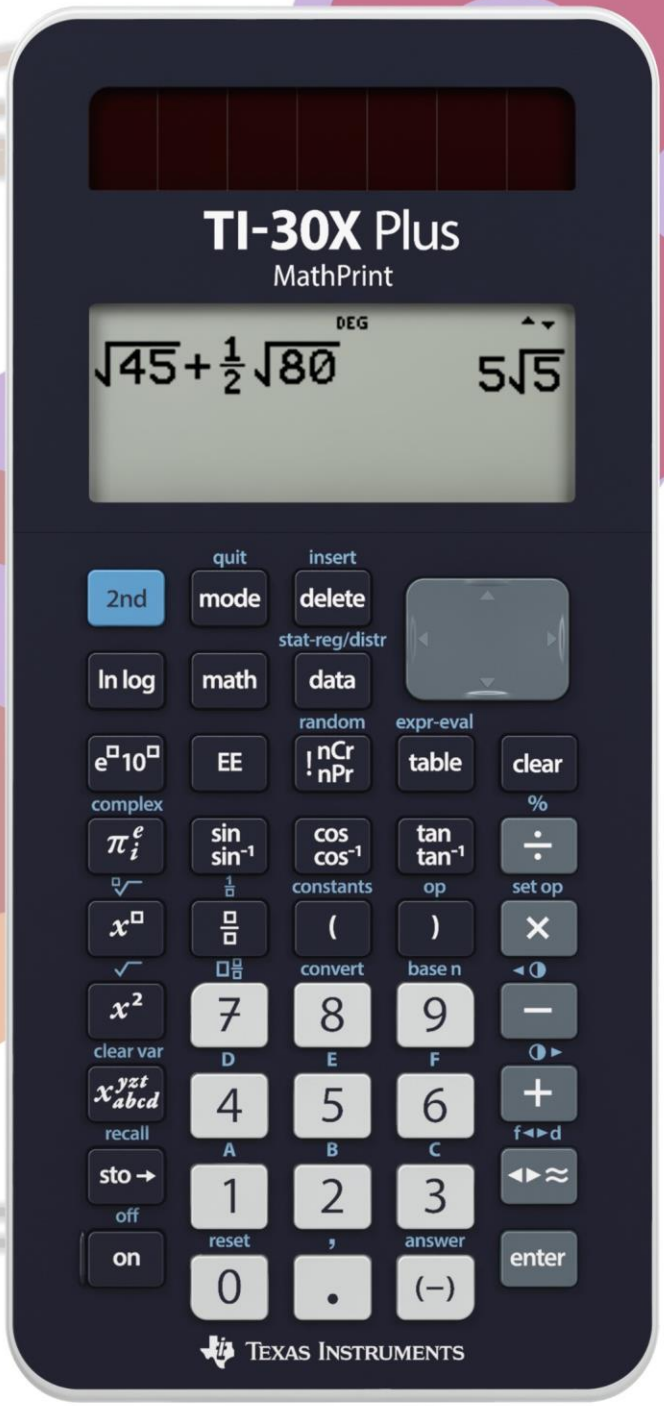

Factoring in some fun

TI-30X Plus MathPrint™

Scientific Calculator

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Factor Trees

Getting Started with Factor Trees

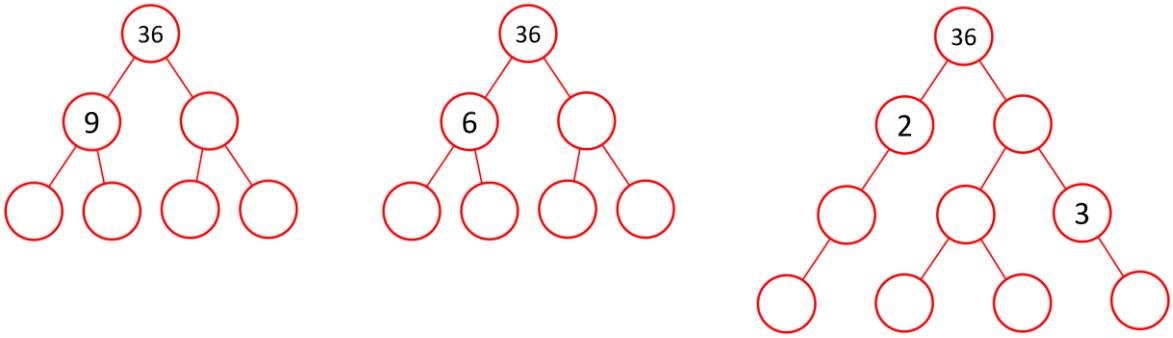
Teacher Notes:

This activity focuses on factor trees and prime factorisation, key learning includes:

- It doesn't matter what factors you choose at the start of a factor tree, the end result is the same, leading to the unique prime factorisation for the selected number. The prime factorisation of a number is it's DNA.
- The product of the numbers in any row of the factor tree is equal to the original number.
- Short-hand notation (index notation) can be used to express the prime factorisation of a number.

Question: 1.

a) Complete the prime factorisation of the number 36 in each of the following:



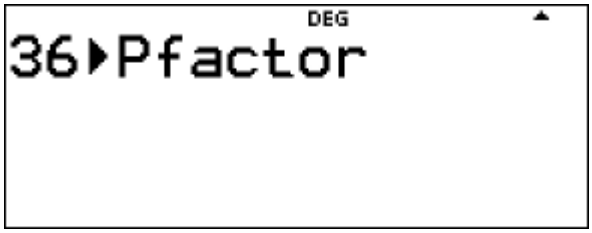
b) Calculate the product of the numbers in each row of each tree. What do you notice about this product?

Answer: The product of the factors is the same in every row.

c) What do you notice about the numbers in the bottom row of each tree?

Answer: The numbers are the same: 2 x 2 x 3 x 3 or 2^2 x 3^2.

d) On your calculator, enter the number 36, then press [math] and select item 4: ▸Pfactor.
Press [enter] to execute the command.
How does the calculator's answer relate to the factor tree?



Answer: The bottom row in the factor tree is: 2 x 2 x 3 x 3, the calculator provides the answer: 2^2 x 3^2. Index notation is being used by the calculator to display the result in a concise format.

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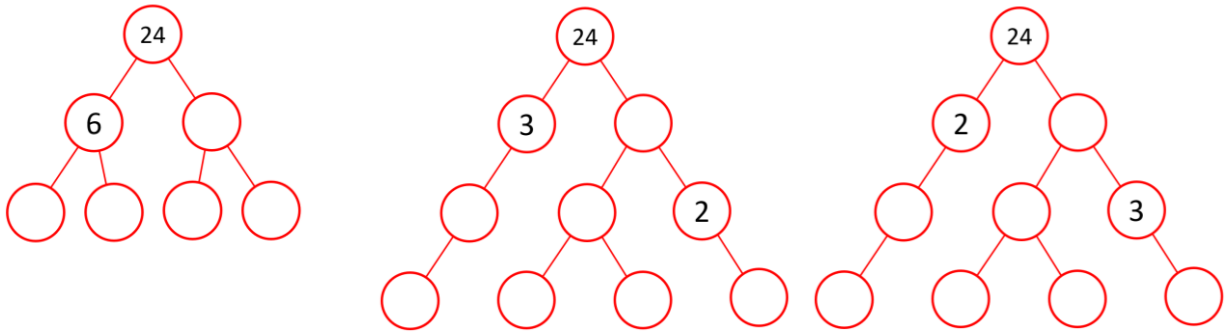
Question: 2.

a) Complete the prime factorisation of the number 24 in each of the following:



b) Calculate the product of the numbers in each row of each tree. What do you notice about this product?

Answer: The product of the factors is the same in every row.

c) What do you notice about the numbers in the bottom row of each tree?

Answer: The numbers are the same: 2 x 2 x 2 x 3 or 2^3 x 3.

d) Use your calculator to determine the prime factorisation of 24 and compare the result to the factor tree.

Answer: The bottom row in the factor tree is: 2 x 2 x 2 x 3, the calculator provides the answer: 2^3 x 3. Index notation is being used by the calculator to display the result in a concise format.

Question: 3.

Generate factor trees for each of the following numbers. Check the bottom row of the factor tree in each case using your calculator.

a) 54 = 2 x 3^3 b) 150 = 2 x 3 x 5^2 c) 126 = 2 x 3^2 x 7 d) 132 = 2^2 x 3 x 11

e) 260 = 2^2 x 5 x 13 f) 350 = 2 x 5^2 x 7 g) 210 = 2 x 3 x 5 x 7 h) 882 = 2 x 3^2 x 7^2

Question: 4.

Generate factor trees for each of the following numbers. Check the bottom row of the factor tree in each case using your calculator.
Hint: Use the factor trees from Question 3 to help.

a) 108 = 2^2 x 3^3 b) 300 = 2^2 x 3 x 5^2 c) 252 = 2^2 x 3^2 x 7 d) 264 = 2^3 x 3 x 11

e) 2600 = 2^3 x 5^2 x 13 f) 3500 = 2^2 x 5^3 x 7 g) 2100 = 2^2 x 3 x 5^2 x 7 h) 8820 = 2^2 x 3^2 x 5 x 7^2

Question: 5.

Use questions 3 & 4 to help write down the prime factorisation of each of the following.

a) 216 = 2^3 x 3^3 b) 600 = 2^3 x 3 x 5^2 c) 504 = 2^3 x 3^2 x 7 d) 528 = 2^4 x 3 x 11

e) 26000 = 2^4 x 5^3 x 13 f) 35000 = 2^3 x 5^4 x 7 g) 21000 = 2^3 x 3 x 5^3 x 7 h) 88200 = 2^3 x 3^2 x 5^2 x 7^2

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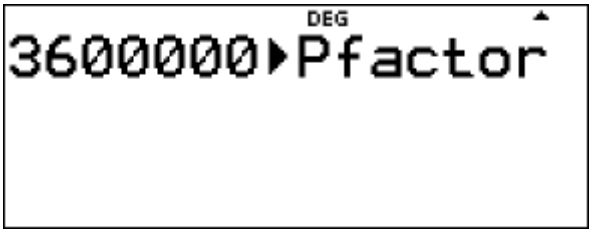
Out smarting the calculator!

The Prime Factorisation command in the calculator is limited to numbers less than 1,000,000.

Try: 3,600,000 ▸ Pfactor



3600000 = 36 x 10 x 10 x 10 x 10 x 10 and 10 = 2 x 5.

Question: 6.

Use the above tip to help determine the prime factorisation of each of the following:

a) 1,080,000
$= 2^2 \times 3^3 \times 10^4$
$= 2^2 \times 3^3 \times 2^4 \times 5^4$
$= 2^6 \times 3^3 \times 5^4$

b) 3,000,000
$= 300 \times 10^4$
$= 2^2 \times 3 \times 5^2 \times 10^4$
$= 2^2 \times 3 \times 5^2 \times 2^4 \times 5^4$
$= 2^6 \times 3 \times 5^4$

c) 2,520,00
$= 252 \times 10^3$
$= 2^2 \times 3^2 \times 7 \times 10^3$
$= 2^2 \times 3^2 \times 7 \times 2^2 \times 5^3$
$= 2^4 \times 3^2 \times 5^3 \times 7$

d) 2,640,000
$= 264 \times 10^4$
$= 2^3 \times 3 \times 11 \times 10^4$
$= 2^7 \times 3 \times 5^4 \times 11$

Climbing the tree

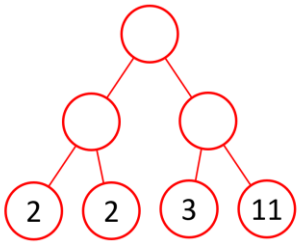
It is possible to start at the base of the factor tree and climb back up!

Question: 7.

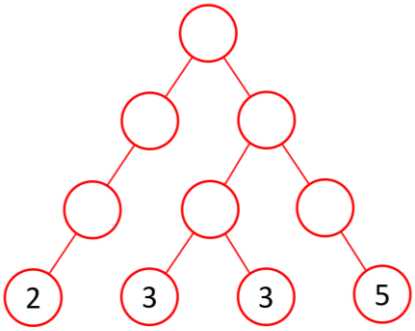
Complete the reverse factor trees (below) to determine the original number and to answer the subsequent questions:

a) Complete the reversed factor trees (below)

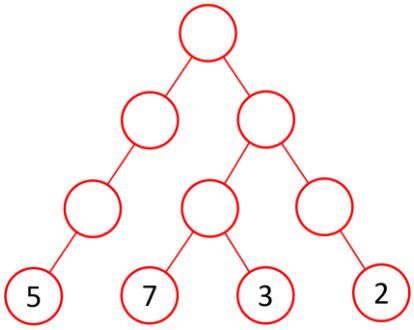
(i)



(ii)



(iii)



b) Which one or more of the original values is a multiple of 7?

Answer: Factor tree (iii) since 7 is one of its prime factors.

c) Which one or more of the original values is a multiple of 9?

Answer: Factor tree (ii) since it is the only one with 3 listed twice amongst its prime factors.

d) Which one or more of the original values is a multiple of 35?

Answer: Factor tree (iii) since it is the only one with 3 and 7 amongst its prime factors.

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Question: 8.

Which one (or more) of the following numbers is NOT a factor of $2^2 \times 3^3 \times 5 \times 7^2 \times 13$. [Multiple Choice]

a) 49 b) 45 c) 260 d) 11

Answer: (d) – The prime factorisation does not include 11, therefore it cannot be a factor of the original number. $49 = 7^2$, $45 = 3^2 \times 5$ and $260 = 2^2 \times 5 \times 13$

Question: 9.

Two numbers are multiplied together: $2^2 \times 3^2 \times 5$ and $3^2 \times 5 \times 7$. Which one of the following number is NOT a factor of this product? [Multiple Choice]

a) 49 b) 45 c) 150 d) 1575

Answer: (a) – While the prime factorisation includes 7, it only appears once, but $49 = 7^2$. Furthermore: $45 = 3^2 \times 5$ (included in the overall prime factorisation), $150 = 2 \times 3 \times 5^2$ which appears in the prime factorisation and $1575 = 3^2 \times 5^2 \times 7$, all of which are a subset of the prime factors.

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HCF & LCM

HCF = Highest Common Factor

Teacher Notes:

HCF = Highest Common Factor, the calculator refers to this as GCD = Greatest Common Divisor.
LCM = Lowest Common Multiple.
These values are extremely important for a range of mathematical operations, particularly common tasks such as adding fractions.
In addition to learning about HCF and LCM, this activity introduces Venn diagrams and factorising, specifically in reference to index notation.

Question: 1.

a) List all the factors of 36.

Answer: 1, 2, 3, 4, 6, 9, 12, 18, 36

b) List all the factors of 42.

Answer: 1, 2, 3, 6, 7, 14, 21, 42

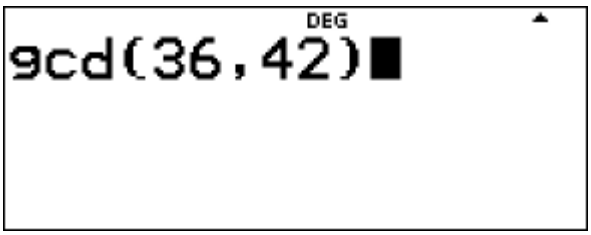
c) Comparing your answers to parts (a) & (b), what factors do 36 and 42 have in common?

Answer: 1, 2, 3, 6

d) Using your answer to part (c), what is the highest (largest) factor that 36 and 42 have in common?

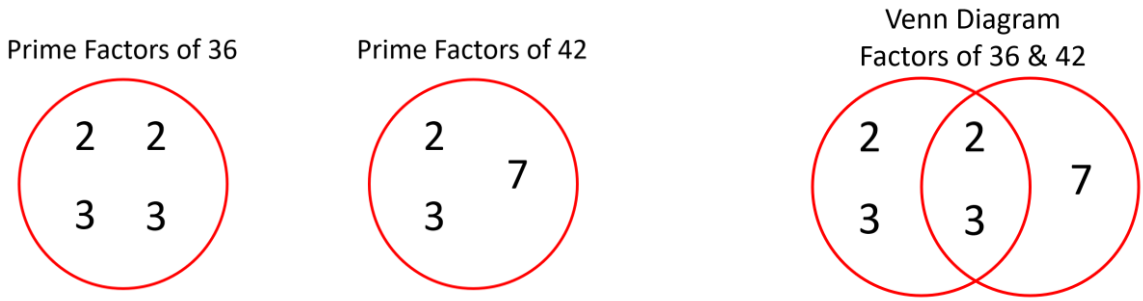
Answer: 6

e) The TI-30XPlus MathPrint calculator has a command called: gcd(#,#). This command can be accessed by pressing: [math] . Perform the calculation shown opposite and write down the result. [1]



Answer: 6

The prime factors of 36 and 42 have been placed into two separate circles (below left). These circles have been moved together so they overlap (below right). The section where the circles overlap holds the 'prime' factors that these two numbers have in common.



[1] GCD = Greatest Common Divisor is the same as Highest Common Factor

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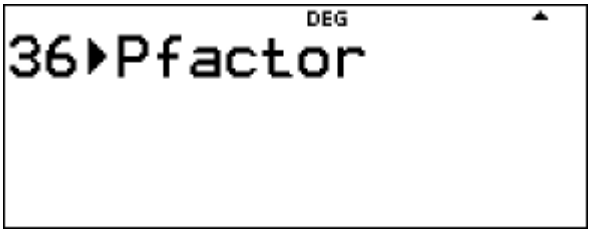
f) What is the product of the two prime factors that 36 and 42 have in common?

Answer: 2 x 3 = 6.



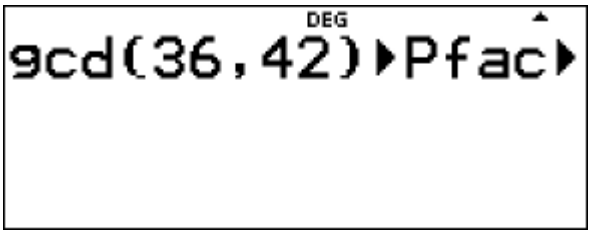
g) Use your calculator to express 36 and 42 as products of their prime factors.

Answer: 36 = 2 x 2 x 3 x 3 or 2^2 x 3^2.

42 = 2 x 3 x 7

h) Multiple commands can be used in a single line on the calculator. Combine gcd(36,42) with ▸Pfactor and explain how this relates to the Venn diagram.

Answer: The calculator writes: 2 x 3, the prime factorisation of the Highest Common Factor.



Teacher Notes:

This is an opportunity to show that the highest common factor of 2^2 x 3^2 and 2 x 3 x 7 is 2 x 3, working towards the algebraic factorisation of **expressions such as:** $6a^2b^2$ and 7ab. Students may make reference to this in their response to the previous question, however there are several more examples of this to come.

Question: 2.

Complete the table below. The first row has been done for you.

First Number #1	Second Number #2	Highest Common Factor	Prime Factorisation #1	Prime Factorisation #2	Prime Factorisation HCF
48	60	12	2^4 x 3	2^2 x 3 x 5	2^2 x 3
90	210	30	2 x 3^2 x 5	2 x 3 x 5 x 7	2 x 3 x 5
105	42	21	3 x 5 x 7	2 x 3 x 7	3 x 7
66	220	22	2 x 3 x 11	2^2 x 5 x 11	2 x 11
60	220	20	2^2 x 3 x 5	2^2 x 5 x 11	2^2 x 5
260	195	65	2^2 x 5 x 13	3 x 5 x 13	5 x 13
306	360	18	2 x 3^2 x 17	2^3 x 3^2 x 5	2 x 3^2
207	460	23	2^2 x 5 x 23	3^2 x 23	23
660	420	60	2^2 x 3 x 5 x 11	2^2 x 3 x 5 x 7	2^2 x 3 x 5

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Question: 3.

The table below is similar to the previous question, however, this time the prime factorisation has been done. In each case, try and calculate the highest common factor **before** determining the actual numbers.
Note: Once you have the 'First' and 'Second' number, use your calculator to check the highest common factor.

Prime Factorisation #1	Prime Factorisation #2	Prime Factorisation HCF	Highest Common Factor	First Number #1	Second Number #2
$2^3 \times 3 \times 7$	$2^2 \times 3 \times 5$	$2^2 \times 3$	12	168	60
$2^2 \times 3^2 \times 11$	$2^3 \times 5 \times 7$	2^2	4	396	280
$3^3 \times 5 \times 7$	$2 \times 3^2 \times 13$	3^2	9	945	234
$2^2 \times 3 \times 11^2$	$2^3 \times 5 \times 11$	$2^2 \times 11$	44	1452	440
$2^5 \times 3 \times 5$	$2^3 \times 5^2 \times 11$	$2^3 \times 5$	40	480	2200
$2^2 \times 5 \times 13^2$	$2 \times 3 \times 5 \times 13$	$2 \times 5 \times 13$	130	3380	390
$2^2 \times 3^2 \times 17$	$2^3 \times 3^2 \times 5$	$2^2 \times 3^2$	36	612	360
$2^2 \times 5^2 \times 23$	$2 \times 3^2 \times 23$	2×23	46	2300	414
$2^2 \times 3^2 \times 7 \times 11$	$2^3 \times 3 \times 5 \times 7$	$2^2 \times 3 \times 7$	84	2772	840

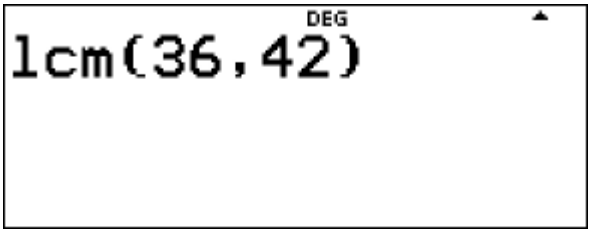
Question: 4.

Two numbers: m & n have a highest common factor of 54. Write down two possible numbers for m & n.

Answer: There are infinitely many answers, all of the form 54 x a and 54 x b where a and b are mutually prime, in other words: hcf(a,b)=1.

LCM = Lowest Common Multiple

Question: 5.



a) Use the calculator to determine the lowest common multiple (LCM) of 36 and 42.

Answer: 252

b) Write down the prime factorisation of 36, 42 and the lowset common multiple of 36 and 42.

Answer(s): $36 = 2^2 \times 3^2$, $42 = 2 \times 3 \times 7$ and
Lowest Common Multiple: $252 = 2^2 \times 3^2 \times 7$.

c) By referring to the Venn diagram for 36 and 42 (Page 1), explain how the Lowest Common Multiple can be calculated.

Answer: The LCM is equal to the product of all the values in the Venn Diagram. Consider the two prime factorisations: (36) $2^2 \times 3^2$ and (42) $2 \times 3 \times 7$. LCM = $2^2 \times 3^2 \times 7$. The factors of 36 and 42 are subsets or $2^2 \times 3^2 \times 7$. Example: $(2^2 \times 3^2) \times 7 = 36 \times 7 = 252$ and $2^2 \times 3^2 \times 7 = 2 \times 3 \times (2 \times 3 \times 7) = 6 \times 42$.

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Question: 6.

The table below is similar to the ones completed in Question 2 and 3, however this one refers to the Lowest Common Multiple. The first row has been done for you.

First Number #1	Second Number #2	Lowest Common Multiple	Prime Factorisation #1	Prime Factorisation #2	Prime Factorisation LCM
48	60	240	$2^4 \times 3$	$2^2 \times 3 \times 5$	$2^4 \times 3 \times 5$
90	210	630	$2 \times 3^2 \times 5$	$2 \times 3 \times 5 \times 7$	$2 \times 3^2 \times 5 \times 7$
105	42	210	$3 \times 5 \times 7$	$2 \times 3 \times 7$	$2 \times 3 \times 5 \times 7$
66	220	660	$2 \times 3 \times 11$	$2^2 \times 5 \times 11$	$2^2 \times 3 \times 5 \times 11$
60	220	660	$2^2 \times 3 \times 5$	$2^2 \times 5 \times 11$	$2^2 \times 3 \times 5 \times 11$
260	195	780	$2^2 \times 5 \times 13$	$3 \times 5 \times 13$	$2^2 \times 3 \times 5 \times 13$
306	360	6120	$2 \times 3^2 \times 17$	$2^3 \times 3^2 \times 5$	$2^3 \times 3^2 \times 5 \times 17$
207	460	4140	$2^2 \times 5 \times 23$	$3^2 \times 23$	$2^2 \times 3^2 \times 5 \times 23$
660	420	4620	$2^2 \times 3 \times 5 \times 11$	$2^2 \times 3 \times 5 \times 7$	$2^2 \times 3 \times 5 \times 7 \times 11$

Question: 7.

Explain, in reference to the prime factorisation of two numbers, how the Lowest Common Multiple can be determined.

Answer: Each prime factor, including its exponent, must be included in the prime factorisation of the Lowest Common Multiple.

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Factors that Count

Teacher Notes:

Prior to commencing this activity, students should have a basic familiarity with prime factorsiation and factor trees. By the conclusion of the investigation students should have developed a rule for determing the quantity of factors for any number, given its prime factorisation. There are much broader goals, well beyond establishing a specific formula.

Along the journey students should have:

- Engaged in systematic exploration
- Used tables to summarise information
- Developed and tested hypothesis
- Generalised their findings (formula)

Students often ask "Why are we doing this?" or "How is this going to help us get a job?" Relevance is important to students and their learning. Developing a formula to predict the quantity of factors for any given number is the sort of task likely to generate these familiar questions. It is important to help students understand that while the formula itself is not likely to help them, the process and concept is very relevant. One of the many rolls mathematics plays in our soceity is to identify patterns and then use these patterns to make predictions. The Covid pandemic presents a very recent and relevant example. "During the early days of the Covid pandemic, daily cases were monitored closely. Mathematicians modelled the growing case numbers using a range of different functions, the most popular being a logistics curve. These formulas were used to make predictions about potential infections so the relevant authorities could prepare for what might happen. Restrictions were put in place with a view to reducing the spread of the virus. The effectiveness of the restrictions could be gaged by comparing actual cases against the predicted values; from here authorities could either tighten or losen restrictions accordingly." This level of modelling is extremely sophisticated (mathematically speaking), as are most real world scenarios, they involve many variables and uncertainties.

This activity involves a similar processes, collect and analyse data, engage in systematic exploration, develop and test a variety of hypothesis with a view to establishing a relationship that can be used to make predictions. The artificial nature of the activity reduces the complexity of the mathematics, reduces the quantity of variables and makes testing, analysing and predicting much more accesible. Even if students do not want a career that involves this type of investigation, understanding the process means they will at least be educated enough to appreciate the complexity of mthematical modelling in any scenario.

In one section of the activity, students are encouraged to use playing cards or numbered counters while trying to determine the quantity of factors. The playing cards or counters have a one to one correspondence with the prime factorisation and provide a physical mechanism by which students identify the manipulation of the prime factors. This manipulating is designed to help them realise that the actual 'base' (prime factor) is not as important as the quantity of these elements (index), therefore see the similarity between: $2^3 \times 3^2$ and something like: $5^3 \times 7^2$.

Students may attempt to 'generalise' as: (some number)^2 x (another number)^2 produces 9 factors, however it is very important that they realise that 'some number' must be a prime number! Students should be encouraged to generalise using mathematical notation such as: $n^2 \times m^2$ where $m \neq n$ and m and n are prime, resuts in 9 factors.

Students can now make predictions for other numbers such as 20,449 which can be represented as: $11^2 \times 13^2$, as this falls into the symbolic representation: $n^2 \times m^2$, student can conclude that 20,449 has exactly 9 factors.

The solutions provided in this document should also act as a guide to how students should set out their work. Notice that the factor trees always show the complete prime factorisation on the bottom row of the factor tree. When a prime factor is identified early in the factor tree, it should continue, the only difference being it no longer splits into two prime factors. This representation means every horizontal row of the tree will have a product equal to the original number.

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Warm up Factors

The number 12 has six factors: 1, 2, 3, 4, 6 and 12. The number 36 has more factors. Which number would have the greatest number of factors: 36 or 5929? This investigation explores how to determine the quantity of factors for any given number without listing each individual factor.

Question: 1.

Construct factor trees for each of the following numbers:

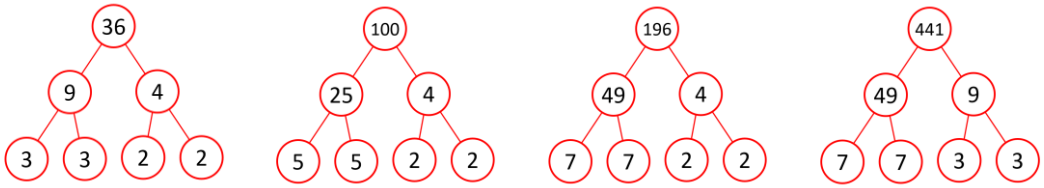
a) 36

b) 100

c) 196

d) 441

Answers: Individual branches may differ, however the 'prime factorisation' at the base of the tree will be the same.



Question: 2.

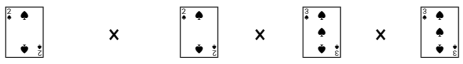
Express each of the numbers: 36, 100, 196 and 441 as a product of its prime factors.

Answers: $36 = 3^2 \times 2^2$ $100 = 5^2 \times 2^2$ $196 = 7^2 \times 2^2$ $441 = 7^2 \times 3^2$

Question: 3.

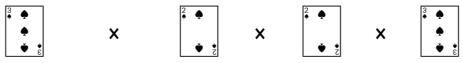
Using a pack of playing cards (or numbered counters), determine all the factors for each of the numbers 36, 100, 196 and 441. Record all arrangements and groupings of the cards used to determine the factors for each number. *Remember to include one and the original number in the factor count.*

Examples: Factors of 36



2 × (2 × 3 × 3)

Factor Pair: 2 × 18



3 × (2 × 2 × 3)

Factor Pair: 3 × 12

Answers:

36 = (2 x 2) x (3 x 3) = 4 x 9	36 = (2 x 2 x 3) x 3 = 12 x 3	36 = (2 x 3) x (2 x 3) = 6 x 6
36 = 2 x (2 x 3 x 3) = 2 x 18	Factors: 1, 2, 3, 4, 6, 9, 12, 18, 36	Quantity: 9
100 = (2 x 2) x (5 x 5) = 4 x 25	100 = (2 x 2 x 5) x 5 = 20 x 5	100 = (2 x 5) x (2 x 5) = 10 x 10
100 = 2 x (2 x 5 x 5) = 2 x 50	Factors: 1, 2, 4, 5, 10, 20, 25, 50, 100	Quantity: 9
196 = (2 x 2) x (7 x 7) = 4 x 49	196 = (2 x 2 x 7) x 7 = 28 x 7	196 = (2 x 7) x (2 x 7) = 14 x 14

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196 = 2 x (2 x 7 x 7) = 2 x 98	Factors: 1, 2, 4, 7, 14, 28, 49, 98, 196	Quantity: 9
441 = (3 x 3) x (7 x 7) = 9 x 49	441 = (3 x 3 x 7) x 7 = 63 x 7	441 = (3 x 7) x (3 x 7) = 21 x 21
441 = 3 x (3 x 7 x 7) = 3 x 147	Factors: 1, 3, 7, 9, 21, 49, 63, 147, 441	Quantity: 9

Question: 4.

What do you notice about the quantity of factors for the numbers 36, 100, 196 and 441?

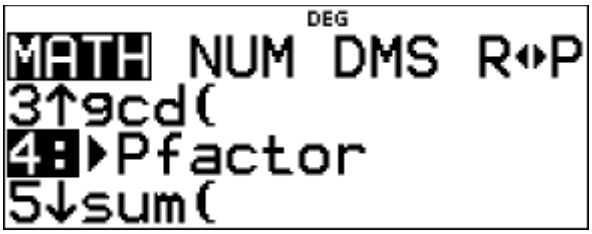
Answer: 36, 100, 196 and 441 all have exactly 9 factors. Students may also note that it is 'unusual' for a number to have an odd quantity of factors, given factors generally occur in pairs. The reason for the odd quantity of factors is that all of these numbers are perfect squares.

Calculator Instructions

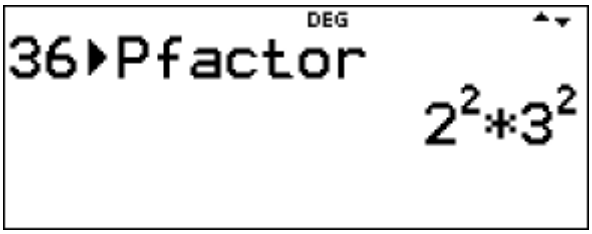
The TI-30XPlus MathPrint calculator includes a command called: Pfactor.

Type the value 36 on the home screen.

Press the MATH key and scroll down to item 4 in the MATH menu. (Shown opposite)



The calculator will generate the Prime Factorisation. This command can be used to expedite the following investigation.



Investigation – The factor Sleuth:

Question: 5.

Use questions 1 to 4 as a guide to investigate the following numbers: 24, 54, 250 and 1029. Determine the prime factorisation and the quantity of factors for each of these numbers.

Answers: 24, 54, 250 and 1029 all have exactly 8 factors. Each number can be represented in the form $a^3 \times b^1$, where a and b are prime and $a \neq b$.

Question: 6.

Once again, using questions 1 to 4 as a guide, investigate the following numbers: 80, 162, 405 and 1250. Determine the prime factorisation and the quantity of factors for each of these numbers.

Answers: 80, 162, 405 and 1250 all have exactly 10 factors. Each number can be represented in the form $a^4 \times b^1$, where a and b are prime and $a \neq b$.

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Question: 7.

Let the numbers in question 1 belong to group 1, the numbers in question 5 belong to group 2 and the numbers in question 6 belong to group 3. Determine which group each of the following numbers belongs:

a) 3025 Group 1	b) 891 Group 3	c) 8281 Group 1	d) 6655 Group 2
e) 4459 Group 2	f) 34969 Group 1	g) 1875 Group 2	h) 1375 Group 2

Question: 8.

The number 675 belongs to a new group. Identify the quantity of factors for this number and at least three other numbers that have the same quantity of factors.

Answer: $675 = 3^3 \times 5^2$. 675 has 12 factors. Other numbers will be of the form: $a^3 b^2$ where a & b are both prime. Examples include: $72 = 2^3 \times 3^2$, $200 = 2^3 \times 5^2$, $392 = 2^3 \times 7^2$, $675 = 3^2 \times 5^2$, $968 = 2^3 \times 11^2$, $1323 = 3^3 \times 7^2$ …

Wrapping up the case.

Question: 9.

It's time to summarise and present your findings. Your aim is to identify a 'rule' that predicts the quantity of factors for any given number, based on its prime factorisation. Explore a range of numbers. Tabulate your results. Develop an hypothesis (formula) and test it, justify any conclusions.

Teacher Notes:

This is where students need to use what they have discovered so far to guide their investigation, this includes tabulating their results in a meaningful way .The following table illustrates one way students may summarise their results. They key to formulating a 'solution' is to focus on the relationship between the indices and the quantity of factors, not the base (prime factors).

The purpose of using manipulatives, such as playing cards, and 'grouping' in previous questions was to highlight that whilst the bases may vary, when the indices format is the same, so too is the quantity of factors, leading students to focus on the relationship between the indices and the quantity of factors.
Sample results are included below, whilst the examples use 2 and 3 as the base, any other combination of prime numbers in the base will yeild the same results.

Number	Prime Factorisation	Indices	Qty Factors
18	$3^2 \times 2$	2, 1	6
36	$2^2 \times 3^2$	2, 2	9
24	$2^3 \times 3$	3, 1	8
48	$2^4 \times 3$	4, 1	10
72	$2^3 \times 3^2$	3, 2	12
216	$2^3 \times 3^3$	3, 3	16
648	$2^3 \times 3^4$	3, 4	20

If students consider the indicies and quantity of factors (final two columns) they will likely establish the relationship. "Adding '1' to each exponent and then calculate the product". Example: $648 = 2^3 \times 3^4$. Indices: 3, 4. Add 1 to each indice gives: 4, 5, and 4 x 5 = 20, the quantity of factors. Student's may describe the pattern using words (and examples) or express it symbolically or as a mixture: "If $q = a^m \times b^n$ where a & b are prime, $a \neq b$ and $m > 0$ and $n > 0$ then the quantity of factors is given by $(m + 1)(n + 1)$".

Students should test numbers that have more than 2 unique prime factors and indices .

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Example:

$2^3 \times 3^3 \times 5^2 = 5400$ Quantity of factors: 4 x 4 x 3 = 48.

$2^3 \times 3^2 \times 5 = 360$ Quantity of factors: 4 x 3 x 2 = 24
[Students should also consider an exponent of 1 for one or more of the prime factors]

23 x 33 x 52 x 72 = 264,600 Quantity of factors: 4 x 4 x 3 x 3 = 144.
[Students may use digital means to determine the quantity of factors, this may include writing a program where possible.]

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Anti Primes

Teacher Notes:

This investigation assumes completion of the "Factors that Count" activity whereby students establish a rule that relates the prime factorisation of a number to the quantity of factors. This rule will expedite calculations in this activity and help students in pursuit of answers. If students have established the rule, using it in this activity validates previous work and helps them understand how the build up of knowledge (and skills) enables more and more complex problems to be resolved.

Question: 1.

Prime numbers have exactly two factors, composite numbers have more than two factors. Numbers such as: 4, 9, and 25 just make it into the composite category having just three factors. Identify other numbers that have exactly three factors.

Answers: 49, 121, 169, 289 … in general p^2 where p is any prime number.

Highly Composite Numbers = Anti-Primes

Some numbers seem to be inundated with factors, particularly in comparison to surrounding numbers. Consider the number 36. The number 36 has 9 factors: {1, 2, 3, 4, 6, 9, 12, 18, 36}. The two numbers either side of 36: 35 and 37 have much fewer factors. The number 35 has 4 factors and 37 is prime (2 factors). Looking slightly further afield, 33, 34, 38 and 39 also have just 4 factors each. Indeed, 36 has more factors than every number less than 36 and is given the title: "highly composite" or "Anti-Prime!"

Anti-Prime: A number that has more factors than any one of its predecessors.

By default, the number 1 is the first anti-prime (1 factor) as there are no positive integers less than 1. The number 2 is the next anti-prime, it has 2 factors. The number 3 is not anti-prime, it has 2 factors, a title already held by the number 2. Performing slightly better is the number 4 having 3 factors. So the first three anti-primes are: 1, 2 & 4.

Question: 2.

Write down the first 7 anti-primes and record the quantity of factors for each.

Answer: The quanity of factors is written in the parenthesis. 1 (1), 2 (2), 4 (3), 6 (4), 12 (6), 24 (8) & 36 (9).

Calculator Instructions

The TI-30XPlus MathPrint calculator, combined with the formula for determining the quantity of factors is an efficient methods for identifying anti-primes. This process uses a combination of the answer (ans) tool and the prime factor (PFactor) command as shown below.

The 'ANS' key is a great way to generate a sequence recursively.

Example: Type in the number 1 and press enter

Type in ANS + 1 followed by enter

Now keep pressing enter, the sequence of whole numbers is generated.

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Start this process again. Type in 1 and press [enter], so the number 1 is stored in "ans".

Now type in: ans + 1 ▸ Pfactor followed by [enter].

Now keep pressing [enter], keep track of where you are up to and record each prime factorisation.

While it is not necessary to express an exponent of 1, it is a useful reminder when calculating the quantity of factors. Example: $180 = 2^2 \times 3^2 \times 5^1$ rather than $2^2 \times 3^2 \times 5$.

Question: 3.

Generate a table for the first 100 integers including their prime factorisation and corresponding quantity of factors for each number. Use the table to identify all the anti-primes less than 100.

Number	Prime Factor	Qty Factors
1	1^1	1 (A)
2	2^1	2 (A)
3	3^1	2
4	2^2	3 (A)
5	5^1	2
6	$2^1 \times 3^1$	4 (A)
7	7^1	2
8	2^3	4
9	3^2	3
10	$2^1 \times 5^1$	4
11	11^1	2
12	$2^2 \times 3$	6 (A)
13	13^1	2
14	$2^1 \times 7^1$	4
15	$3^1 \times 5^1$	4
16	2^4	5
17	17^1	2
18	$2^1 \times 3^2$	6
19	19^1	2
20	$2^2 \times 5^1$	6

Number	Prime Factor.	Qty Factors
51	$3^1 \times 17^1$	4
52	$2^2 \times 13^1$	4
53	53^1	2
54	$2^1 \times 3^3$	6
55	$5^1 \times 11^1$	4
56	$2^3 \times 7^1$	8
57	$3^1 \times 19^1$	4
58	$2^1 \times 29^1$	4
59	59^1	2
60	$2^2 \times 3^1 \times 5^1$	12 (A)
61	61^1	2
62	$2^1 \times 31^1$	4
63	$3^2 \times 7^1$	6
64	2^6	7
65	$5^1 \times 13^1$	4
66	$2^1 \times 3^1 \times 11^1$	8
67	67^1	2
68	$2^2 \times 17^1$	6
69	$3^1 \times 23^1$	4
70	$2^1 \times 5^1 \times 7^1$	8

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Number	Prime Factor	Qty Factors
21	$3^1 \times 7^1$	4
22	$2^1 \times 11^1$	4
23	23^1	2
24	$2^3 \times 3^1$	8 (A)
25	5^2	3
26	$2^1 \times 13^1$	4
27	3^3	4
28	$2^2 \times 7^1$	6
29	29^1	2
30	$2^1 \times 3^1 \times 5^1$	8
31	3^1	2
32	2^5	6
33	$3^1 \times 11^1$	2
34	$2^1 \times 17^1$	4
35	$5^1 \times 7^1$	4
36	$2^2 \times 3^2$	9 (A)
37	37^1	2
38	$2^1 \times 19^1$	4
39	$3^1 \times 13^1$	2
40	$2^3 \times 5^1$	8
41	41^1	2
42	$2^1 \times 3^1 \times 7^1$	8
43	43^1	2
44	$2^2 \times 11^1$	6
45	$3^2 \times 5^1$	6
46	$2^1 \times 23^1$	4
47	47^1	2
48	$2^4 \times 3^1$	10 (A)
49	7^2	3
50	$2^1 \times 5^2$	6

Number	Prime Factor.	Qty Factors
71	71^1	2
72	$2^3 \times 3^2$	12
73	73^1	2
74	$2^1 \times 37^1$	4
75	$3^1 \times 5^2$	6
76	$2^2 \times 19^1$	6
77	$7^1 \times 11^1$	2
78	$2^1 \times 3^1 \times 13^1$	8
79	79^1	2
80	$2^4 \times 5^1$	10
81	3^4	5
82	$2^1 \times 41^1$	4
83	83^1	2
84	$2^2 \times 3 \times 7$	12
85	$5^1 \times 17^1$	4
86	$2^1 \times 43^1$	4
87	$3^1 \times 29^1$	4
88	$2^3 \times 11^1$	8
89	89^1	2
90	$2^1 \times 3^2 \times 5^1$	12
91	$7^1 \times 13^1$	4
92	$2^2 \times 23^1$	6
93	$3^1 \times 31^1$	4
94	$2^1 \times 47^1$	4
95	$5^1 \times 19^1$	4
96	$2^5 \times 3$	12
97	97^1	2
98	$2^1 \times 7^2$	6
99	$3^2 \times 11^1$	6
100	$2^2 \times 5^2$	9

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Question: 4.

What is different about 60 (anti-prime) compared to all the anti-primes less than 60?

Answer: Anti-primes less than 60 only have 2 and 3 as their factors. 1 (1), 2 = 2^1 (2), 4 = 2^2 (3), 6 = $2^1 \times 3^1$ (4) 12 = $2^2 \times 3^1$ (6), 24 = $2^3 \times 3^1$ (8), 36 = $2^2 \times 3^2$ (9) and 48 = $2^4 \times 3^1$ (10) but 60 = $2^2 \times 3^1 \times 5^1$ (12).

Teacher Notes: This is an important characteristic. Following the pattern of the prime factorisations less than 60 show successive increases in the exponents of 2 and 3. Following 48 = $2^4 \times 3^1$, it may have seemed reasonable to have the next anti-prime as $2^3 \times 3^2$ = 72; however, this would produce 12 factors, up from 10 factors for 48. The number 60 has 10 factors and is the first number to have 3 prime factors.

Ask students to work out "what is the smallest number with 4 **different** prime factors? (2 x 3 x 5 x 7 = 210) Students may also start to review the quantity of factors for the anti-primes: 1, 2, 3, 4, 6, 8, 9, 10 and therefore ponder how many factors the next anti-prime may hold. It is not likely to be 11 factors since such a number would need to be a perfect square.

Question: 5.

Terry has been searching for anti-primes. He identifies the number 504. Terry's friend Tania thinks Terry has made a mistake. Without working out all the anti-primes up to 504, determine who is correct and why?

Answer: Tania is correct. 504 = $2^3 \times 3^2 \times 7$. A number with exactly the same number of factors would be: $2^3 \times 3^2 \times 5$ which is equal to 360. The number 360 has 24 factors, so too does 504, but 360 < 540 so it would be an anti-prime before 504. Note: This logic does not mean that 360 must be an anti-prime, however, if it is not an anti-prime, then by the same argument, 504 would not be an anti-prime.

Question: 6.

Paula has been searching for anti-primes. She identifies the number 8820. John says "that can't be an anti-prime". Without working out all the anti-primes up to 8820, explain why John is correct.

Answer: 8820 = $2^2 \times 3^2 \times 5 \times 7^2$. This fits the pattern for successive prime factorisation, but the exponent on the 7 is a problem. The number: 6300 = $2^2 \times 3^2 \times 5^2 \times 7$ will have exactly the same quantity of factors but is smaller than 8820.

Question: 7.

Julie and Dale are discussing anti-primes. Dale says he found 14 anti-primes between 1 and 200. He went on to say that the largest anti-prime in this range is 180. Without finding all the anti-primes, Julie immediately says that Dale has made a mistake! Explain how Julie can make this claim without finding all the anti-primes between 1 and 200.

Answer: 180 has 18 factors. We can already see that an anti-prime with exactly 5 factors doesn't exist, nor is there one with 7 factors. For a number to have an odd quantity of factors it must be a perfect square. Perfect squares greater than 36 have the following quantity of factor counts: 49 (3), 81 (5), 100 (9), 121 (4), 144* (15), 169 (3) and 196 (9). Of these only 144 is a *candidate* to be an anti-prime. So anti-primes between 36 and 200 must have either 10, 12, 14, 15, 16 or 18 factors. There are 7 anti-primes from 1 to 36 (Question 2) and only 6 possibilities between

Conclusion

Discuss your findings with regards to anti-primes. Suggest strategies for identifying more anti-primes and quick ways to determine if a number is a 'likely' candidate for anti-prime selection.

Teacher Notes: This is not intended to be a comprehensive study/review of anti-primes. Mathematician, Srinivasa Ramanujan explored anti-primes and highlighted several key features in the process of identifying anti-primes (highly composite numbers), some students, reflecting on the questions posed in this investigation, will be able to identify some anti-prime numbers.

Answers:

Anti-primes must have prime factorisations consisting of consecutive prime numbers.
Example: 2 x 3 x 5 could be an anti-prime but 2 x 3 x 7 could not. These two prime factorisations result in the same quantity of factors, however 2 x 3 x 5 < 2 x 3 x 7.

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The exponents of anti-primes will be incremental.
Example: $2^1 \times 3^1 \times 5^2$ cannot be an anti-prime but $2^2 \times 3^1 \times 5^1$ could.

Advanced – There is a 'pay-off' between increasing the exponent versus the inclusion of another prime factor. Example: $2^1 \times 3^1 \times 5^1$ works for the previous two rules, prime factors have not been skipped (rule 1) and the exponents on the prime factors are incremental (or constant). However, start with $2^1 \times 3^1$ (4 factors), multiplying by 5 produces: $2^1 \times 3^1 \times 5^1$ which has 8 factors. The original number: $2^1 \times 3^1$ could have been multiplied by 4 (which produces a smaller number than multiplying by 5) and results in the prime factorisation: $2^3 \times 3^1 = 24$, which also has 8 factors.

Extend this to other numbers such as: $4620 = 2^2 \times 3^1 \times 5^1 \times 7^1 \times 11^1$ (48 factors).

Compare this with $4320 = 2^5 \times 3^3 \times 5^1$ (48 factors). Multiplying by 7 x 11 (77) compare this with multiplying by $2^3 \times 3^2$ (72).

Students may also comment on the overall quantity of factors and work backwards to identify smaller numbers that have the same quantity of factors.

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Common Sequence

Starting the Sequence

Teacher Notes:

This investigation looks into generating a sequence that involves 'highest common factors'. The sequence is recursive where each subsequent term is dependent on the highest common factor of the current term number and the previous term. The rules for this sequence may seem somewhat obscure and not likely to lead to anything in particular, however, if pursued, the long term result is extremely surprising!

Problem Statement

There are all sorts of patterns created by sequences such as the Fibonacci sequence: 1, 1, 2, 3, 5, 8, 13 …

In this investigation we look at a very unusual sequence with an aim to see if any observable patterns appear as the sequence progresses. The sequence starts with the first two terms defined: $t_0 = 1$ and $t_1 = 1$.

Term Number:	n	0	1	2
Term:	t_n	1	1	

Just like the Fibonacci sequence, this sequence is recursive; this means means that each successive term depends on the previous term or terms. To calculate the next term in this sequence we check to see if there are any common factors between the two highlighted values:

Term Number:	n	0	1	2
Term:	t_n	1	1	

If the highest common factor is 1 then we add the two highlighted values then and add 1 to the result. The highest common factor of 1 and 2 is 1, therefore we apply the rule: 1 + 2 + 1 = 4.

Term Number:	n	0	1	2	3
Term:	t_n	1	1	4	

We're ready to calculate the next term. We apply the same criteria. The highest common factor between the two highlighted terms 3 and 4 is 1, so again we apply the rule: 4 + 3 + 1 = 8.

Term Number:	n	0	1	2	3	4
Term:	t_n	1	1	4	8	

We're ready to calculate the next term. Applying the same criteria we see that the highest common factor between the highlighted terms 8 and 4 is no longer 1. [hcf(8,4)=4]. When the highest common factor is not equal to 1, the term (8) is divided by the highest common factor, in this case 4. Our new term becomes: 8 ÷ 4 = 2.

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We'll do two more terms for practice.

Term Number:	n	0	1	2	3	4	5
Term:	t_n	1	1	4	8	2	

The highest common factor of 2 and 5 is 1, therefore the next term is 2 + 5 + 1 = 8.

Term Number:	n	0	1	2	3	4	5	6
Term:	t_n	1	1	4	8	2	8	

The highest common factor 8 and 6 is not 1, therefore the alternative rule is applied. The highest common factor of 8 and 6 is 2; so the next term is equal to 8 ÷ 2 = 4

Check the next two terms in the table below.

Term Number:	n	0	1	2	3	4	5	6	7	8
Term:	t_n	1	1	4	8	2	8	4	12	3

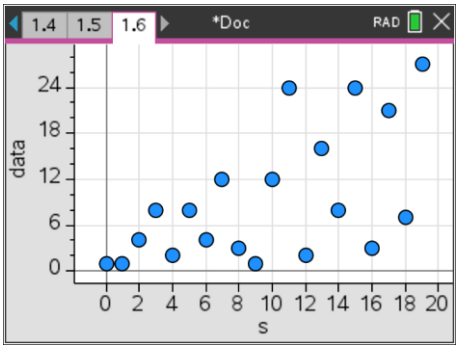
Question: 1.

Complete the next series of terms in the table. (Check the answers with your teacher before continuing)

Term Number:	n	8	9	10	11	12	13	14	15	16
Term:	t_n	3	1	12	24	2	16	8	24	3

Question: 2.

Create a scatterplot for the data collected so far with the term number (n) on the x axis (independent variable) and the term on the y axis (dependent variable). Are there any patterns appearing at this stage?



Students may draw the scatterplot by hand or use a spreadsheet or graphing calculator.

[Graphing calculator result shown opposite.]

Answer: In this early scatterplot there are no obvious patterns. Students may observe that almost all the points lie under the line y = 2x + 1, or that prime numbers cause terms to increase in value.

Examples: $t_7 = 12$, $t_{11} = 24$ and $t_{13} = 16$.

Question: 3.

Generate the first 100 terms of the sequence. Remember that you can use the GCD (Greatest Commond Divisor or Highest Common Factor command to check your answers.)

Answer: Rather than list all the answers here, a screen shot from a spreadsheet has been included.

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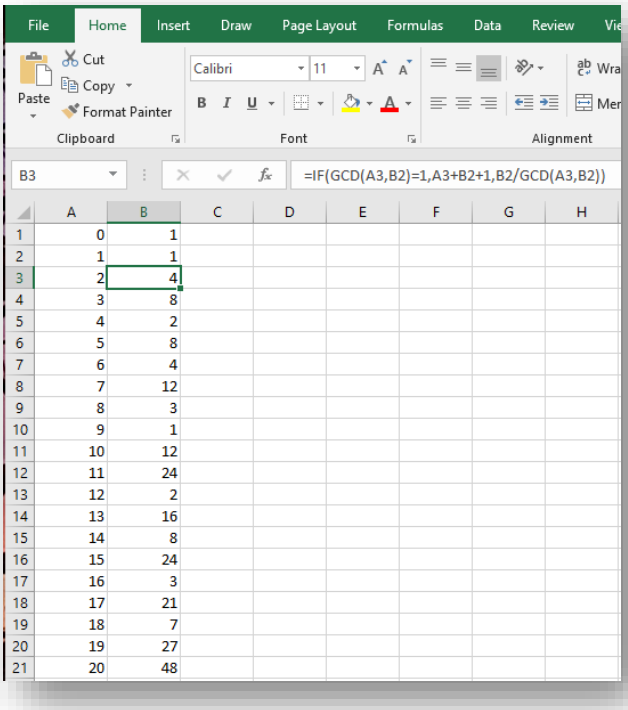
=IF(GCD(A3,B2)=1,A3+B2+1,B2/GCD(A3,B2))



	A	B
1	0	1
2	1	1
3	2	4
4	3	8
5	4	2
6	5	8
7	6	4
8	7	12
9	8	3
10	9	1
11	10	12
12	11	24
13	12	2
14	13	16
15	14	8
16	15	24
17	16	3
18	17	21
19	18	7
20	19	27
21	20	48

Mathemtical Investigation - Flying Straight

One of the challenges faced in any mathematical investigation is that we are not sure where the investigation may lead. Our first clue in this investigation is that our sequence is based on divisibility. When divisibility exists (common factors), the value of the next term in the sequence decreases, the reverse is true when divisibility is not present.

It's now time to start investigating. You may need to explore well beyond the first 100 terms so a spreadsheet or simple program can help generate terms in the sequence. Look for clues such as:

- What happens if terms or term numbers that are highly composite?
- What happens if terms or term numbers are prime number?
- What is the typical up / down run length? [Run = Quantity of consecutive term increases or decreases.]
- How often do 1's or 2's appear as terms and where do they occur?
- Progressively increase the quantity of data that you are analysing. Sometimes patterns start to appear when we get to stand back from the data.

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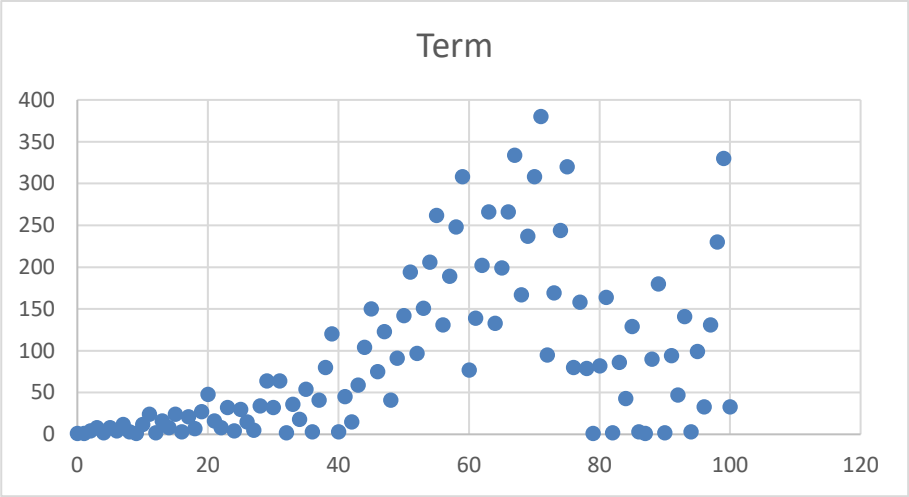
Answer:

Sample Chart (Using Excel) – First 100 terms



Based on the first 100 terms there are not any clear patterns. There is a suggestion that perhaps the sequence cycles, building up between 1 and 80 then dropping only to start rebuilding again. However, as students push the sequence further, this pattern disappears. Indeed, the remarkable part about this sequence is that it almost complete void of patterns until it reaches term 638.

Term 638 is the first even term to equal 1. Year 7 will not be able to generalise this situation; however, it is unique and important (shown algebraically below). Students in Year 7 however can return to the calculations 'by hand' to gain an insight as to what is going on here and at least see that the ability to 'generalise' (algebra) would be powerful.

Term No.	2n	2n + 1	2n + 2	2n + 3	2n + 4	2n + 5	2n + 6	2n + 7
Term.	1	2n + 3	4n + 6	2	1	2n + 7	4n + 14	2

The first 750 terms of the sequence have been graphed below. Students can now see that there is a distinct pattern. This pattern will now continue forever. The sudden flip to a distinct pattern can be used to help illustrate the importance of 'algebra'. "Trying lots of numbers" (first 500 or so terms) is not sufficient evidence to prove something, in contrast, algebra can be used to prove something is always true.

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Author: P. Fox