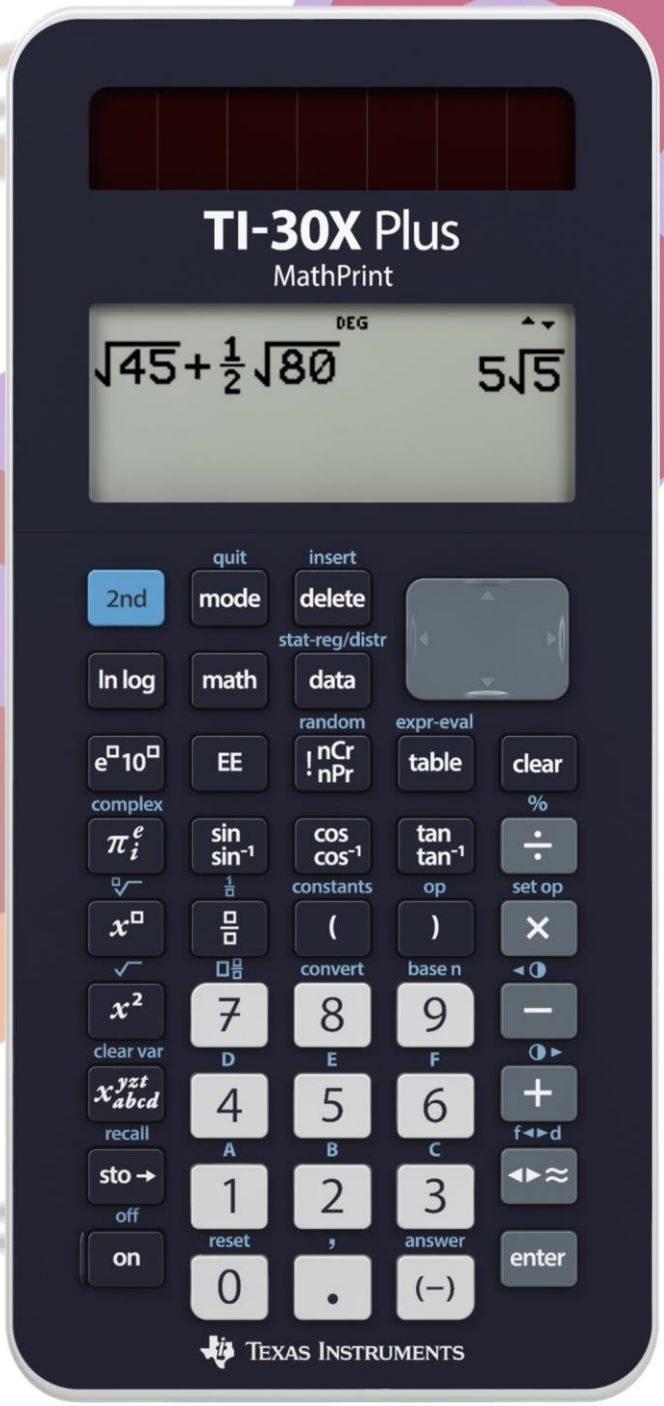

Factoring in some fun

TI-30X Plus MathPrint™

Scientific Calculator

Contents

Factor Trees 3
Getting Started with Factor Trees 3
Teacher Notes: ..... **Error! Bookmark not defined.**
Out smarting the calculator! 4
Climbing the tree 5
HCF & LCM 6
HCF = Highest Common Factor 6
Teacher Notes: ..... **Error! Bookmark not defined.**
Teacher Notes: ..... **Error! Bookmark not defined.**
LCM = Lowest Common Multiple 8
Factors that Count 9
Teacher Notes: ..... **Error! Bookmark not defined.**
Warm up Factors 9
Calculator Instructions 9
Investigation – The factor Sleuth: 10
Teacher Notes: ..... **Error! Bookmark not defined.**
Anti Primes 11
Teacher Notes: ..... **Error! Bookmark not defined.**
Highly Composite Numbers = Anti-Primes 11
Calculator Instructions 11
Conclusion 12
Common Sequence 13
Starting the Sequence 13
Teacher Notes: ..... **Error! Bookmark not defined.**
Problem Statement 13
Mathemtical Investigation - Flying Straight 14

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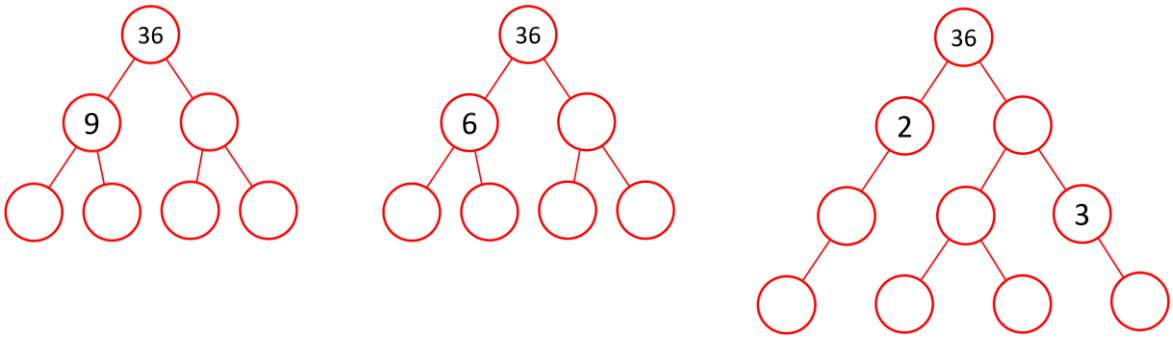
Author: P. Fox

Factor Trees

Getting Started with Factor Trees

Question: 1.

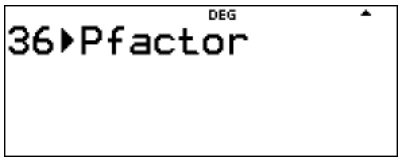
a) Complete the prime factorisation of the number 36 in each of the following:



b) Calculate the product of the numbers in each row of each tree. What do you notice about this product?

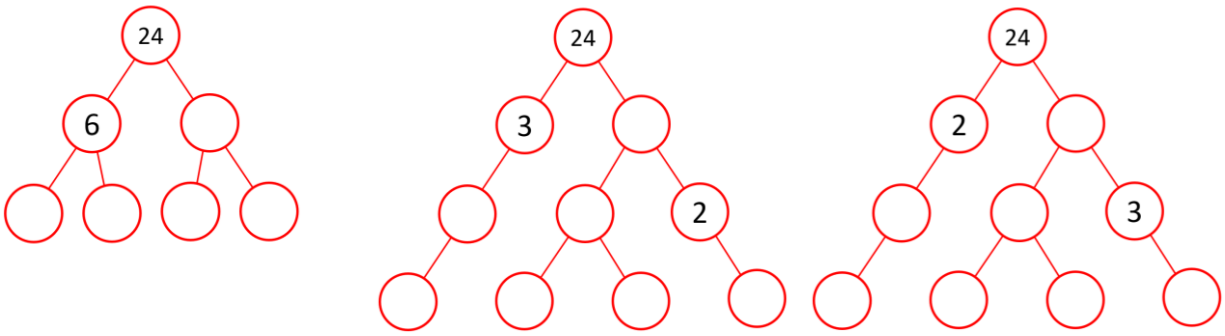
c) What do you notice about the numbers in the bottom row of each tree?

d) On your calculator, enter the number 36, then press [math] and select item 4: ▸Pfactor.
Press [enter] to execute the command.
How does the calculator's answer relate to the factor tree?



Question: 2.

a) Complete the prime factorisation of the number 24 in each of the following:



b) Calculate the product of the numbers in each row of each tree. What do you notice about this product?

c) What do you notice about the numbers in the bottom row of each tree?

d) Use your calculator to determine the prime factorisation of 24 and compare the result to the factor tree.

Question: 3.

Generate factor trees for each of the following numbers. Check the bottom row of the factor tree in each case using your calculator.

a) 54	b) 150	c) 126	d) 132
e) 260	f) 350	g) 210	h) 882

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Question: 4.

Generate factor trees for each of the following numbers. Check the bottom row of the factor tree in each case using your calculator.
Hint: Use the factor trees from Question 3 to help.

a)	108	b)	300	c)	252	d)	264
e)	2600	f)	3500	g)	2100	h)	8820

Question: 5.

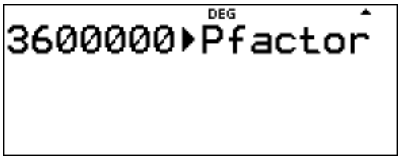
Use questions 3 & 4 to help write down the prime factorisation of each of the following.

a)	216	b)	600	c)	504	d)	528
e)	26000	f)	35000	g)	21000	h)	88200

Out smarting the calculator!

The Prime Factorisation command in the calculator is limited to numbers less than 1,000,000.

Try: 3,600,000 ▸ Pfactor



3600000 = 36 x 10 x 10 x 10 x 10 x 10 and 10 = 2 x 5.

Question: 6.

Use the above tip to help determine the prime factorisation of each of the following:

a)	1,080,000	b)	3,000,000	c)	2,520,00	d)	2,640,000

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Climbing the tree

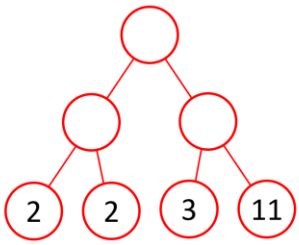
It is possible to start at the base of the factor tree and climb back up!

Question: 7.

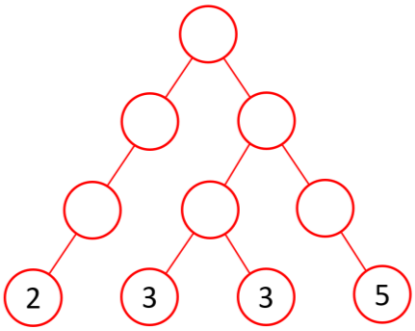
Complete the reverse factor trees (below) to determine the original number and to answer the subsequent questions:

a) Complete the reversed factor trees (below)

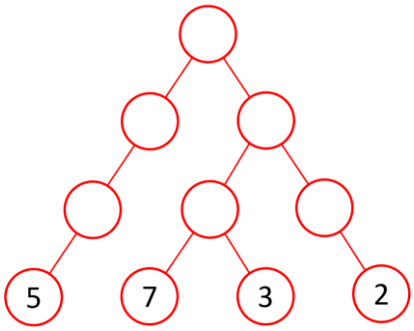
(i)



(ii)



(iii)



b) Which one or more of the original values is a multiple of 7?

c) Which one or more of the original values is a multiple of 9?

d) Which one or more of the original values is a multiple of 35?

Question: 8.

Which one (or more) of the following numbers is NOT a factor of $2^2 \times 3^3 \times 5 \times 7^2 \times 13$. [Multiple Choice]

a) 49 b) 45 c) 260 d) 11

Question: 9.

Two numbers are multiplied together: $2^2 \times 3^2 \times 5$ and $3^2 \times 5 \times 7$. Which one of the following number is NOT a factor of this product? [Multiple Choice]

a) 49 b) 45 c) 150 d) 1575

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HCF & LCM

HCF = Highest Common Factor

Question: 1.

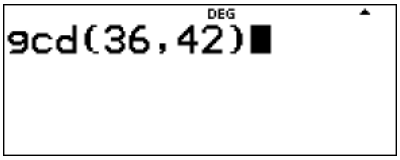
a) List all the factors of 36.

b) List all the factors of 42.

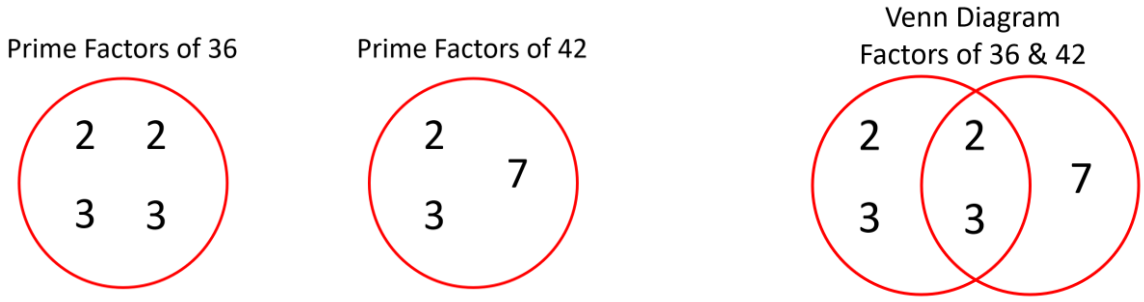
c) Comparing your answers to parts (a) & (b), what factors do 36 and 42 have in common?

d) Using your answer to part (c), what is the highest (largest) factor that 36 and 42 have in common?

e) The TI-30XPlus MathPrint calculator has a command called: gcd(#,#). This command can be accessed by pressing: [math] . Perform the calculation shown opposite and write down the result. [1]

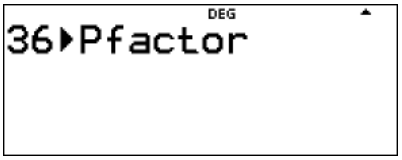


The prime factors of 36 and 42 have been placed into two separate circles (below left). These circles have been moved together so they overlap (below right). The section where the circles overlap holds the 'prime' factors that these two numbers have in common.

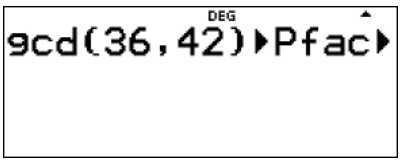


f) What is the product of the two prime factors that 36 and 42 have in common?

g) Use your calculator to express 36 and 42 as products of their prime factors.



h) Multiple commands can be used in a single line on the calculator. Combine gcd(36,42) with ▸Pfactor and explain how this relates to the Venn diagram.



[1] GCD = Greatest Common Divisor is the same as Highest Common Factor

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Question: 2.

Complete the table below. The first row has been done for you.

First Number #1	Second Number #2	Highest Common Factor	Prime Factorisation #1	Prime Factorisation #2	Prime Factorisation HCF
48	60	12	$2^4 \times 3$	$2^2 \times 3 \times 5$	$2^2 \times 3$
90	210				
105	42				
66	220				
60	220				
260	195				
306	360				
207	460				
660	420				

Question: 3.

The table below is similar to the previous question, however, this time the prime factorisation has been done.
In each case, try and calculate the highest common factor **before** determining the actual numbers.
Note: Once you have the 'First' and 'Second' number, use your calculator to check the highest common factor.

Prime Factorisation #1	Prime Factorisation #2	Prime Factorisation HCF	Highest Common Factor	First Number #1	Second Number #2
$2^3 \times 3 \times 7$	$2^2 \times 3 \times 5$	$2^2 \times 3$	12	168	60
$2^2 \times 3^2 \times 11$	$2^3 \times 5 \times 7$				
$3^3 \times 5 \times 7$	$2 \times 3^2 \times 13$				
$2^2 \times 3 \times 11^2$	$2^3 \times 5 \times 11$				
$2^5 \times 3 \times 5$	$2^3 \times 5^2 \times 11$				
$2^2 \times 5 \times 13^2$	$2 \times 3 \times 5 \times 13$				
$2^2 \times 3^2 \times 17$	$2^3 \times 3^2 \times 5$				
$2^2 \times 5^2 \times 23$	$2 \times 3^2 \times 23$				
$2^2 \times 3^2 \times 7 \times 11$	$2^3 \times 3 \times 5 \times 7$				

Question: 4.

Two numbers: m & n have a highest common factor of 54. Write down two possible numbers for m & n.

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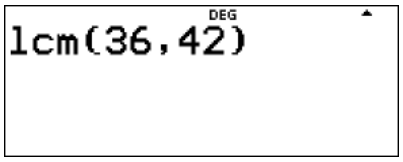
LCM = Lowest Common Multiple

Question: 5.

a) Use the calculator to determine the lowest common multiple (LCM) of 36 and 42.

b) Write down the prime factorisation of 36, 42 and the lowset common multiple of 36 and 42.



c) By referring to the Venn diagram for 36 and 42 (Page 1), explain how the Lowest Common Multiple can be calculated.

Question: 6.

The table below is similar to the ones completed in Question 2 and 3, however this one refers to the Lowest Common Multiple. The first row has been done for you.

First Number #1	Second Number #2	Lowest Common Multiple	Prime Factorisation #1	Prime Factorisation #2	Prime Factorisation LCM
48	60	240	$2^4 \times 3$	$2^2 \times 3 \times 5$	$2^4 \times 3 \times 5$
90	210				
105	42				
66	220				
60	220				
260	195				
306	360				
207	460				
660	420				

Question: 7.

Explain, in reference to the prime factorisation of two numbers, how the Lowest Common Multiple can be determined.

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Factors that Count

Warm up Factors

The number 12 has six factors: 1, 2, 3, 4, 6 and 12. The number 36 has more factors. Which number would have the greatest number of factors: 36 or 5929? This investigation explores how to determine the quantity of factors for any given number without listing each individual factor.

Question: 1.

Construct factor trees for each of the following numbers:

a) 36

b) 100

c) 196

d) 441

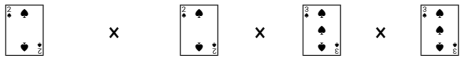
Question: 2.

Express each of the numbers: 36, 100, 196 and 441 as a product of its prime factors.

Question: 3.

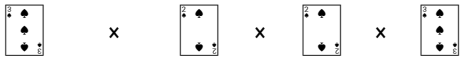
Using a pack of playing cards (or numbered counters), determine all the factors for each of the numbers 36, 100, 196 and 441. Record all arrangements and groupings of the cards used to determine the factors for each number. *Remember to include one and the original number in the factor count.*

Examples: Factors of 36



2 × (2 × 3 × 3)

Factor Pair: 2 × 18



3 × (2 × 2 × 3)

Factor Pair: 3 × 12

Question: 4.

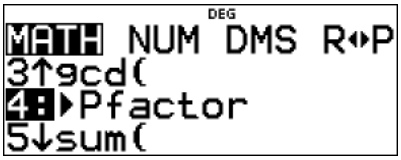
What do you notice about the quantity of factors for the numbers 36, 100, 196 and 441?

Calculator Instructions

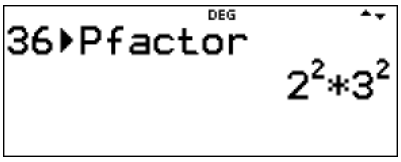
The TI-30XPlus MathPrint calculator includes a command called: Pfactor.

Type the value 36 on the home screen.

Press the MATH key and scroll down to item 4 in the MATH menu. (Shown opposite)



The calculator will generate the Prime Factorisation. This command can be used to expedite the following investigation.



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Investigation – The factor Sleuth:

Question: 5.

Use questions 1 to 4 as a guide to investigate the following numbers: 24, 54, 250 and 1029. Determine the prime factorisation and the quantity of factors for each of these numbers.

Question: 6.

Once again, using questions 1 to 4 as a guide, investigate the following numbers: 80, 162, 405 and 1250. Determine the prime factorisation and the quantity of factors for each of these numbers.

Question: 7.

Let the numbers in question 1 belong to group 1, the numbers in question 5 belong to group 2 and the numbers in question 6 belong to group 3. Determine which group each of the following numbers belongs:

a) 3025	b) 891	c) 8281	d) 6655
e) 4459	f) 34969	g) 1875	h) 1375

Question: 8.

The number 675 belongs to a new group. Identify the quantity of factors for this number and at least three other numbers that have the same quantity of factors.

Wrapping up the case.

Question: 9.

It's time to summarise and present your findings. Your aim is to identify a 'rule' that predicts the quantity of factors for any given number, based on its prime factorisation. Explore a range of numbers. Tabulate your results. Develop an hypothesis (formula) and test it, justify any conclusions.

Number	Prime Factorisation	Indices	Qty Factors

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Anti Primes

Question: 1.

Prime numbers have exactly two factors, composite numbers have more than two factors. Numbers such as: 4, 9, and 25 just make it into the composite category having just three factors. Identify other numbers that have exactly three factors.

Highly Composite Numbers = Anti-Primes

Some numbers seem to be inundated with factors, particularly in comparison to surrounding numbers. Consider the number 36. The number 36 has 9 factors: {1, 2, 3, 4, 6, 9, 12, 18, 36}. The two numbers either side of 36: 35 and 37 have much fewer factors. The number 35 has 4 factors and 37 is prime (2 factors). Looking slightly further afield, 33, 34, 38 and 39 also have just 4 factors each. Indeed, 36 has more factors than every number less than 36 and is given the title: "highly composite" or "Anti-Prime!"

Anti-Prime: A number that has more factors than any one of its predecessors.

By default, the number 1 is the first anti-prime (1 factor) as there are no positive integers less than 1. The number 2 is the next anti-prime, it has 2 factors. The number 3 is not anti-prime, it has 2 factors, a title already held by the number 2. Performing slightly better is the number 4 having 3 factors. So the first three anti-primes are: 1, 2 & 4.

Question: 2.

Write down the first 7 anti-primes and record the quantity of factors for each.

Calculator Instructions

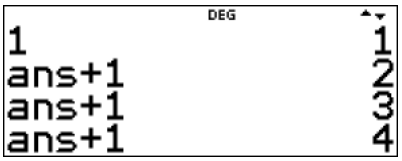
The TI-30XPlus MathPrint calculator, combined with the formula for determining the quantity of factors is an efficient methods for identifying anti-primes. This process uses a combination of the answer (ans) tool and the prime factor (PFactor) command as shown below.

The 'ANS' key is a great way to generate a sequence recursively.

Example: Type in the number 1 and press enter

Type in ANS + 1 followed by enter

Now keep pressing enter, the sequence of whole numbers is generated.



Start this process again. Type in 1 and press enter, so the number 1 is stored in "ans".

Now type in: ans + 1 ▸ Pfactor followed by enter.

Now keep pressing enter, keep track of where you are up to and record each prime factorisation.

While it is not necessary to express an exponent of 1, it is a useful reminder when calculating the quantity of factors. Example: $180 = 2^2 \times 3^2 \times 5^1$ rather than $2^2 \times 3^2 \times 5$.

Question: 3.

Generate a table for the first 100 integers including their prime factorisation and corresponding quantity of factors for each number. Use the table to identify all the anti-primes less than 100.

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Number	Prime Factor	Qty Factors

Number	Prime Factor.	Qty Factors

Question: 4.

What is different about 60 (anti-prime) compared to all the anti-primes less than 60?

Question: 5.

Terry has been searching for anti-primes. He identifies the number 504. Terry's friend Tania thinks Terry has made a mistake. Without working out all the anti-primes up to 504, determine who is correct and why?

Question: 6.

Paula has been searching for anti-primes. She identifies the number 8820. John says "that can't be an anti-prime". Without working out all the anti-primes up to 8820, explain why John is correct.

Question: 7.

Julie and Dale are discussing anti-primes. Dale says he found 14 anti-primes between 1 and 200. He went on to say that the largest anti-prime in this range is 180. Without finding all the anti-primes, Julie immediately says that Dale has made a mistake! Explain how Julie can make this claim without finding all the anti-primes between 1 and 200.

Conclusion

Discuss your findings with regards to anti-primes. Suggest strategies for identifying more anti-primes and quick ways to determine if a number is a 'likely' candidate for anti-prime selection.

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Common Sequence

Starting the Sequence

Problem Statement

There are all sorts of patterns created by sequences such as the Fibonacci sequence: 1, 1, 2, 3, 5, 8, 13 …

In this investigation we look at a very unusual sequence with an aim to see if any observable patterns appear as the sequence progresses. The sequence starts with the first two terms defined: $t_0 = 1$ and $t_1 = 1$.

Term Number:	n	0	1	2
Term:	t_n	1	1	

Just like the Fibonacci sequence, this sequence is recursive; this means means that each successive term depends on the previous term or terms. To calculate the next term in this sequence we check to see if there are any common factors between the two highlighted values:

Term Number:	n	0	1	2
Term:	t_n	1	1	

If the highest common factor is 1 then we add the two highlighted values then and add 1 to the result. The highest common factor of 1 and 2 is 1, therefore we apply the rule: 1 + 2 + 1 = 4.

Term Number:	n	0	1	2	3
Term:	t_n	1	1	4	

We're ready to calculate the next term. We apply the same criteria. The highest common factor between the two highlighted terms 3 and 4 is 1, so again we apply the rule: 4 + 3 + 1 = 8.

Term Number:	n	0	1	2	3	4
Term:	t_n	1	1	4	8	

We're ready to calculate the next term. Applying the same criteria we see that the highest common factor between the highlighted terms 8 and 4 is no longer 1. [hcf(8,4)=4]. When the highest common factor is not equal to 1, the term (8) is divided by the highest common factor, in this case 4. Our new term becomes: 8 ÷ 4 = 2.

We'll do two more terms for practice.

Term Number:	n	0	1	2	3	4	5
Term:	t_n	1	1	4	8	2	

The highest common factor of 2 and 5 is 1, therefore the next term is 2 + 5 + 1 = 8.

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Term Number:	n	0	1	2	3	4	5	6
Term:	t_n	1	1	4	8	2	8	

The highest common factor 8 and 6 is not 1, therefore the alternative rule is applied. The highest common factor of 8 and 6 is 2; so the next term is equal to 8 ÷ 2 = 4

Check the next two terms in the table below.

Term Number:	n	0	1	2	3	4	5	6	7	8
Term:	t_n	1	1	4	8	2	8	4	12	3

Question: 1.

Complete the next series of terms in the table. (Check the answers with your teacher before continuing)

Term Number:	n	8	9	10	11	12	13	14	15	16
Term:	t_n	3								

Question: 2.

Create a scatterplot for the data collected so far with the term number (n) on the x axis (independent variable) and the term on the y axis (dependent variable). Are there any patterns appearing at this stage?

Question: 3.

Generate the first 100 terms of the sequence. Remember that you can use the GCD (Greatest Commond Divisor or Highest Common Factor command to check your answers.)

Mathemtical Investigation - Flying Straight

One of the challenges faced in any mathematical investigation is that we are not sure where the investigation may lead. Our first clue in this investigation is that our sequence is based on divisibility. When divisibility exists (common factors), the value of the next term in the sequence decreases, the reverse is true when divisibility is not present.

It's now time to start investigating. You may need to explore well beyond the first 100 terms so a spreadsheet or simple program can help generate terms in the sequence. Look for clues such as:

- What happens if terms or term numbers that are highly composite?
- What happens if terms or term numbers are prime number?
- What is the typical up / down run length? [Run = Quantity of consecutive term increases or decreases.]
- How often do 1's or 2's appear as terms and where do they occur?
- Progressively increase the quantity of data that you are analysing. Sometimes patterns start to appear when we get to stand back from the data.

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