Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1; an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$

Question 1

The equation $3x^2 + 3y^2 - 7by + 3 = 0$, where b is a real constant, will represent a circle if

A. $a < -\frac{6}{7}$ only

B. $a > \frac{6}{7}$ only

C. $a = \pm\frac{6}{7}$ only

D. $a < -\frac{6}{7}$ or $a > \frac{6}{7}$

E. $-\frac{6}{7} < a < \frac{6}{7}$

Question 2

The number of straight line asymptotes of the graph of $y = \dfrac{3x^3 - x^2 + 1}{x^2 + x - 2}$ is

A. 0
B. 1
C. 2
D. 3
E. 4

Question 3

The gradient of the hyperbola $\dfrac{y^2}{8} - \dfrac{x^2}{2} = 2$ at any point is an element of the set

A. $\mathbb{R}$

B. $\mathbb{R} \setminus [-2,2]$

C. $\mathbb{R} \setminus (-2,2)$

D. $(-2,2)$

E. $[-2,2]$

Question 4

For $x \in (0, \pi) \setminus \left\{\frac{\pi}{2}\right\}$, the solutions to $2\sin(x) > \frac{1}{2}\sec(x)$ are given by

A. $x \in \left(0, \frac{\pi}{12}\right)$

B. $x \in \left(\frac{\pi}{12}, \frac{5\pi}{12}\right)$

C. $x \in \left(\frac{5\pi}{12}, \frac{\pi}{2}\right)$

D. $x \in \left(\frac{\pi}{12}, \frac{5\pi}{12}\right) \cup \left(\frac{\pi}{2}, \pi\right)$

E. $x \in \left(0, \frac{5\pi}{12}\right) \cup \left(\frac{\pi}{2}, \pi\right)$

Question 5

If $g : \left(0, \frac{\pi}{6}\right) \to \mathbb{R}$, $g(x) = \operatorname{cosec}^2(3x) + \sec^2(3x)$, which one of the following statements is false?

A. g has range $[4, \infty)$

B. g is identical to the function $g : \left(0, \frac{\pi}{6}\right) \to \mathbb{R}$ where $g(x) = \operatorname{cosec}^2(3x)\sec^2(3x)$

C. g is identical to the function $g : \left(0, \frac{\pi}{6}\right) \to \mathbb{R}$ where $g(x) = \frac{8}{\cos(12x) - 1}$

D. $g'\left(\frac{\pi}{12}\right) = 0$

E. g is identical to the function $g : \left(0, \frac{\pi}{6}\right) \to \mathbb{R}$ where $g(x) = \frac{8}{1 - \cos(12x)}$

Question 6

Given that $A, B, C, D \in \mathbb{R} \setminus \{0\}$, the partial fraction form for the expression $\frac{3x^2 + 10x + 8}{(3x+4)^3(x^2-4)}$ is

A. $\frac{A}{x-2} + \frac{B}{3x+4} + \frac{C}{(3x+4)^2}$

B. $\frac{A}{x^2-4} + \frac{B}{3x+4} + \frac{C}{(3x+4)^2} + \frac{D}{(3x+4)^3}$

C. $\frac{Ax}{x^2-4} + \frac{B}{3x+4} + \frac{C}{(3x+4)^2} + \frac{D}{(3x+4)^3}$

D. $\frac{Ax+B}{x^2-4} + \frac{Cx+D}{(3x+4)^3}$

E. $\frac{A}{x-2} + \frac{B}{3x+4} + \frac{Cx}{(3x+4)^2}$

Question 7

The number of distinct roots of the equation $(z^4 - 16)(z^2 - 2iz + 8)$, where $z \in \mathbb{C}$, is

A. 3
B. 4
C. 5
D. 6
E. 7

Question 8

The set of points in the complex plane defined by $|z| = |z+4|$ is

A. The point $z = -2$
B. The line $\text{Re}(z) = 2$
C. The line $\text{Re}(z) = -2$
D. The circle with centre $(4,0)$ and radius 4
E. The circle with centre $(-4,0)$ and radius 4

Question 9

The polar form of the complex number $i - \sqrt{3}$ is

A. $2\text{cis}\left(-\frac{5\pi}{6}\right)$
B. $2\text{cis}\left(\frac{5\pi}{6}\right)$
C. $2\text{cis}\left(-\frac{\pi}{6}\right)$
D. $4\text{cis}\left(\frac{5\pi}{6}\right)$
E. $2\text{cis}\left(\frac{2\pi}{3}\right)$

Question 10

On an argand diagram, a set of points which lies on a circle of radius 3 centred at the origin is

A. $\{z \in \mathbb{C} : z\bar{z} = 3\}$
B. $\{z \in \mathbb{C} : z^2 = 3\}$
C. $\{z \in \mathbb{C} : \text{Re}(z^2) + \text{Im}(z^2) = 9\}$
D. $\{z \in \mathbb{C} : (z+\bar{z})^2 - (z-\bar{z})^2 = 36\}$
E. $\{z \in \mathbb{C} : (\text{Re}(z))^2 + (\text{Im}(z))^2 = 36\}$

Question 11

For the parametric equations $x=\sin(2t)-\cos(2t)$ and $y=\frac{1}{2}\sin(4t)$, $\frac{dy}{dx}$ in terms of t is

A. $\cos(2t)+\sin(2t)$

B. $\cos(2t)-\sin(2t)$

C. $\sec(2t)+\operatorname{cosec}(2t)$

D. $\sec(2t)-\operatorname{cosec}(2t)$

E. $\dfrac{\cos(4t)}{\cos(2t)-\sin(2t)}$

Question 12

Let $f:\left[-\frac{\pi}{2},\frac{3\pi}{2}\right]\to\mathbb{R}$, $f(x)=\cos^3(x)$. Using the substitution $u=\sin(x)$, the area bounded by the graph of f and the x-axis could be found by evaluating

A. $\int_{-\frac{\pi}{2}}^{\frac{3\pi}{2}}\left(1-u^2\right)du$

B. $\int_{-1}^{1}\left(1-u^2\right)du$

C. $2\int_{-1}^{1}\left(1-u^2\right)du$

D. $2\int_{-1}^{1}\left(u^2-1\right)du$

E. $2\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(1-u^2\right)du$

Question 13

For the relation $e^x\sin^{-1}(y)+e^y\sin^{-1}(x)=0$, the value of $\frac{d^2y}{dx^2}$ at the origin is

A. 1
B. –1
C. 4
D. –4
E. 0

Question 14

The solution of the differential equation $\sqrt{4+x^2}\,\frac{dy}{dx}=2$, with $y(0)=0$, can be approximated using Euler's method with step size 0.1. Using this method, the value obtained for y when $x=0.3$ is

A. 0.1000
B. 0.1999
C. 0.2989
D. 0.2994
E. 0.3994

Question 15

Two forces $\underset{\sim}{F}_1$ and $\underset{\sim}{F}_2$ act on an object. $\underset{\sim}{F}_1$ acts in the positive $\underset{\sim}{j}$ direction with magnitude 2 Newtons and $\underset{\sim}{F}_2$ acts in the direction of $\sqrt{3}\underset{\sim}{i} + \underset{\sim}{j}$ with magnitude 6 Newtons. The magnitude of the total force acting on the object, in Newtons, is

A. 8
B. $2\sqrt{43}$
C. $2\sqrt{13}$
D. $2\sqrt{7}$
E. $2\sqrt{2}$

Question 16

$ABCD$ is a parallelogram. The position vectors, respectively, of the points A, B C, and D are $\underset{\sim}{a} = -3\underset{\sim}{k}$, $\underset{\sim}{b} = \underset{\sim}{i} + n\underset{\sim}{j}$, $\underset{\sim}{c} = 5\underset{\sim}{i} + 2m\underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{d} = n\underset{\sim}{i} - 2\underset{\sim}{k}$. The values of m and n are

A. $m = 0$, $n = 5$
B. $m = 2$, $n = 4$
C. $m = 3$, $n = 6$
D. $m = 8$, $n = 6$
E. $m = 12$, $n = 6$

Question 17

A small pebble is projected vertically upwards with an initial velocity of $\frac{7\sqrt{2}}{3}$ ms^{-1}. It is subjected to gravity and air resistance. The acceleration of the pebble is described by the differential equation $\frac{d^2x}{dt^2} = -\left(g + 0.3v^2\right)$, where x m and v ms^{-1} are the pebble's vertical displacement and velocity respectively at time t seconds. The time taken for the pebble to reach its maximum height is

A. 5π seconds
B. $5\sqrt{6}\pi$ seconds
C. $\frac{5\sqrt{6}\pi}{126}$ seconds
D. $\frac{5\sqrt{6}\pi}{12}$ seconds
E. $\frac{6\sqrt{5}\pi}{126}$ seconds

Question 18
The length L cm and width W cm of a rectangle are independent normally distributed random variables, where $L \sim N\left(7, 3^2\right)$ and $W \sim N\left(5, 2^2\right)$. In terms of the standard normal variable Z, the probability that the rectangle's perimeter is greater than 50 cm is equivalent to

A. $\Pr(Z > 50)$

B. $\Pr(Z < 50)$

C. $\Pr\left(Z > 2\sqrt{13}\right)$

D. $\Pr\left(Z > \sqrt{26}\right)$

E. $\Pr\left(Z > \sqrt{13}\right)$

Question 19
A random sample of 400 potatoes from a farm has a total mass of 10 kg and a standard deviation of 4 g. Assuming the standard deviation of the sample is a sufficiently accurate estimate of the population standard deviation, an approximate 90% confidence interval for the mean mass of potatoes, in grams, produced on this farm is given by

A. $(24.67, 25.33)$

B. $(24.61, 25.39)$

C. $(9999.67, 10000.33)$

D. $(9999.61, 10000.39)$

E. $(23.68, 26.32)$

Question 20
The length of time a runner takes to complete the Melbourne Marathon is normally distributed with a mean of 4 hours and a standard deviation of 1 hour. The probability that the average time taken by a random sample of 10 runners is less than 3.5 hours is closest to

A. 0.3085
B. 0.0569
C. 0.9431
D. 0.6915
E. 0.4431