# TI-30X Plus MathPrint ${ }^{\text {" }}$ Scientific Calculator 

## Guidebook for HSC Mathematics Standard



## TI-30X Plus MathPrint ${ }^{\text {TM }}$ Scientific Calculator Guidebook NSW Stage 6 Mathematics Standard

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## About this guidebook

This guidebook is designed to show ways in which the TI-30X Plus MathPrint ${ }^{T M}$ scientific calculator can augment and enhance the teaching and learning of NSW Stage 6 Mathematics Standard. The Year 12 course featured in the guidebook is the Mathematics Standard 2 Year 12 course.

The first chapter of the guidebook is a getting started chapter which provides an overview of features such as display settings, modes and menus. In addition, the getting started chapter gives some general guidance for navigating around the calculator, on calculator syntax and tips for efficient and accurate calculation.

Throughout the chapters on Algebra, Measurement, Financial Mathematics and Statistical Analysis (note that Networks is not covered), TI-30X Plus MathPrint ${ }^{T M}$ features and menus relevant to the subject matter are introduced and explained. In terms of features (math tools), for example, the conversions feature is introduced on page 14, the data editor and list formulas feature is introduced on page 21, the stored operation feature is introduced on page 22 , the function feature is introduced on page 24 , and the expression evaluation feature is introduced on page 26. In terms of menus, for example, the statistics - regressions (STAT - REG) menu is introduced at the start of the section on Statistical Analysis (page 70).

The examples showcased in this guidebook are relevant to NSW Stage 6 Mathematics Standard. Most examples include a brief teaching note. Such teaching notes can provide a mathematical purpose for the example, highlight the mathematical concepts being developed in the example and describe how that example might fit in with the aims and outcomes of using calculators judiciously in Mathematics teaching and learning.

The examples generally follow a two-column table format. In most examples, the left-hand column displays step-bystep keystrokes that demonstrate TI-30X Plus MathPrint ${ }^{\text {TM }}$ functionality accompanied by a solution outline and notes. Where applicable, the right-hand column displays accompanying screenshots.

All examples in this guidebook assume the default settings as shown in Section 0.5 (page 8) on modes. If desired, the $\mathrm{TI}-30 \mathrm{X}$ Plus MathPrint ${ }^{\mathrm{TM}}$ can be reset so that all students start at the same point.

To do this, press 2nd [reset] 2].

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0 Getting started
This chapter provides an overview of features such as display settings, modes and menus.
It also gives some general guidance on navigation around the calculator, on calculator syntax and tips for efficient and accurate calculation.
0.1 Switching the calculator on and off

Press on to turn the TI-30X Plus MathPrint ${ }^{T M}$ on.
Press 2nd [off] to turn it off.
While the display is cleared, the history settings and memory are retained.
If no key is pressed for approximately 3 minutes, the APDTM (Automatic Power DownTM) feature turns off the TI30X Plus MathPrint ${ }^{\top M}$ automatically.
Press on after APDTM and the display, pending operations, settings and memory are retained.
0.2 Display contrast

To adjust the contrast:
(1) Press and release the 2nd key.
(2) Press $[\bullet \cdot \cdot]$ to darken the screen or press $[\cdot \square]$ to lighten the screen.

Note: This adjusts the contrast one level at a time. Repeat the above steps as needed.

### 0.3 Home screen

The TI-30X Plus MathPrint ${ }^{T M}$ can display a maximum of 4 lines with a maximum of 16 characters per line.

## Keystrokes description:

For entries and expressions longer than the visible screen area, scroll left and right (©) and (1)) to view the entire entry or expression.

Depending on space, the answer is displayed either directly to the right of the entry or on the right side of the next line.
randint $(1,6)+r a b$
1)+randint $(1,6)$

In MathPrint ${ }^{\text {TM }}$ mode, you can enter up to four levels of consecutive nested functions and expressions, which include fractions, square roots, exponents with $\wedge, \sqrt[x]{y}, \mathrm{e}^{x}$ and $10^{x}$.

Special indicators and cursors may display on the screen to provide additional information concerning functions or results.

| Indicator | Definition |
| :---: | :---: |
| 2ND | 2nd function. |
| FIX | Fixed-decimal setting. |
| SCI, ENG | Scientific or engineering notation. |
| DEG, RAD, GRAD | Angle mode (degrees, radians or gradians). |
| L1, L2, L3 | Displays above the lists in data editor. |
| H, B, O | Indicates HEX, BIN or OCT number-base mode. No indicator is displayed for default DEC mode. |
| 哥 | The calculator is performing an operation. Press on to break the calculation. |
| $\pm$ V | An entry is stored in memory before and/or after the visible screen area. Press and $\Theta$ to scroll. |
| - | Indicates that the multi-tap key is active. |
|  | Normal cursor. Shows where the next item you type will appear. Replaces any current character. |
| 糹 | Entry-limit cursor. No additional characters can be entered. |
| - | Insert cursor. A character is inserted in front of the cursor location. |
| E': | Placeholder box for empty MathPrintTM template. Use arrow keys to move into the box. |
| D | MathPrintTM cursor. Continue entering in the current MathPrintTM template or press (1) to exit the template. |

### 0.4 2nd functions

Press 2nd to activate the secondary function of a given key. Note that 2ND appears as an indicator on the screen. To cancel before pressing the next key, press 2nd again.

## Example: Activating a 2 nd function

Press 2nd $[r]$ to calculate the square root of a non-negative value.
Use the TI-30X Plus MathPrint ${ }^{T M}$ to calculate $\sqrt{25}$.

## Keystrokes and solution:



### 0.5 Modes

Press mode to choose modes.
Press $\Theta \odot(1)$ (1) to choose a mode and press enter to select it.
Press clear or 2 [nd [quit] to return to the home screen and perform your calculations using the chosen mode settings.

Default mode settings are highlighted in the following two screenshots.


DEGREE RADIAN GRADIAN sets the angle mode.
NORMAL SCI ENG sets the numeric notation mode.

- NORMAL displays results with digits to the left and right of the decimal point. For example, 123456.78.
- SCI expresses numbers with one digit to the left of the decimal point and the appropriate power of 10 . For example, 1.2345678 E 5 which is equivalent to $1.2345678 \times 10^{5}$.
- ENG displays results as a number from $1-999$ times 10 to an integer power. The integer power is always a multiple of 3 . For example, 123.45678E3.

Note: EEE is a shortcut key to enter a number in scientific notation format.
FLOAT 0123456789 sets the decimal notation mode.

- FLOAT (floating decimal point) displays up to 10 digits, plus the sign and decimal point.
- 0123456789 (fixed decimal point) specifies the number of digits ( 0 through 9 ) to display to the right of the decimal point.

REAL a+bir $\angle \theta$ sets the format of complex number results.

- REAL real results.
- a+bi rectangular results.
- $\mathrm{r} \angle \theta$ polar results.

DEC HEX BIN OCT sets the number base.

- DEC decimal (base 10).
- HEX hexadecimal (base 16). To enter hex digits A through F, use [2nd [A] etc.
- BIN binary (base 2).
- OCT octal (base 8).


## MATHPRINT CLASSIC

- MATHPRINT mode displays most inputs and outputs in textbook format.

| $2^{5}$ | DEG | 32 | $\cos (15)$ |
| :--- | ---: | ---: | ---: |
| $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{\sqrt{6}+\sqrt{2}}{4}$ |  |

- CLASSIC mode displays inputs and outputs in a single line.

0.6 Multi-tap keys

When pressed, a multi-tap key cycles through multiple functions.
Press (1) to stop multi-tap.
For example, press Enn to access $\boldsymbol{\operatorname { s i n }}, \boldsymbol{s i n}^{-1}, \boldsymbol{\operatorname { s i n h }}$ and $\boldsymbol{\operatorname { s i n }}^{-1}$.
Press the key repeatedly to display the function you wish to enter.
Multi-tap keys include

### 0.7 Menus

Menus provide access to a number of calculator functions.
Some menu keys, such as [2nd [recall], display a single menu.
Others, such as math, display multiple menus.
Press $(1)$ and $\odot$ to scroll and select a menu item or press the corresponding number next to the item.
To return to the previous screen without selecting the item, press clear.
To exit a menu and return to the home screen, press 2nd [quit].

## Keystrokes description:

Press 2nd [recall] (key with a single menu) to access RECALL VAR.


Press math (key with multiple menus) to access MATH, NUM, $D M S$ and $R \triangleleft P$.

0.8 Scrolling expressions and history

Press (1) or © $(1)$ to move the cursor within an expression that you are entering or editing.
Press 2nd © to move the cursor directly to the beginning of the expression.
Press [2nd (1) to move the cursor directly to the end of the expression.
From an expression or edit, press $\Theta$ or $\Theta$ to move the cursor through previous entries in the history. Pressing enter from an input or output in history will paste that expression back to the cursor position on the edit line.

Press 2 nd $\Theta$ from the denominator of a fraction in the expressions edit to move the cursor to the history. Pressing enter from an input or output in the history will paste that expression to the denominator.

Example: Scrolling expressions and history
Press $x^{2}$ to calculate the square of a value.
Use the TI-30X Plus MathPrint ${ }^{T M}$ to calculate
(a) $17^{2}-7^{2}$.
(b) $\sqrt{17^{2}-7^{2}}$, giving your answer in exact form.

Teacher Note: Students need sound mental computation strategies to determine the value of $17^{2}-7^{2}$ or sound estimation strategies to obtain a good estimate of its value.

## Keystrokes and solution:

(a) Enter 17 and press $x^{2} \square 7 x^{2}$ enter.

$$
17^{2}-7^{2}=240
$$

(b) Press [2nd $[v] \odot \odot$ enter enter.

$\sqrt{17^{2}-7^{2}}=4 \sqrt{15}$

### 0.9 Answer toggle

Press $\sim \approx$ to toggle the display result (when possible) between fraction and decimal answers, surd and decimal answers and multiples of $\pi$ and decimal answers.

## Example: Using answer toggle

Use the TI-30X Plus MathPrint ${ }^{\text {TM }}$ to express $4 \sqrt{15}$ in decimal form.

## Keystrokes and solution:

Enter $\mathbf{4} \sqrt{\mathbf{1 5}}$ (or use the last output from the previous example by pressing $\Theta$ enter ).

Press $\omega \approx$ to toggle between exact form and decimal form.
Press enter.

$4 \sqrt{15}=15.49 \ldots$
Note: $\sim \approx$ is also available to toggle number formats for values
in cells in the Function table and in the Data Editor.

### 0.10 Last answer

Press 2 nd [answer].
The last entry performed on the home screen is stored to the variable ans. This variable is retained in memory even after the TI-30X Plus MathPrint ${ }^{T M}$ is turned off.

To recall the value of ans:

Press 2nd [answer] (ans displays on the screen), or:
Press any operation key ( $\square, \square$ etc.) in most edit lines as the first part of an entry.
The variable ans and the operator are both displayed.
The variable ans is stored and pastes in full precision which is 13 digits.

## Example: Using last answer

The following example shows how the variable ans can preserve continuity in calculations.

## Keystrokes description:

Enter 2 and press 区 2 enter.

$$
2 \times 2=4
$$


$2 \times 2 \times 2=8$
Enter 3 and press 2nd [ $\stackrel{\rightharpoonup}{ }-$ ] 2nd [answer] enter.

$$
\sqrt[3]{2 \times 2 \times 2}=2
$$

### 0.11 Order of operations

## Order of operations hierarchy:

(1st) Expressions inside parentheses.
(2nd) Functions that need a closing bracket and precede the argument such as $\sin , \log$ and all $R \longleftrightarrow P$ menu items.
(3rd) Functions that are entered after the argument, such as $\boldsymbol{x}^{\mathbf{2}}$ and angle unit modifiers.
(4th) Exponentiation ( $\wedge$ ) and roots.
In MathPrint ${ }^{T M}$ mode, exponentiation using the $x$ key is evaluated from right to left. For example, $\mathbf{2}^{\mathbf{3}^{\mathbf{2}}}$ is evaluated as $2^{\left(3^{2}\right)}=512$.

The TI-30X Plus MathPrint ${ }^{T M}$ evaluates expressions entered with $x^{2}$ and [ 10 ] from left to right in both Classic and MathPrintTM modes.

For example, pressing $2 \times x^{2}$ enter is calculated as $\left(2^{2}\right)^{2}=16$.


Note: In Classic mode, exponentiation using the $x$ key is evaluated from left to right. For example, $2^{\wedge} 3 \wedge 2$ is evaluated as $\left(\mathbf{2}^{\wedge} 3\right)^{\wedge} \mathbf{2}=\mathbf{6 4}$.

（5th）Negation $(-)$ ．
（6th）Fractions．
（7th）Permutations（ nPr ）and combinations（ nCr ）．
（8th）Multiplication，implied multiplication，division and angle indicator $\angle$ ．
（9th）Addition and subtraction．
（10th）Logic operators and，nand．
（11th）Logic operators or，xor，xnor．
（12th）Conversions such as $\ \mathrm{n} / \mathrm{d}<$ Un／d， $\mathrm{F} \triangleleft \mathrm{D}, ~ D M S$ ．
（13th）sto $\rightarrow$
（14th）enter evaluates the input expression．
Note：End of expression operators and angle conversion DMS，for example，are only valid in the home screen． They are ignored in wizards，function table display and data editor features where the expression result，if valid，will display without a conversion．

## Example：Order of operations

Use the TI－30X Plus MathPrint ${ }^{\text {TM }}$ to calculate
（a） $42+3 \times-14$ ．
（b） $2+-6+9$ ．
（c）$\sqrt{27+37}$ ．
（d） $5 \times(3+4)$ ．
（e） $5(3+4)$ ．
（f）$\sqrt{8^{2}+15^{2}}$ ．
（g）$(-4)^{2}$ and $-4^{2}$ ．

## Keystrokes and solution：

（a）$\times \div+-$
Enter 42 and press $⿴ 囗+\boxed{\square} 14$ enter
$42+3 \times-14=0$
（b）（－）
Enter 2 and press $⿴ 囗 6$－- enter．
$2+-6+9=5$

（c）$\sqrt{ }$ and +
Press［2nd［ $r-$ ］and enter 27 ＠ 37 enter．
$\sqrt{27+37}=8$

| $27+37$ | DEG |
| ---: | ---: |
|  |  |
|  |  |
|  |  |

(d) ()

Enter 5 and press $\boxtimes \square 3 母 4 \square$ enter.
$5 \times(3+4)=35$
(e) () and +

Enter 5 and press $\square 3 \square 4 \square$ enter.
$5(3+4)=35$

## (f) $\wedge$ and $\sqrt{ }$

Press 2nd $[r]$ and enter 8 x $x^{2}$ (15 $x^{2}$ enter.

$\sqrt{8^{2}+15^{2}}=17$
(g) ( ) and -

Press $\square \boxed{\square}$ and enter $4 \square x^{2}$ enter.
$(-4)^{2}=16$


Press $\left[(-)\right.$ and enter $4 x^{2}$ enter.
$-4^{2}=-16$

### 0.12 Clearing and correcting

Press [2nd [quit] to return the cursor to the home screen.
Press clear to clear an error message. It also clears characters on an author line.
Press delete to delete the character at the cursor. When the cursor is at the end of an expression, it will backspace and delete.

Press [2nd [insert] to insert (rather than replace) a character at the cursor.
Press [2nd [clear var] 1 to clear variables $\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}, \boldsymbol{t}, \boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}, \boldsymbol{d}$ back to their default values of 0 .
Press 2nd [reset] 2 to return the TI-30X Plus MathPrint to default settings, clears memory variables, pending operations, all entries in history and statistical data; clears any stored operation and ans.

### 0.13 Memory and stored variables

The TI-30X Plus MathPrint has eight memory variables, $\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}, \boldsymbol{t}, \boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ and $\boldsymbol{d}$.
The following can be stored to a memory variable:

- real (or complex) numbers.
- expression results.
- calculations from various menus such as Distributions.
- data editor cell values (stored from the edit line).

Features of the TI-30X Plus MathPrint that use variables will use stored values.

Press $s t 0 \rightarrow$ to store a variable and press $x_{a b i c d}^{v z e d}$ (a multi-tap key that cycles through the variables $\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}, \boldsymbol{t}, \boldsymbol{a}, \boldsymbol{b}$, $\boldsymbol{c}$ and $\boldsymbol{d}$ ) to select the variable to store.

Press enter to store the value in the selected variable. If the selected variable already has a stored value, that value is replaced by the new one.

Press $x_{a b e}^{x=2 \pi} t$ to recall and use the stored values for these variables. The variable, say $\boldsymbol{y}$, is inserted into the current entry and the value assigned to $\boldsymbol{y}$ is used to evaluate the expression. To enter two or more variables in succession, press (1) after each.
Press 2nd [recall] to display a menu of variables and their stored values. Select the variable you wish to recall and press enter. The value assigned to the variable is inserted into the current entry and used to evaluate the expression.
Press [2nd [clear var] and select 1: Yes to clear all variable values. Any computed Stat Vars will no longer be available in the Stat Vars menu and would require recalculation.

## Example: Using stored variables

Given that $x=5$ and $y=12$, use the TI-30X Plus MathPrint to find the value of $x^{2}+y^{2}$.

## Keystrokes and solution:

| Press [2nd [clear var] 1 to clear variables. | $\begin{aligned} & \underset{12 \rightarrow x}{12 \rightarrow y} \end{aligned}$ | DEG | - 12 |
| :---: | :---: | :---: | :---: |
|  |  |  |  |
| Two possible approaches: |  |  |  |


| Approach 1: |  | DEG | $\stackrel{\square}{*}$ |
| :---: | :---: | :---: | :---: |
|  | $x^{2}+y^{2}$ |  | 169 |

## Approach 2:

Press [2nd [recall] $1 x^{2}$ [2nd [recall] 2 这 enter.


Note: This calculation can also be performed directly.

### 0.14 Unit conversions

The TI-30X Plus MathPrint ${ }^{\text {TM }}$ has a conversions feature that allows a total of 20 conversions (or 40 if converting both ways).

The conversion occurs at the end of an expression and can be stored as a variable.
Press $2 n d$ [convert] to access the CONVERSIONS menu.
The five conversion categories are:
(1) English-Metric.
(2) Temperature.
(3) Speed, length.
(4) Pressure.
(5) Power, Energy.

Example: Performing unit conversions
Use the TI-30X Plus MathPrint conversions feature to convert
(a) 100 degrees Fahrenheit to degrees Celsius, giving your answer correct to one decimal place.
(b) $20 \mathrm{~m} / \mathrm{s}$ to $\mathrm{km} / \mathrm{h}$.

## Keystrokes and solution:

(a) Enter 100 and press [2nd [convert] 2 to select Temperature.
CONVERSIONS
1: English-Metric
2:Temperature
$3 \downarrow$ Speed, Length

100 degrees Fahrenheit converts to 37.8 degrees Celsius (correct to one decimal place).

Note: This conversion can also be made using the formula

$$
\begin{array}{cc}
100 & \circ \\
& { }^{\mathrm{F}}{ }^{\circ}{ }^{\circ} \mathrm{CEG} \\
\hline
\end{array}
$$

DEG
CONVERSIONS

1:English-Metric 2: Temperature 3WSpeed, Length
(b) Enter 20 and press 2nd [convert] 3 to select Speed, Length.

Press (1) to select $\mathrm{m} / \mathrm{s} \boldsymbol{k m} / \mathrm{h}$. Press enter enter.
$20 \mathrm{~m} / \mathrm{s}$ converts to $72 \mathrm{~km} / \mathrm{h}$
Note: This conversion can be made by multiplying by 3.6.


## 1 Basic mathematical functions

### 1.1 Fractions

In MathPrint mode, press 圖.
Fractions with 圖 can include real (and complex numbers), operation keys ( $\square$, $\boxtimes$ etc.) and most function keys ( $x^{2}$, [2nd [ [\%], etc.).

Press $\Theta$ or $\Theta$ to move the cursor between the numerator and denominator.
Fraction results are automatically simplified, and the output is in improper fraction form.
When a mixed number output is required, press math $\square$ to access the $n / d$ Un/d conversion feature.
Press [2nd [ $\square$ 음 to enter a mixed number. Use the arrow keys to cycle through the unit, numerator and denominator.

## Example: Adding fractions

This example shows how to use a calculator to add fractions, including mixed numbers and fractions with different denominators.

Use the TI-30X Plus MathPrint to calculate $\frac{3}{4}+1 \frac{7}{12}$.
Give your answer as an improper fraction and as a mixed number.
Teacher Note: Students need to be able to convert an improper fraction to a mixed number and vice versa.

## Keystrokes and solution:


$\frac{3}{4}+1 \frac{7}{12}=\frac{7}{3}$
To give the answer as a mixed number:
Press math 1 enter.
$\frac{3}{4}+1 \frac{7}{12}=2 \frac{1}{3}$


Note: Parentheses are added automatically.


If decimal numbers are used or calculated in a fraction's numerator or denominator, the result will display as a decimal.

Press [2nd [ $\mathrm{f} \triangleright \mathrm{d}$ ] when wanting to convert a fraction to a decimal.

## Example: Converting a decimal number to an improper fraction

This example shows how to convert a decimal number to an improper fraction.
Use the TI-30X Plus MathPrint to calculate $\frac{1.2+1.3}{4}$.
Give your answer as a decimal number and an improper fraction.
Teacher Note: Students need to be able to convert a decimal number to an improper fraction and vice versa.

## Keystrokes and solution:

Press 圖 and enter $1.2 \oplus 1.3 \odot 4$ enter .

$$
\frac{1.2+1.3}{4}=0.625
$$

To express the answer as a fraction:

| $\frac{1.2+1.3}{4}$ | 0.625 |  |
| :--- | :--- | :--- |
|  |  |  |

$\qquad$
Approach 2:
Press math 1 enter.

$$
0.625=\frac{5}{8}
$$



Pressing 圖 before or after numbers or functions are entered may pre-populate the numerator with parts of your expression. Watch the screen as you press keys to ensure your expression is entered exactly as required.

To paste a previous entry from history in the numerator or mixed number unit, place the cursor in the numerator or unit, press $\Theta$ to scroll to the desired entry and press enter to paste the entry to the numerator or unit.

To paste a previous entry from history in the denominator, place the cursor in the denominator, press 2nd $\Theta$ to jump into history. Press $\Theta$ to scroll to the desired entry and press enter to paste the entry to the denominator.

### 1.2 Percentages

To perform a calculation involving a percentage, press 2nd [\%] after entering the value of the percentage.

## Example: Calculating the percentage of a quantity

This example shows how to use a calculator to calculate the percentage of a quantity.
Use the TI-30X Plus MathPrint to calculate $7.5 \%$ of 150 .
Teacher Note: Students need to be able to estimate the magnitude of the resulting quantity. For example, $10 \%$ of 150 is 15 .

## Keystrokes and solution:

Enter 7.5 and press 2nd [\%] 区 150 enter.
$7.5 \%$ of 150 is 11.25


### 1.3 Scientific notation

A number in scientific notation is made up of the following two parts multiplied together.
A number, $a$, where $1 \leq a<10$ and a power of 10 . Hence numbers of the form $a \times 10^{n}$ where $n$ is an integer.
Press mode. NORMAL SCI ENG sets the numeric notation mode. In SCI mode, numbers are expressed with one digit to the left of the decimal point and the appropriate power of 10.

EE is a shortcut key to enter a number in scientific notation format.

## Example: Entering numbers in scientific notation format

This example shows how to use a calculator to enter a number in scientific notation format.
Use the TI-30X Plus MathPrint to enter $1.3 \times 10^{-5}$ as $1.3 \mathrm{E}-5$.

## Keystrokes and solution:

Enter 1.3 and press 画 $-(-) 5$ enter.
$1.3 \times 10^{-5}=0.000013$
To change to SCI mode, press mode $\Theta$ (1) enter.


Press clear enter.
The $a^{\square} 10^{0}$ key is a multi-tap key. Pressing $\left.e^{\square} 10^{0}\right]$ pastes the base 10 to the power function.
Hence another way of entering a number in scientific notation format, is to press $0^{\square 100}$. The result obtained is displayed according to the numeric notation mode setting. Use parentheses to ensure correct order of operation.

Another way of entering a number in scientific notation format, is to enter $\mathbf{1 0}$ and press $x^{\text {D }}$.

## Example

This example shows how to use a calculator to find the quotient of two numbers expressed in scientific notation.
Use the TI-30X Plus MathPrint to calculate $\frac{5 \times 10^{3}}{8 \times 10^{-2}}$. Give your answer in scientific notation.
Teacher Note: Students need to be able to estimate the size of a quotient (or a product). This is an important skill when monitoring outputs from calculations performed with technology.

## Keystrokes and solution:



## Approach 3: <br>  enter.

$$
\frac{5 \times 10^{3}}{8 \times 10^{-2}}=6.25 \times 10^{4}
$$

### 1.4 Powers, roots and reciprocals

Press $x^{2}$ to calculate the square of a value.
Press $x$ to raise a value to the power indicated. Press (1) to move the cursor out of the power in MathPrint mode.

## Example: Raising a value to a power

This example shows how to use a calculator to raise a value to the power indicated.
Use the TI-30X Plus MathPrint to calculate
(a) $10^{3}+9^{3}$.
(b) $12^{3}+1^{3}$.

Teacher Note: The Hardy-Ramanujan Number, 1729, is the smallest number which can be expressed as the sum of two different cubes in two different ways.

## Keystrokes and solution:



Press 2 nd $[r]$ to calculate the square root of a non-negative value. [In complex number modes, $\mathbf{a}+\mathbf{b i}$ and $\mathbf{r} \angle \boldsymbol{\theta}$, press $2 \mathrm{znd}[r]$ to calculate the square root of a negative real value.]

Press 2nd [ $\square^{\square}$ ] to calculate the $x$ th root of any non-negative value and any odd integer root of a negative value.

## Example: Finding the $x$ th root of a non-negative value

This example shows how to use a calculator to find the $x$ th root of a non-negative value.
Use the TI-30X Plus MathPrint to calculate $\sqrt[6]{729}$.
Teacher Note: Students should recognise that 3 raised to the power of 6 gives 729 .

## Keystrokes and solution:

Enter 6 and press 2nd [ ${ }^{\square} \checkmark$ - 729 enter .


$$
\sqrt[6]{729}=3
$$

Press $\left[\frac{1}{\square}\right]$ to calculate the reciprocal of a value.

## Example: Finding the reciprocal of a fraction

This example shows how to use a calculator to verify numerically that $\left(\frac{a}{b}\right)^{-1}=\frac{b}{a}$ for $a=5$ and $b=7$.
Use the TI-30X Plus MathPrint to calculate the reciprocal of $\frac{5}{7}$. Give your answer as a
(a) fraction.
(b) decimal.

Teacher Note: Students need to recognise that the reciprocal of $\frac{a}{b}$ is $\left(\frac{a}{b}\right)^{-1}$ or $\frac{1}{\frac{a}{b}}=\frac{b}{a}$.

## Keystrokes and solution:

(a) Enter 5 and press 圖 7 and press (1) 2nd [ 10 ] enter.
$\frac{1}{\frac{5}{7}}=\left(\frac{5}{7}\right)^{-1}=\frac{7}{5}$

(b) Press $\omega \approx$ to express as a decimal.
$\left(\frac{5}{7}\right)^{-1}=1.4$


Alternatively, press [2nd [ $\mathrm{f} \stackrel{\mathrm{d}}{ }$ ] (convert fraction to decimal) enter.
1.5 Pi (symbol pi)

The TI-30X Plus MathPrint can be used to perform calculations involving $\pi$.
To access $\pi$, press $\pi_{i}$ (a multi-tap key).
Note that $\pi \approx 3.14159265359$ for calculations and $\pi \approx 3.141592654$ for display in Float mode.

## Example: Area of a circle

This example shows how to use a calculator to find the area of a circle given its radius and the radius of a circle given its area.

Use the TI-30X Plus MathPrint to find
(a) the area of a circle whose radius is 8 cm . Give your answer correct to one decimal place.
(b) the radius of a circle whose area is $60 \mathrm{~m}^{2}$. Give your answer correct to one decimal place.

Teacher Note: When using technology, students need to have a sense of the magnitude of the expected answer. Hence students need to carefully monitor their calculations when using a calculator.

## Keystrokes and solution:

(a) $A=\pi r^{2}$

Press $\pi^{\mathrm{e}} \boldsymbol{x}$ and enter $8 x^{2}$ enter.

$A=64 \pi\left(\mathrm{~cm}^{2}\right)$
Press $\sim \approx$ to convert to a decimal.
$A=201.1\left(\mathrm{~cm}^{2}\right)$ correct to 1 decimal place .

| (b) $60=\pi r^{2}$ and so $r=\sqrt{\frac{60}{\pi}}(r>0)$ | $\sqrt{\frac{60}{\pi}}$ |
| :--- | :--- |
| 4.370193722 |  |
| Press 2nd $[r]$ 圖 and enter $60 \odot \pi_{i}^{\text {e }}$ enter]. |  |

$r=4.4(\mathrm{~m})$ correct to 1 decimal place.

## 2 Algebra

### 2.1 Formulae and equations (MS-A1)

We introduce two TI-30X Plus MathPrint ${ }^{T M}$ features, namely, the data editor and list formulas feature and the stored operations feature.

## TI-30X Plus MathPrint ${ }^{\text {TM }}$ data editor and list formulas feature

Press data to access the data editor.
Data can be entered in up to three lists (L1, L2 and L3). Each list can contain up to 50 items.
When editing a list, press data to access the CLR, FORMULA and OPS menus.
Use (1) (1) $\odot \odot$ to select a cell in the data editor and then enter a value.
Mode settings affect the display of a cell value. Fractions, radicals and $\pi$ values will display.
Press:
sto $\rightarrow$ to store the value of the cell to a variable.
$\Delta \approx$ to toggle the number format when a cell is highlighted.
delete to delete a cell.
enter clear to clear the edit line of a cell.
2nd [quit] to return to the home screen.
2nd $\Theta$ to go to the top of a list.
2nd $\Theta$ to go to the bottom of a list.
Use the CLR menu to clear the data from a list or lists.

## FORMULA menu:

In the data editor, press data (1) to display the FORMULA menu. Select the appropriate menu item to add or edit a list formula in the highlighted column or clear formulas from a particular list.

When a data cell is highlighted, pressing sto $\rightarrow$ is a shortcut to open the formula edit state.
In the formula edit state, pressing data displays a menu to paste L1, L2 or L3 in the formula.
Formulas cannot contain a circular reference such as $\mathbf{L 1}=\mathbf{L} 1$.
When a list contains a formula, the edit line will display the reversed cell name. Cells will update if referenced lists are updated.

To clear a formula list, clear the formula first and then clear the list.
If $s t \rightarrow$ is used in a list formula, the last element of the computed list is stored to the variable. Lists cannot be stored.

List formulas accept all TI-30X Plus MathPrint ${ }^{T M}$ functions and real numbers.

## Options (OPS menu):

In the data editor, press data (1) to display the OPS menu.
This allows you to sort values from smallest to largest or largest to smallest, create a sequence of values to fill a list or sum the elements in a list which can then be stored to a variable for further use.

## TI-30X Plus MathPrint ${ }^{\text {TM }}$ stored operations feature

Press 2nd [set op] to store an operation.
Press $2 \mathrm{nd}[\mathrm{op}]$ to paste an operation to the home screen.
To set an operation and then recall it:
Press 2nd [set op].
Enter any combination of numbers, operations, and/or data values.
Press enter to store the operation.
Press 2nd [op] to recall the stored operation and apply it to the last answer or the current entry.
If you apply 2nd [op] directly to a 2nd [op] result, a $\mathbf{n}=1$ iteration counter is incremented.
Students are expected to:

- substitute numerical values into linear algebraic expressions and equations.
- evaluate the subject of a formula, given the value of other pronumerals in the formula.
- change the subject of a formula.
- develop and solve linear equations, including those derived from substituting values into a formula, or those developed from a word description.


## Example: Solving a linear equation

The TI-30X Plus MathPrint ${ }^{T M}$ data editor and list formulas feature, and the stored operations feature can both be used to solve problems involving linear expressions and equations. This example could also be solved using the function feature (see page 24) and, of course, the conversions feature (see page 14).

On a particular July day, a weather forecast listed the following predicted maximum temperatures.
Canberra $13^{\circ} \mathrm{C}$
Sydney $18^{\circ} \mathrm{C}$
Thredbo $2^{\circ} \mathrm{C}$
The function $F(C)=\frac{9}{5} C+32$ can be used to convert degrees Celsius to degrees Fahrenheit.
(a) Convert these temperatures from degrees Celsius to degrees Fahrenheit using the TI-30X Plus MathPrint
(i) data editor and list formulas feature.
(ii) stored operations feature.
(b) If Katoomba is predicted to have a maximum temperature of $9^{\circ} \mathrm{C}$, use the $\mathrm{TI}-30 \mathrm{X}$ Plus MathPrint to convert this temperature to degrees Fahrenheit.

Teacher Note: This example showcases the different ways that the TI-30X Plus MathPrint can be used to solve these types of problems.

Keystrokes and solution:
(a) (i) Using the data editor and list formulas feature:

Press data. Press data 4 to clear all lists.
Enter 13 and press $\odot$. Repeat for 18 and 2.
The three temperatures should now be displayed in L1.
Press (1) to scroll across to the top of L2. Press data (1) to select
FORMULA and press 1 . Enter the temperature conversion formula to L2.

Enter $\frac{9}{5}$ using the division key to ensure decimal outputs.
Enter 9 and press $\square_{\square} \mathbf{x}$. Press data enter to paste $\mathbf{L 1}$ into the author line. Press $\square$ and enter 32 enter.

L2 should now display the converted temperatures $55.4^{\circ} \mathrm{F}$, $64.4^{\circ} \mathrm{F}$ and $35.6^{\circ} \mathrm{F}$.

(a) (ii) Using the stored operations feature:

Press 2nd [setop].
[If required, press clear to clear any previously stored operations.]

Press $\times$ and enter $1.8 \oplus 32$ enter 13 2nd [op].
Repeat for 18 and 2.
The three converted temperatures are $55.4^{\circ} \mathrm{F}, 64.4^{\circ} \mathrm{F}$ and

## OP=*1. $8+32 \square$

## $\downarrow$

 $35.6^{\circ} \mathrm{F}$.

(b) Using the data editor and list formulas feature:

Press data.
Move to L1(4) =, enter 9 and press enter.
L2 should now display Katoomba's converted temperature of $48.2^{\circ} \mathrm{F}$.

Using the stored operations feature:
Press 2nd [quit]. Enter 9 and press 2nd [op].


Students are expected to:

- substitute numerical values into non-linear algebraic expressions and equations.
- solve problems involving formulae.

Here we introduce the TI-30X Plus MathPrint ${ }^{\text {TM }}$ function feature.
TI-30X Plus MathPrint ${ }^{\text {TM }}$ function feature
Press table to access the function table.


The function table menu contains the following options:

## 1: Add/Edit Func

Lets you define a function $f(x)$ or $g(x)$ or both and generates a table of values.


## 2: f(



## 3: g(

Pastes $\mathbf{g}$ ( to an input area such as the home screen to evaluate the function at a point (for example, $\mathbf{g}(\mathbf{2})$ ).
Press $\Theta$ and $\Theta$ to move around the function table feature.
To set up a function table:
Press table 1 to select Add/Edit Func. [Press clear if required.]
Enter one or two functions as appropriate and press enter.
TABLE SETUP contains the options Start, Step, Auto, or $\boldsymbol{x}=$ ?
Start: Specifies the starting value for the independent variable, $\boldsymbol{x}$. It is set to start at $\mathbf{0}$.
Step: Specifies the step value for the independent variable, $\boldsymbol{x}$. The step can be positive or negative but cannot be zero. It is set at $\mathbf{1}$.

Auto: Automatically generates a series of values for the dependent variable, $\boldsymbol{y}$, based on the table start and the table step values.
$\boldsymbol{x}=\boldsymbol{?}$ : Lets you build a table manually for the dependent variable, $\boldsymbol{y}$, by allowing entry of specific values for the independent variable, $\boldsymbol{x}$.

To display a table, input the desired settings, select CALC and press enter.
In function table view, press clear to display and edit the TABLE SETUP wizard as needed.

## Example: Solving a non-linear equation

This example shows how a calculator can be used to solve a question involving a non-linear equation (quadratic) derived from a likely unfamiliar context.

All people attending a party shook hands with each other as a way of exchanging a greeting. The number of handshakes, $N$, exchanged between $x$ people at the party is given by $N=\frac{x}{2}(x-1)$ where $x \in \mathbb{Z}^{+}$.
(a) Use the TI-30X Plus MathPrint ${ }^{T M}$ function feature to find the number of handshakes that would be exchanged between 5,10 and 50 people respectively.
(b) Given that 136 handshakes were exchanged, use the TI-30X Plus MathPrint ${ }^{T 1}$ function feature to determine how many people at the party shook hands.

Teacher Note: In part (b), it is important for students to check their answer by substitution.

## Keystrokes and solution:

(a) Press table 1 to access the function table.
[If required, press clear.]
 $\odot \odot$.

Move the cursor to select $\boldsymbol{x}=\boldsymbol{?}$ and press enter (CALC) enter.
Enter 5 and press enter 10 enter 50 enter.
With 5 people, there are 10 handshakes.
With 10 people, there are 45 handshakes.
With 50 people, there are 1225 handshakes.
Alternatively, press [2nd [quit] to go to the home screen.
Press table 2 and enter 5 enter.
So $f(5)$ is 10 as before.
Press $\Theta \odot$ to select $\mathbf{f}(\mathbf{5})$. Press enter.
Change $f(5)$ to $f(10)$ and press enter.


Change $\mathbf{f} \mathbf{( 1 0 )}$ to $\mathbf{f}(\mathbf{5 0})$ and press enter. Thus, confirming our results.
(b) From part (a), we conclude that $x>10$.

By entering values for $\boldsymbol{x}$, starting with 15 , for example, the last two screenshots show that 17 people exchanged 136 handshakes.


Here we introduce TI-30X Plus MathPrint ${ }^{T M}$ expression evaluation feature.

### 2.2 TI-30X Plus MathPrint ${ }^{\text {TM }}$ expression evaluation feature

Press [2nd [expr-eval] to input and calculate an expression using numbers, functions and variables/parameters. Pressing [2nd [expr-eval] from a populated home screen expression pastes the content to Expr =.

If variables $\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}, \boldsymbol{t}, \boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ and $\boldsymbol{d}$ are used in the expression, you will be prompted for values or use the stored values displayed for each prompt. The number stored in the variables will update in $\mathrm{TI}-30 \mathrm{X}$ Plus MathPrint ${ }^{\top \mathrm{TM}}$.

Students are expected calculate stopping distances of vehicles using a suitable formula.

A vehicle's stopping distance is the distance it travels from the time the vehicle's driver sees an event occurring to the time the vehicle is brought to a stop.

The general formula for stopping distance is $d_{S}=d_{R}+d_{B}$, where $d_{S}$ is the stopping distance, $d_{R}$ is the reaction distance and $d_{B}$ is the braking distance. All distances are measured in metres.

## Example: Stopping distances

This example shows how to use a calculator to calculate the stopping distance of a vehicle.
Brian was driving at a speed of $110 \mathrm{~km} / \mathrm{h}$ when he needed to apply the brakes and come to a stop.
His reaction time, $t$, is known to be 1.5 seconds.
The stopping distance, $d_{S}$ metres, can be modelled by the formula $d_{S}=\frac{5 v t}{18}+\frac{v^{2}}{150}$.
Use the formula for $d_{S}$ and the $\mathrm{TI}-30 \mathrm{X}$ Plus MathPrint ${ }^{\text {TM }}$ expression evaluation feature to find Brian's stopping distance, $d_{s}$. Give your answer correct to the nearest metre.

Teacher Note: When using the TI-30X Plus MathPrint ${ }^{T M}$ expression evaluation feature, it is important for students to be clear that $x$ is not representing a distance. In this context, it is representing the speed, $v$.

## Keystrokes and solution:

$d_{S}=\frac{5 v t}{18}+\frac{v^{2}}{150}$
As the Tl-30X Plus MathPrint ${ }^{T M}$ expression evaluation feature does not house the variable $v$, we will use the variable $x$ instead.
$d_{S}=\frac{5 x t}{18}+\frac{x^{2}}{150}$
Press 2 nd [expr-eval]. [If required, press clear.]
$x_{a b i c d}^{x=\pi}$ is a multi-tap key that cycles through the variables $\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}$, $\boldsymbol{t}, \boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ and $\boldsymbol{d}$.

 and continue to press $x_{a b e d}^{v 2 \pi}$ until $t$ appears.
 clear 110 enter clear 1.5 enter.

Substituting $x=110$ and $t=1.5$ into $d_{S}=\frac{5 x t}{18}+\frac{x^{2}}{150}$ gives $d_{S}=126.5$.

Brian's stopping distance, $d_{S}$, is 127 metres, correct to the nearest metre.


Alternatively, Brian's stopping distance can be calculated as shown at right.

Students are expected to:

- calculate and interpret blood alcohol content (BAC) based on drink consumption and body weight.
- determine the number of hours required for a person to stop consuming alcohol in order to reach zero BAC.

Blood alcohol content (BAC) is a measure of the amount of alcohol in your blood. The measurement is the number of grams of alcohol in 100 millilitres ( mL ) of blood.

For example, a BAC of 0.07 means 0.07 grams or 70 mg of alcohol in every 100 mL of blood.
A person's BAC is influenced by the number of standard drinks consumed in a given amount of time and the person's body weight.

## Example: Blood alcohol content (BAC) (1)

This example shows how to use a calculator to calculate a person's blood alcohol content (BAC) based on their drink consumption and body weight.

A male's BAC can be estimated using the formula $B A C=\frac{10 N-7.5 H}{6.8 M}$ where $N$ is the number of standard drinks consumed, $H$ is the number of hours drinking and $M$ is the body weight in kilograms.

Peter is 92 kg and has consumed nine standard drinks in three hours.
Use the BAC formula and the TI-30X Plus MathPrint ${ }^{\text {TM }}$ expression evaluation feature to estimate Peter's BAC three hours after he started drinking. Give your answer correct to two decimal places.
Teacher Note: It is a good idea to discuss with students some of the limitations in estimating a person's BAC. A person's $B A C$ is measured with a breathalyser or by analysing a blood sample. Factors such as gender, fitness and liver function can affect a person's BAC. It is also worth highlighting the various blood alcohol limits applied to drivers in NSW.

## Keystrokes and solution:

$$
B A C=\frac{10 N-7.5 H}{6.8 M}
$$

As the TI-30X Plus MathPrint ${ }^{T M}$ expression evaluation feature does not house the variables $N, H$ and $M$, we will use the variables $x, t$ and $y$ respectively instead.

$$
B A C=\frac{10 x-7.5 t}{6.8 y}
$$

Press 2 2nd [expr-eval]. [lf required, press clear .]
$x_{a b a d}^{y z e d}$ is a multi-tap key that cycles through the variables $\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}$, $\boldsymbol{t}, \boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ and $\boldsymbol{d}$.
 $\square$ and enter 7.5 and continue to press $\underbrace{v 25}_{\text {abad }}$ until $t$ appears.



| Press (1) (1) enter clear 9 enter clear 3 enter clear 92 enter. | $\frac{10: * x-7.5 * t}{6.8:+9}$ |
| :---: | :---: |
|  |  |
| $B A C=\frac{10 x-7.5 t}{6.8 y} \text { gives } B A C=0.107896$ | \% |
| Peter's BAC is estimated to be 0.11 , correct to two decima places. | $\frac{\frac{10: * 9-7.5: * 3}{6.8 *: 92}}{0.107896419}$ |

## Example: Blood alcohol content (BAC) (2)

This example shows how to use a calculator to determine the number of hours required for a person to stop consuming alcohol in order to reach zero BAC.

The number of hours a person should wait before driving can be estimated from the formula $t=\frac{B A C}{0.015}$ where $t$ is the time in hours.

Tahlia's BAC is 0.140 and Toby's BAC is 0.091 . The legal BAC limit for both drivers is 0.05 .
Use the formula $t=\frac{B A C}{0.015}$ and the TI-30X Plus MathPrint ${ }^{T M}$ stored operations feature to calculate the number of hours each should wait before they can drive. Give your answers correct to the nearest hour.

Teacher Note: Knowledge of this formula is very useful when making informed decisions as to when to drive after consuming alcohol.

## Keystrokes and solution:

Using the stored operations feature:
Press [2nd [set op]. [lf required, press clear to clear any previously stored operations.]

Press $\square_{-}$and enter 0.015 enter.
Enter $\mathbf{0 . 1 4 0}$ and press 2nd [op] 0.091 [2nd [op].
To the nearest hour, Tahlia should wait 10 hours before driving and
Toby should wait 7 hours before driving.


### 2.3 Linear relationships (MS-A2)

Students are expected to:

- model, analyse and solve problems involving linear relationships.
- review the linear function $y=m x+c$ and understand the geometrical significance of $m$ and $c$.
- construct and analyse a linear model, graphically or algebraically, to solve practical problems.

The TI-30X Plus MathPrint ${ }^{T M M}$ can be used to model, analyse and solve problems involving linear relationships.

## Example: Linear relationships (1)

This example shows how to use a calculator to solve a problem involving a linear relationship.
Daisy's car has a petrol tank with a capacity of 54 litres. Her car's average fuel consumption is 6 litres $/ 100 \mathrm{~km}$. She fills the petrol tank to capacity and drives 700 km to stay with friends.

Let $L$ litres be the amount of petrol remaining in the car's petrol tank after travelling $x$ hundred kilometres. For example, $x=1$ denotes a travel distance of 100 km .
(a) Find an expression for $L$, in terms of $x$. Give your answer in the form $L=m x+c$.
(b) Interpret, in context, the value of $m$ and the value of $c$ found in part (a).
(c) Use the TI-30X Plus MathPrint ${ }^{\text {TM }}$ function feature to calculate
(i) how much petrol was left in the tank when Daisy arrived at her friend's house.
(ii) the maximum distance Daisy's car can travel before running out of petrol.

Teacher Note: Students need to be able to formulate a linear relationship from worded information. It is also important that students understand the meaning, in context, of $m$ and $c$ in the linear function $y=m x+c$.

## Keystrokes and solution:

(a) $L=54-6 x$
(b) $m=-6$ represents 6 litres of fuel being used per 100 km .
$c=54$ represents the initial amount of fuel in the car's petrol tank.
(c) (i) A distance of 700 km corresponds to $x=7$.

Press table 1 to access the function table.
[If required, press clear.]

Move the cursor to select $\boldsymbol{x}=$ ? , press enter (CALC) enter.
Enter 7 and press enter.
When $x=7, L=12$ and so Daisy had 12 litres in her petrol
 tank when she arrived at her friend's house.
(c) (ii) Enter guess(es) for the value of $\boldsymbol{x}$ and press enter.

When $x=8, L=6$ and when $x=9, L=0$.
Alternatively, press 2nd [quit] to go to the home screen.
Press table 2 and enter guess(es) for the value of $\boldsymbol{x}$.
When $x=9, L=0$ and so Daisy's car can travel 900 km before running out of petrol.


Students are expected to:

- recognise that a direct variation relationship produces a straight-line graph.
- determine a direct variation relationship from a written description, a straight-line graph or a linear function in the form $y=m x$.
- recognise the gradient of a direct variation graph as the constant of variation.

A variable $y$ varies directly with a variable $x$ if $y=m x$, where $m$ is a non-zero constant of proportionality. The graph of $y=m x$ is a linear function that passes through the origin with a gradient $m$.

## Example: Direct variation

This example shows how to use a calculator to help solve a problem involving direct variation.
A district nurse who often travels by car delivering medical supplies receives a travelling allowance. This travelling allowance, $A$ dollars, varies directly with the number of kilometres, $n$, travelled in a week. During a particular week, the district nurse travelled 60 kilometres and received an allowance of $\$ 15$. The following week, the district nurse travelled a distance of 400 kilometres. Find the district nurse's travelling allowance for that week.

Teacher Note: This example illustrates how, given the constant of proportionality, to establish an equation which can be used to determine the value of an unknown quantity. When solving direct variation problems, it is important to link the given equation with a corresponding table of values or a corresponding graph.

## Keystrokes and solution:

$A \propto n$ and so $A=k n$ where $k$ is the constant of proportionality.
Substitute $n=60$ and $A=15$ into $A=k n$ and solve to find $k$.
$15=60 k$
$k=0.25$
Enter 15 and press 圖 $60(1) \omega \approx$ enter.
$k=0.25$ and hence $A=0.25 n$.
Substitute $n=400$ into $A=0.25 n$.
Press [2nd [answer] ख and enter 400 enter.

$A=0.25 \times 400$
$=100$
The district nurse's travelling allowance for that week was $\$ 100$.

### 2.4 Types of relationships (MS-A4)

### 2.4.1 Simultaneous linear equations

Students are expected to:

- solve a pair of simultaneous linear equations by finding the point of intersection between two straight-line graphs.
- develop a pair of simultaneous linear equations to model a practical situation.
- solve practical problems that involve determining and interpreting the break-even point of a simple business problem where cost and revenue are represented by linear equations.


## Example: Simultaneous linear equations

This example shows how to use a calculator to solve a practical problem that involves determining and interpreting the break-even point of a simple business problem where cost and revenue are represented by linear equations. This example could also be solved using the TI-30X Plus MathPrint ${ }^{T M}$ function feature.

A company that manufactures and sells paper straws has fixed costs of $\$ 400$ per week.
It costs the company $\$ 2.50$ to make a carton of 500 straws.
The company sells a carton for $\$ 5.00$.
(a) If $\$ C$ is the cost of producing $S$ cartons of straws per week, express $C$ in terms of $S$.
(b) If $\$ I$ is the income received for selling $S$ cartons of straws per week, express $I$ in terms of $S$.
(c) Use the TI-30X Plus MathPrint ${ }^{\text {TM }}$ data editor and list formulas feature to find the break-even point.

Teacher Note: Encourage students to sketch the two graphs on the same set of axes to determine the approximate location of the point of intersection. This helps to construct a table of values that contains the solution to the two equations. Students should relate the solution found to the intersection point of the two graphs and to the context of the problem being solved.

Keystrokes and solution:
(a) $C=2.5 S+400$
(b) $I=5 S$
(c) Press data. Press data 4 to clear all lists.

In L1, enter 100 as an initial guess and press $\odot$.
Press (1) to scroll across to the top of L2.
Press data (1) to select FORMULA and press enter.
Enter the list formula $\mathbf{2 . 5} \mathbf{~ * ~ L 1 ~ + ~ 4 0 0 ~ t o ~} \mathbf{L 2}$.
Enter 2.5 and press $\boxtimes$. Press data enter to paste $\mathbf{L} 1$ into the author line. Press $\dagger$ and enter 400 enter.
650 should now be displayed in $\mathbf{L} 2$.


Press (1) to scroll across to the top of L3.

Press data ( () to select FORMULA and press enter.


Enter 5 and press $x$ data enter enter.
500 should now be displayed in L3.
Move to $\mathbf{L 1 ( 2 )}=$ and enter an appropriate guess for the $S$ value, for example, 150. Press enter.

Continue to enter $S$-values that (hopefully) approach the required solution.

After each $S$-value is entered, press enter.
The two screenshots at right show a set of $S$-values used to determine the solution.

$S=160, C=I=800$, which is the break-even point.

### 2.4.2 Non-linear relationships

Students are expected to graph and recognise an exponential function in the form $y=a^{x}$ and $y=a^{-x}$ where y $a>0$.

## Example: Exponential models (1)

This example shows how to use a calculator to construct a table of values which can be used to help sketch the graph of a function of the form $y=a^{x}$.

Consider the function $y=3^{x}$.
Use the TI-30X Plus MathPrint ${ }^{T M}$ data editor and list formulas to complete the following table of values for $y=3^{x}$.

| $x$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $y$ |  |  |  |  |  |  |  |

Teacher Note: Students should recognise that the graph of $y=a^{x}$ always passes through the point $(0,1)$ i.e. when $x=0, y=a^{0}=1$. They should also recognise that the $x$-axis is an asymptote.

## Keystrokes and solution:

Press data. Press data 4 to clear all lists.
[Note: Press $(-)$ to enter a negative number.]
In L1 enter $\mathbf{- 3}$ and press $\Theta$. Enter the values -2, -1, 0, 1, 2, and 3.

These seven $x$-values should now be displayed in L1.


Press (a) to scroll across to the top of L2. and the table can be completed.


Students are expected to use an exponential (growth or decay) model to solve problems.

## Example: Exponential models (2)

The number of flies, $N$, in a population at time, $t$ days where $t \geq 0$, can be modelled by $N=40(1.2)^{t}$.
Use the TI-30X Plus MathPrint ${ }^{\text {TM }}$ to find, correct to the nearest whole number, the number of flies in the population after 3 days.

Teacher Note: The growth factor of 1.2 means that there are 1.2 times as many flies after each day passes. This is the same as saying that after each day there are $20 \%$ more flies (the population of flies grows at the rate of $20 \%$ per day).

## Keystrokes and solution:

Substitute $t=3$ into $N$.
Enter 40 and press $\times 1.2 x$ enter.
$N=69.12$ and so the number of flies after 3 days is 69 , correct to the nearest whole number.


Students are expected to solve practical problems involving quadratic functions.
They are also expected to interpret the turning point of a quadratic in a practical context and consider the range of values for $x$ and $y$ for which the quadratic model makes sense in a practical context.

## Example: Quadratic model

This example shows how to use a calculator to solve a practical problem involving a quadratic function.
A farmer has 500 metres of fencing with which to enclose a rectangular paddock.
Use the $\mathrm{TI}-30 \mathrm{X}$ Plus MathPrint ${ }^{\text {TM }}$ function feature to find the maximum area that can be enclosed.
Teacher Note: The TI-30X Plus MathPrint ${ }^{T M}$ data editor and list formulas feature can also be used here.

## Keystrokes and solution:

Let $x \mathrm{~m}$ be the length of the paddock, $y \mathrm{~m}$ be the width of the paddock and $A \mathrm{~m}^{2}$ be the area of the paddock.
From the perimeter:
$2 x+2 y=500$

$$
\begin{aligned}
2 y & =500-2 x \\
y & =250-x
\end{aligned}
$$

$A=x(250-x)$
As $x$ is the length of the paddock, $x>0$.
Also $250-x>0$ so $x<250$.
The quadratic is valid for $0<x<250$.

| Press table 1 to access the function table. [If required, press clear .] | $f(x)=x *(250-x) \square$ |
| :---: | :---: |
|  |  |
|  | $\downarrow$ |

Move the cursor to select $\boldsymbol{x}=$ ? and press enter (CALC) enter.
As we have a quadratic function in factorised form, the maximum value of $A$ will occur when $x=125$ (midpoint of 0 and 250).

When $x=125, A=125(250-125)$.
Enter 124 and press enter 125 enter 126 enter.


The maximum area is $15625 \mathrm{~m}^{2}$ and the rectangle that gives this area has dimensions 125 m by 125 m .

Thus, the rectangle is a square.
Students are expected to recognise that $y=\frac{k}{x}$, where $k$ is a constant, represents inverse variation.
A variable $y$ varies inversely with a variable $x$ if $y=\frac{k}{x}$, where $k$ is a non-zero constant of proportionality.
The graph of $y$ as a function of $x$ is a rectangular hyperbola whose asymptotes are the $x$-axis and the $y$-axis.

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## Example: Inverse variation

This example shows how to use a calculator to help solve a problem involving inverse variation.
In electrical circuits, Ohm's law states that, for a given voltage, the current, $I$ amperes, in an electrical component is inversely proportional to its resistance, $R$ ohms.

An electrical component has a resistance of 2.4 ohms and passes a current of 5 amperes when connected to a battery.

If the same battery is used, use the TI-30X Plus MathPrint ${ }^{\text {TM }}$ to find the current passing through an electrical component whose resistance is 1.5 ohms.

Teacher Note: It is important for students to recognise that if $R$ increases then $I$ decreases and if $R$ decreases then $I$ increases.

Keystrokes and solution:
$I \propto \frac{1}{R}$ and so $I=\frac{k}{R}$ where $k$ is the constant of proportionality.
Substitute $R=2.4$ and $I=5$ into $I=\frac{k}{R}$ and solve to find $k$.
$5=\frac{k}{2.4}$
$k=5 \times 2.4$

Enter 5 and press 区 2.4 and press enter.
$k=12$ and hence $I=\frac{12}{R}$.
Substitute $R=1.5$ into $I=\frac{12}{R}$.
Press 2nd [answer] © and enter 1.5 enter.

$$
\begin{aligned}
I & =\frac{12}{1.5} \\
& =8
\end{aligned}
$$

The current passing through the electrical component is 8 amperes.

## 3 Measurement

### 3.1 Applications of measurement (MS-M1)

### 3.1.1 Practicalities of measurement

Students are expected to calculate conversions between common units of measurement, for example kilometres to metres or litres to millilitres.

## Example: Length conversion

This example shows how to use a calculator to perform a unit conversion.
Use the Tl-30X Plus MathPrint ${ }^{T M}$ stored operations feature to convert 4850 metres to kilometres.
Teacher Note: Students should know that converting metres to kilometres involves dividing by 1000. The operation is division when converting from a smaller unit to a larger unit.

## Keystrokes and solution:



Students are expected to calculate the percentage error of a reported measurement using:

$$
\text { Percentage error }=\frac{\text { Absolute error }}{\text { Measurement }} \times 100 \%
$$

## Example: Percentage error

This example shows how to use a calculator to calculate a percentage error.
It is known that the height of a tower is exactly 42 metres. Anne measured the height of the tower to 42.3 metres.
Use the TI-30X Plus MathPrint ${ }^{T M}$ to calculate the percentage error of Anne's measurement. Give your answer correct to three decimal places.
Teacher Note: Students should recognise that the relative error indicates how accurate a measurement is relative to the magnitude of the quantity being measured. The relative error is often expressed as a percentage error.

## Keystrokes and solution:

Percentage error is $\frac{42.3-42}{42} \times 100 \%$.
Press 圖 and enter $\mathbf{4 2 . 3} \square 42 \ominus 42$ © © $\mathbf{~} 100$ enter.
The percentage error is $0.714 \%$, correct to three decimal places.

$$
\begin{array}{|c|}
\hline \frac{42.3-42}{42} * 100^{0.6} \\
0.714285714
\end{array}
$$

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### 3.1.2 Perimeter, area and volume

Students are expected to solve practical problems involving the calculation of perimeters and areas of various shapes including composite shapes.

## Example: Area of an annulus

This example shows how to use a calculator to solve a problem involving an annulus.
The area, $A \mathrm{~m}^{2}$, of a circular path of outer radius $R \mathrm{~m}$ and inner radius $r \mathrm{~m}$ is given by $A=\pi R^{2}-\pi r^{2}$.
A particular circular path has an outer radius of 76 m and an inner radius of 60 m .
Use the TI-30X Plus MathPrint ${ }^{\text {TM }}$ expression evaluation feature to find the area of the circular path, giving your answer correct to one decimal place.

Teacher Note: A good activity would be to withhold the formula for the area of an annulus and ask students to derive it. An alternative form for the formula is $A=\pi\left(R^{2}-r^{2}\right)$.

## Keystrokes and solution:

$A=\pi R^{2}-\pi r^{2}$
As the TI-30X Plus MathPrint ${ }^{T M}$ expression evaluation feature does not house the variables $R$ and $r$, we will use the variables $y$ and $x$ instead.
$A=\pi y^{2}-\pi x^{2}$
Press [2nd [expr-eval]. [lf required, press clear.]
$x_{a b c d}^{z z t}$ is a multi-tap key that cycles through the variables $\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}, \boldsymbol{t}$, $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ and $\boldsymbol{d}$.

Press $\pi_{i}^{\text {e }}$ 区 $x^{2}$.

Press enter clear and enter 76 enter clear 60 enter $\sim \approx$.
Substituting $y=76$ and $x=60$ into $A=\pi y^{2}-\pi x^{2}$ gives $A=6836.105 \ldots(\mathrm{~m} 2)$.

The area of the circular path is $6836 \mathrm{~m}^{2}$, correct to one decimal place.

Alternatively, the area of the circular path can be calculated as shown at right.



Students are expected to use:

- Pythagoras' theorem to solve problems involving right-angled triangles.
- a scale factor to find unknown lengths in similar figures.


## Example: Pythagoras' theorem and a scale factor

This example shows how to use a calculator to solve a problem involving use of Pythagoras' theorem and a scale factor.

An 8 m ladder leans against a wall.
The foot of the ladder is 6.4 m from the base of the wall on level ground.
(a) How far up the wall is the ladder?

A person is one quarter of the way up the ladder.
(b) (i) How far above the ground is the person?
(ii) How far away from the wall is the person?

Teacher Note: It is important to stress to students the need to draw a labelled diagram(s) when attempting to solve such problems.

## Keystrokes and solution:

(a) Let $h m$ be the distance up the wall of the ladder.

Using Pythagoras' theorem:
$8^{2}=6.4^{2}+h^{2}$
$h^{2}=8^{2}-6.4^{2}$
$h=\sqrt{8^{2}-6.4^{2}} \quad(h>0)$
Press 2nd [ -7 and enter 8 x $x^{2}-6.4 x^{2}$ enter.
$h=4.8$
The ladder is 4.8 m up the wall.
(b) (i) The triangles are similar (all corresponding angles equal).

$$
\begin{aligned}
\frac{y}{4.8} & =\frac{2}{8} \\
y & =\frac{2}{8} \times 4.8
\end{aligned}
$$



Enter 2 and press 圖 8 (1) 区 2nd [answer] enter.
$y=1.2$. The person is 1.2 m above the ground .

（b）（ii）

$$
\begin{aligned}
\frac{x}{6.4} & =\frac{2}{8} \\
x & =\frac{2}{8} \times 6.4
\end{aligned}
$$

Edit the previous author line．


Press $\Theta \odot$ enter $(1)$ and enter 6.4 enter．
$x=1.6$
The distance from the person to the wall is：
$(6.4-1.6)=4.8(\mathrm{~m})$
Students are expected to solve practical problems involving the calculation of surface area of solids．

## Example：Surface area of a cylinder

This example shows how to use a calculator to calculate the surface area of a cylinder．
The total surface area，$S \mathrm{~cm}^{2}$ ，of a cylinder with radius $r \mathrm{~cm}$ and height $h \mathrm{~cm}$ is given by $S=2 \pi r^{2}+2 \pi r h$ ．
Use the TI－30X Plus MathPrint ${ }^{\text {TM }}$ expression evaluation feature to find the surface area of a cylinder whose base radius is 4 cm and height is 6 cm ．

Give your answer correct to one decimal place．
Teacher Note：An alternative form for the formula is $S=2 \pi r(r+h)$ ．

## Keystrokes and solution：

$S=2 \pi r^{2}+2 \pi r h$
As the TI－30X Plus MathPrint ${ }^{T M}$ expression evaluation feature does not house the variables $r$ and $h$ ，we will use the variables $x$ and $y$ instead．
$S=2 \pi x^{2}+2 \pi x y$
Press 2 nd［expr－eval］．［lf required，press clear ．］
$x_{\text {abed }}^{v z e d}$ is a multi－tap key that cycles through the variables $\boldsymbol{x}$ ， $\boldsymbol{y}, \boldsymbol{z}, \boldsymbol{t}, \boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ and $\boldsymbol{d}$ ．

Enter $\mathbf{2}$ and press 区 $\pi_{i}^{\text {e }}$ 区．Press $x_{a b c a d}^{v z e d ~}$ to paste $\boldsymbol{x}$ ．
Press $x^{2}$ and enter 2 区 $\pi_{i}^{\text {e }}$ 区


$80 \pi$
251.3274123

Substituting $x=4$ and $y=6$ into $S=2 \pi x^{2}+2 \pi x y$ gives $A=251.327 \ldots$ (cm ${ }^{2}$ ).

The surface area of the cylinder is $251.3 \mathrm{~cm}^{2}$, correct to one

## $2 * \pi * 4^{2}+2 * \pi * 4 * 6 *$ <br> 251.3274123

 decimal place.Alternatively, the surface area of the cylinder can be calculated as shown at right.

## Example: Surface area of a prism (problem solving)

This example requires the use of a calculator and problem-solving strategies to determine integer side lengths of a rectangular prism.

The surface area, $S \mathrm{~m}^{2}$, of a rectangular box with a lid has length $l \mathrm{~m}$, width $w \mathrm{~m}$ and height $h \mathrm{~m}$ is given by $S=2(l w+h l+h w)$.

Given that $l<w<h$, determine the dimensions of a rectangular box of integer length, width and height measurements with a surface area between $120 \mathrm{~m}^{2}$ and $130 \mathrm{~m}^{2}$.

Teacher Note: This example is different to a standard textbook problem involving surface areas and dimensions of prisms. Students can use the TI-30X Plus MathPrint ${ }^{\text {TM }}$ to undertake a systematic search for possible integer solutions.

Answer: $l=3, w=5$ and $h=6$
Students are expected to solve practical problems involving the calculation of volume and capacity of solids.

## Example: Volume of a composite solid

This example shows how to use a calculator to find the volume of a composite solid.
An ice-cream cone is completely filled with ice-cream and is topped with a hemispherical scoop as shown.


The cone has a height of 12 cm and the diameter at the top of the cone is 5 cm .
Use the TI-30X Plus MathPrint ${ }^{\text {TM }}$ expression evaluation feature to calculate the total volume of ice-cream, giving your answer correct to the nearest cubic centimetre.

Teacher Note: Students need be able to determine the solids that make up a composite solid and calculate the volume of each. In this instance, adding the two volumes together.

## Keystrokes and solution：

$V=\frac{1}{3} \pi r^{2} h+\frac{2}{3} \pi r^{3}$
As the TI－30X Plus MathPrint ${ }^{T M}$ expression evaluation feature does not house the variables $r$ and $h$ ，we will use the variables $x$ and $y$ instead．

Let $V_{C}$ be the volume of the cone and let $V_{H}$ be the volume of the hemisphere．
The total volume，$V$ ，is given by $V=V_{C}+V_{H}$ ．
$V=\frac{1}{3} \pi x^{2} y+\frac{2}{3} \pi x^{3}$

Press 2nd［expr－eval］．［lf required，press clear．］
$x_{a b c a t}^{y z t}$ is a multi－tap key that cycles through the variables $\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}$ ， $\boldsymbol{t}, \boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ and $\boldsymbol{d}$ ．
 $x_{a b c d}^{y z t}$ to paste $\boldsymbol{x}$ ．Press $x^{2}$ and press $\boldsymbol{y}$ ．

Press $⿴ 囗 十$ and enter 2 回 3 （1）区 $\pi_{i}^{\text {ein }}$ 区
If the diameter is 5 cm ，then the radius is 2.5 cm ．
Press enter clear and enter 2.5 enter clear 12 enter．
Substituting $x=2.5$ and $y=12$ into $V=\frac{1}{3} \pi x^{2} y+\frac{2}{3} \pi x^{3}$

gives $V=111.264 \ldots\left(\mathrm{~cm}^{3}\right)$ ．
Correct to the nearest cubic centimetre，the total volume of ice－ cream $111 \mathrm{~cm}^{3}$ ．

Alternatively，the surface area of the cylinder can be calculated as shown at right．

Students are expected to use the trapezoidal rule to solve a variety of practical problems．
$A=\frac{h}{2}\left(d_{f}+d_{l}\right)$ where $h$ is the height or distance between the parallel sides and $d_{f}$ and $d_{l}$ are the distances of the first and last parallel sides．

## Example: Using the trapezoidal rule to find an approximate area

This example shows how to use a calculator to help determine an approximate area with the trapezoidal rule.
The areas enclosed by contours in a lake are as follows:

| Contour (m) | 150 | 155 | 160 | 165 | 170 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Area $\left(\mathrm{m}^{2}\right)$ | 1550 | 7900 | 15800 | 24100 | 31000 |

Use the trapezoidal rule and the TI-30X Plus MathPrint ${ }^{\text {TM }}$ to find an approximate volume of water in the lake between the contours 150 m and 170 m .

Teacher Note: Where possible, it is a good idea to explore the effect of increasing the number of trapezia used in an approximation.

## Keystrokes and solution:

Let $V$ be the volume.
$V=\frac{5}{2}(1550+31000+2(7900+15800+24100))$

$\square 7900$ 母 $15800 \oplus 24100 \square \square$ Enter.

$$
V=320375\left(\mathrm{~m}^{3}\right)
$$

The approximate volume of water in the lake is $320375 \mathrm{~m}^{3}$.

### 3.1.3 Units of energy and mass

Students are expected to use metric units of energy to solve problems, including calories, kilocalories, joules and kilojoules, their abbreviations and how to convert between them.

The common unit for food energy is the kilojoule (kJ). A kilojoule is an SI unit and is 1000 joules. The previous unit used was the calorie (Cal). 1 calorie is 4.184 kilojoules.

## Example: Food energy unit conversions

This example shows how to use a calculator to perform food energy unit conversions.
Use the TI-30X Plus MathPrint ${ }^{T M}$ conversions feature to convert
(a) 1060 calories to kilojoules.
(b) 284 kilojoules to calories.

Give each answer correct to the nearest whole number.
Teacher Note: Two definitions, a large calorie and a small calorie, are in wide use. In nutrition and food science, the term calorie almost always refers to the large calorie. It is used in food labels to express the energy value of foods in per serving or per mass. Give students opportunities to compare, contrast, interrogate and understand nutrition information displayed on food labels.

## Keystrokes and solution：

（a）Enter $\mathbf{1 0 6 0}$ and press 2nd［convert］．

Press 5 to select Power，Energy．Select cal $\boldsymbol{J}$ and press enter enter．

CONVERSIONS 3个Speed，Length 4：Pressure
1060 calories converts to 4438 kilojoules（correct to the nearest whole number）．

Note：This conversion can be made by multiplying by 4.184 ．

5：Power，Energy

（b）Enter 284 and press 2nd［convert］．
Press 5 to select Power，Energy．Select Jヤcal and press enter enter．

284 kilojoules converts to 68 calories（correct to the nearest whole number）．

## $\begin{aligned} & 284 \\ & \text { J } \text { cal } \\ & 67.83223464\end{aligned}$

Note：This conversion can be made by dividing by 4.184 ．
Students are expected to use units of energy to solve problems involving electricity consumption，for example kilowatt－hours（kWh），and investigate the energy consumption of common electrical appliances．

The kilowatt－hour（ kWh ）is a unit of energy equal to one kilowatt（ 1000 W ）of power sustained for one hour and is commonly used to measure the electricity consumption of an electrical appliance．

## Example：Calculating the cost of using an electrical appliance

This example shows how to use a calculator to calculate the cost of using an electrical appliance．A power provider charges $\$ 0.20$ per kWh for electricity．

Use the Tl－30X Plus MathPrint ${ }^{\text {TM }}$ to calculate the cost of running a 30 W LED light bulb for a week，giving your answer correct to the nearest cent．

Teacher Note：Give students opportunities to compare，contrast，interrogate and understand information displayed on electricity bills and on energy rating labels for electrical appliances．These labels display the amount of electricity（kWh）the appliance typically uses in a year．

## Keystrokes and solution：

The electricity consumed in kWh in a week is $\frac{30}{1000} \times 24 \times 7$ ．
The cost in a week is $\left(\frac{30}{1000} \times 24 \times 7\right) \times 0.2$ ．
区 and enter 0.2 enter．

The cost in a week is $\$ 1.01$（correct to the nearest cent）．

### 3.2 Working with time (MS-M2)

Students are expected to convert units of time, convert between 12-hour and 24-hour clocks and calculate time intervals.

The DMS menu is useful for converting units of time.
Press mode to choose an angle mode from the mode screen. Note that DEG is the default.
Press math (1) (1) to display the DMS menu.


In terms of units of time, the number of degrees displayed can be used to represent the number of hours.
The DMS menu enables you to specify the unit modifier as 'hours' ( ${ }^{\circ}$ ), minutes ('), seconds (") or convert a time expressed as a decimal to a time expressed in hours, minutes and seconds using - DMS.

## Example: Time conversions

This example shows how to use a calculator to convert units of time.
Use the TI-30X Plus MathPrint ${ }^{\text {TM }}$ to convert a time of 2.4 hours to a time in hours and minutes.
Teacher Note: Students need to understand that 3.25 hours, for example, on a calculator display converts to $3 \frac{1}{4}$ hours or 3 hours 15 minutes.

## Keystrokes and solution:

Enter 2.4 and press math (1) (1) 6 enter.
This time is 2 hours 24 minutes.


## Example: Calculating a time interval

This example shows how to use a calculator to calculate a time interval.
At the 1908 Olympic Games, John Hayes (USA) won the marathon in a time of 2:55:18. At the 2016 Olympic Games, Eluid Kipchoge (Kenya) won the marathon in a time of 2:08:44.

Use the TI-30X Plus MathPrint ${ }^{\text {TM }}$ to calculate how much faster Kipchoge's winning time was compared to Hayes' winning time.

Teacher Note: Such calculations can be problematic without the use of a calculator. However, encourage students to attempt questions of this type using mental computation and written methods.

## Keystrokes and solution:

Enter 2 and press math (1) (1) 155 math (1) (1) 218 math
(1) (1) 3 - 2 math (1) (1) 18 math (1) (1) 244 math (1) (1) 3 math (1) (1) 6 enter.

Kipchoge's winning time is 46 minutes and 34 seconds faster.

## $2^{\circ} 55^{\prime} 18^{\prime \prime}-0^{\circ} 2^{\circ} 8^{\prime} 44 i \prime \prime$

## Example: 12 -hour and 24 -hour notation

This example shows how to use a calculator to solve a problem involving 12-hour and 24 -hour notation as well as the use of mixed units (years, months, days, hours and seconds).

On September 30, Anne set her digital watch at 13:00:00.
During October, Anne notices that the watch loses 7 seconds per day.
Use the TI-30X Plus MathPrint ${ }^{\text {TM }}$ to calculate what time would be showing on Anne's watch when it is 13:00:00 on October 31.

Teacher Note: Students need to know the number of days in each month, that there are 60 minutes in an hour and there are 60 seconds in a minute.

## Keystrokes and solution:

There are 31 days in October so Anne's watch will have lost
$31 \times 7=217$ seconds.
Enter 31 and press $\begin{aligned} \\ 7 \text { enter. }\end{aligned}$
Convert 217 seconds into minutes and seconds:
Press [2nd [answer] $\square \square$ and enter 60 ® $60 \square$ math (1)
(1) 6 enter.

217 seconds is 3 minutes and 37 seconds.
Enter 13 and press math (1) (1) 10 and press math (1) (1) 20 math (1) (1) 3 3 [2nd [answer] math (1) (1) 6 enter.

Subtracting 3 minutes and 37 seconds from 13:00:00 gives


12:56:23. This would be the time on Anne's watch.

### 3.3 Non-right-angled trigonometry

Press mode to choose an angle mode from the mode screen. Note that DEG is the default.
Press math (1) (1) to display the DMS menu.


This menu enables you to specify the angle unit modifier as degrees ( ${ }^{\circ}$ ), minutes ('), seconds ("); specify a radian angle ( $\mathbf{r}$ ); specify a gradian angle ( $\mathbf{g}$ ), or convert an angle from decimal degrees to degrees, minutes and seconds using DMS.

Inputs are interpreted and results displayed according to the angle mode setting without the need to enter an angle unit modifier.

## Example: Converting an angle from decimal degrees to degrees and minutes

This example shows how to use a calculator to convert an angle from a decimal to degrees and minutes.
Use the TI-30X Plus MathPrint ${ }^{\text {TM }}$ to convert $62.4^{\circ}$ to an angle expressed in degrees and minutes.
Teacher Note: It is useful for students to know that $0.1^{\circ}$ corresponds to $6^{\prime}$.

## Keystrokes and solution:

Enter 62.4 and press math (1) (1) 6 enter.
$62.4^{\circ}=62^{\circ} 24^{\text {c }}$


Students are expected to know the sign of $\sin A$ and $\cos A$ for $0^{\circ} \leq A \leq 180$.
To create a sequence of values to fill a list in the data editor, press data © to display the Options (OPS) menu. Press 3 and complete the required fields.

## Example: Investigating the sign of $\cos A$ for $0^{\circ} \leq A \leq 180^{\circ}$

This example shows how to use a calculator to investigate the sign of $\cos A$ for $0^{\circ} \leq A \leq 180$.
Use the TI-30X Plus MathPrint ${ }^{\text {TM }}$ data editor and list formulas feature to complete the following table of values for $\cos A$.

Where appropriate, give each value of $\cos A$ correct to two decimal places.

| $A$ | $0^{\circ}$ | $30^{\circ}$ | $60^{\circ}$ | $90^{\circ}$ | $120^{\circ}$ | $150^{\circ}$ | $180^{\circ}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\cos A$ |  |  |  |  |  |  |  |

Teacher Note: The same approach can be used to investigate the $\operatorname{sign}$ of $\sin A$ for $0^{\circ} \leq A \leq 180$.

## Keystrokes and solution:

The angle mode is DEG.
Press data. Press data 4 to clear all lists.
Press data (1) 3 .
Select L1 and press enter.
Enter $\mathbf{3 0}$ and press 区.


EXPR IN $\chi: 30: *^{\text {DEG }} \chi$
START $\chi: 0$
END $x: 6$
STEP SIZE:1
SEQUENCE FILL scroll down to select SEQUENCE FILL and press enter.

These seven $A$-values should now be displayed in L1.
Press (1) to scroll across to the top of $\mathbf{L} \mathbf{2}$.
Press data (1) to select FORMULA and press enter.
Enter the list formula $\cos (\mathbf{L} 1)$ to $\mathbf{L 2}$.
Press ${ }_{\text {coss- }}^{\text {cos. }}$ and press data enter to paste $\mathbf{L 1}$ into the author line. Press $1>\approx$ enter.

L2 should now display the $\cos A$ values and the table can be completed.

For $0^{\circ} \leq A \leq 90^{\circ}, 0 \leq \cos A \leq 1$.
For $90^{\circ} \leq A \leq 180^{\circ},-1 \leq \cos A \leq 0$.


Students are expected to use trigonometric ratios to find the length of an unknown side or the size of an unknown angle in a right-angled triangle. This includes solving practical problems involving Pythagoras' theorem, rightangled and non-right-angled triangle trigonometry, angles of elevation and depression and the use of true and compass bearings.

Students are expected to work with angles correct to the nearest degree and/or minute.
 prior to the calculation.

## Example: Trigonometric ratios (1)

This example shows how to use a calculator to solve a problem involving right-angled triangle trigonometry.
The angle of depression from a drone flying horizontally 100 metres above the water to a buoy at sea is $23^{\circ} 18^{\prime}$.
Find the horizontal distance, $x$ metres, from the drone to the buoy.
Give your answer correct to one decimal place.
Teacher Note: It is important to reinforce the following five steps when solving a right-angled triangle trigonometry problem.

## Keystrokes and solution:

The angle msode is DEG.
Step (1): Draw a diagram.
Step (2): Label all given and required information where $x$ represents the horizontal distance.

Step (3): From TOA, the required trigonometric ratio is tan.
Step (4):

$\tan 23^{\circ} 18^{\prime}=\frac{100}{x}$ and so $x=\frac{100}{\tan 23^{\circ} 18^{\prime}}$


Given two sides of a right-angled triangle, we can use trigonometric ratios to find unknown angles. For example, if $\sin \theta=\frac{1}{2}$, we can find the angle $\theta$ whose sine is equal to $\frac{1}{2}$.

To do this on the TI-30X Plus MathPrintTM, we use the inverse of sine.
Press Sin sin (a multi-tap key) to access $\sin ^{-\mathbf{1}}$. Enter $\frac{\mathbf{1}}{\mathbf{2}}$ and press $\square$ enter.


So $\sin ^{-1} \frac{1}{2}$ is the 'angle whose sine is $\frac{1}{2}$.
Inverse trigonometric ratios can be thought of as 'angle finders'.

## Example: Trigonometric ratios (2)

This example shows how to use a calculator to find the magnitude of an angle, in degrees and minutes, given a trigonometric ratio for the angle.

Use the TI-30X Plus MathPrint ${ }^{\text {TM }}$ to find the value of $\theta$ for $\sin \theta=0.3798$. Give your answer in degrees and minutes.

Teacher Note: It is important to reinforce that $\sin ^{-1}(0.3798)$ means the angle whose sine is 0.3798 .

## Keystrokes and solution:

The angle mode is DEG.
 enter.

In degrees and minutes, $\theta=22^{\circ} 19^{\text {. }}$.
$\qquad$

Students are expected to use the sine rule, cosine rule and area of a triangle formula for solving problems. This can include finding the size of an obtuse angle. The ambiguous case of the sine rule is excluded.


For a triangle $A B C$, the sine rule is given by $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$.
It is used to find unknown lengths and angles when given:
(1) two angles and one side length.
(2) two side lengths and an angle opposite one of the sides.

For a triangle $A B C$, the cosine rule is given by $a^{2}=b^{2}+c^{2}-2 b c \cos A$.
It is used to find unknown lengths and angles when given:
(1) all three side lengths.
(2) two side lengths and the included angle.

The above formula can be rearranged to find the unknown angle $A$ where $\cos A=\frac{b^{2}+c^{2}-a^{2}}{2 b c}$.
If two sides of a triangle and the included angle are given then $\mathrm{Area}=\frac{1}{2} a b \sin C$.

## Example: Sine and cosine rule

This example shows how to solve a problem using the cosine rule and the sine rule.,
Use the TI-30X Plus MathPrint ${ }^{T M}$ to find all the unknown angles and side lengths in triangle $A B C$ for which $a=5, b=4$ and $\mathrm{C}=46^{\circ} 24^{\prime}$. Give your answer for $c$ correct to two decimal places and your answers for $A$ and $B$ correct to the nearest minute.

Teacher Note: When performing such multi-stage calculations, do not round intermediate answers as this is likely to lead to inaccurate final answers.

## Keystrokes and solution:

The angle mode is DEG.
Using the cosine rule:
$c=\sqrt{5^{2}+4^{2}-2 \times 5 \times 4 \times \cos 46^{\circ} 24^{\prime}}$
 46 math (1) (1) (1) 24 math (1) (1) ( 2 ( 1 enter.

Correct to 2 decimal places, $c=3.66$.


Using the sine rule:
$\frac{5}{\sin A}=\frac{3.662 \ldots}{\sin 46^{\circ} 24^{\prime}} \Rightarrow \sin A=\frac{5 \sin 46^{\circ} 24^{\prime}}{3.662 \ldots}$
$A=\sin ^{-1}\left(\frac{5 \sin 46^{\circ} 24^{\prime}}{3.662 \ldots}\right)$
 math (1) (1) $2 \square \odot$ 2nd [answer] (1) $\square$ math (1) (1) 6 enter.

Correct to the nearest minute, $A=81^{\circ} 20^{\prime}$.
Note that careful use of the expression evaluation feature could be used in solving this problem (the pronumerals are $a, b, c, A, B, C$ ).

$B=180^{\circ}-\left(81^{\circ} 20^{\prime}+46^{\circ} 24^{\prime}\right)$
Enter 180 and press $\square \square$ [2nd [answer] +46 math (1) (1) 1 24 math (1) (1) 2 (1) math (1) (1) ( 6 enter.

Correct to the nearest minute, $B=52^{\circ} 16^{\circ}$.

## Example: Trigonometry and Pythagoras' theorem

This example shows how to use Pythagoras' theorem to solve a three-dimensional problem involving right-angled triangles. In particular, solving a problem involving the lengths of the edges and diagonals of a rectangular prism.

Find the length of a straw which will fit diagonally into a child's fruit juice box (a rectangular prism) and extend out of the box by 2 cm . Give your answer correct to one decimal place.


10 cm
Teacher Note: When solving three-dimensional problems involving Pythagoras' theorem, it is important to draw carefully labelled diagrams identifying the right-angled triangles, where the theorem can be applied to find unknown lengths.

## Keystrokes and solution:

$$
\begin{aligned}
x^{2} & =10^{2}+5^{2} \\
& =125 \\
y^{2} & =x^{2}+15^{2} \\
& =125+225 \\
& =350 \\
y & =\sqrt{350}(\mathrm{~cm})
\end{aligned}
$$

The length of the straw is $(y+2)(\mathrm{cm})$.
Enter 10 and press $x^{2}$ - 5 x $x^{2}$ enter 2nd [ $v$ ] 2nd [answer] $\dagger$ $15 x^{2}(1)+2 \square \approx$ enter.

Correct to one decimal place, the length of the straw is 20.7 cm .


### 3.4 Rates and ratios (MS-M7)

Students are expected to use rates to solve and describe practical problems, for example, to compare 'best buys', by comparing price per unit or quantity per monetary unit.

## Example: Calculating the 'best buy'

This example shows how to use a calculator to determine a 'best buy'.
Use the TI-30X Plus MathPrint ${ }^{\text {TM }}$ to determine which of the following options is the 'best buy'.
500 grams of butter for $\$ 4.50$ or 300 grams of butter for $\$ 2.75$.
Teacher Note: This problem can be solved using the unitary method.

## Keystrokes and solution:

500 grams of butter for $\$ 4.50$
What is 1 gram worth?


1 gram is worth 0.9 cents.
300 grams of butter for $\$ 2.75$

## What is 1 gram worth?

Press $\Theta \odot$ enter to bring the previous entry to a new author line.
Press 2nd (1) to move to the beginning of the previous entry and press [2nd [insert] to edit as shown at right. Press enter.

1 gram is worth 0.92 cents (correct to two decimal places).
$0.9<0.92$ and so 500 grams of butter for $\$ 4.50$ is the 'best buy'.

Students are expected to solve practical problems involving speed.
The speed of an object is a rate because it is the distance travelled in a certain time.

## Example: Calculating travel time

This example shows how to solve a real-life problem involving speed and travel time.
A truck is travelling at a constant speed of 90 km per hour.
Use the TI-30X Plus MathPrint ${ }^{\text {TM }}$ to find how long it would take the truck to travel 3.75 km . Give your answer in minutes and seconds.

Teacher Note: This example illustrates how to use the TI-30X Plus MathPrint ${ }^{T M}$ to convert a time into minutes and seconds.

## Keystrokes and solution:

In one hour, the truck would travel 90 km . It would take $\frac{1}{90}$ of an
hour to travel 1 km .
It would take $\frac{3.75}{90}$ of an hour to travel 3.75 km .
Enter 3.75 and press 圆 90 (1) $\approx \approx$ enter.
This gives $\frac{1}{24}$ hours.
Press [2nd [answer] math (1) (1) 6 enter.
This gives an output in minutes and seconds.


The truck would take 2 minutes and 30 seconds to travel
3.75 km.

Students are expected to calculate the amount of fuel used on a trip, given the consumption rate and to compare fuel consumption statistics for various vehicles.

A motor vehicle's fuel consumption is the number of litres of fuel it uses to travel 100 kilometres.

The formula for fuel consumption, $F C$, in $L / 100 \mathrm{~km}$, can be expressed as $F C=\frac{A}{d} \times 100$ where $A$ is the amount of fuel in litres and $d$ is the distance travelled in kilometres.

## Example: Comparing fuel consumption

This example shows how to use a calculator to compare fuel consumption statistics for various vehicles.
Consider the following fuel consumption statistics for three vehicles.
Vehicle A uses 29.4 L of fuel to travel 350 km .
Vehicle B uses 77.3 L of fuel to travel 840 km .
Vehicle C uses 45.0 L of fuel to travel 490 km .
Use the TI-30X Plus MathPrint ${ }^{T M}$ data editor and list formulas feature to determine which of the three vehicles has the most economical fuel consumption.

Teacher Note: When comparing fuel consumption, in L/100 km, for different vehicles, students should recognise that the most economical (best) fuel consumption corresponds to the smallest value.

## Keystrokes and solution:

Press data. Press data 4 to clear all lists.
Enter 29.4 and press $\Theta$. Repeat for 77.3 and 45.0.
The three fuel amounts should now be displayed in L1.
Press (1) to scroll across to the top of L2.
Enter 350 and press $\odot$. Repeat for 840 and 490.
The three distances should now be displayed in $\mathbf{L} 2$.
Press (1) to scroll across to the top of L3.


Press data (1) to select FORMULA and press 1.
Enter the list formula L1 / L2 * $\mathbf{1 0 0}$ to L3.
Press data enter to paste L1 into the author line.
Press $\div$ and press data 2 to paste $\mathbf{L} \mathbf{2}$ into the author line.
Press $\times$ and enter 100 enter.
L3 should now display the fuel consumption statistics for the three vehicles.

Vehicle A is $8.4 \mathrm{~L} / 100 \mathrm{~km}$.
Vehicle B is $9.20 \ldots \mathrm{~L} / 100 \mathrm{~km}$.
Vehicle C is $9.18 \ldots \mathrm{~L} / 100 \mathrm{~km}$.


As $8.4<9.18 \ldots<9.20 \ldots$, vehicle A is the most fuel efficient of the three vehicles.

Students are expected to solve practical problems involving ratio, for example, measurements from scale drawings.

## Example: Interpreting a scale in a house plan

This example shows how to use a calculator to help interpret a scale in a house plan.
Using an architect's house plans, a builder measured the width of the house to be 350 mm . If the house has an actual width of 14.5 m , use the $\mathrm{Tl}-30 \mathrm{X}$ Plus MathPrint ${ }^{\mathrm{TM}}$ to find the scale diagram ratio of the house plans.

Teacher Note: Choosing the most appropriate unit of measurement is important. Afford students opportunities to interpret and use scales in photographs, plans and drawings used in everyday life.

## Keystrokes and solution:

350 mm on the house plan represents an actual width of 14.5 m .
Convert the units so that the scale measurement and actual measurement are expressed in the same unit.
Here, change the larger unit to the smaller unit. Hence convert 14.5 m to mm .
The ratio is $350(\mathrm{~mm}): 14.5(\mathrm{~m})$.
As 1 m is $10^{3} \mathrm{~mm}$ and considering the above ratio as a part-to-whole ratio we obtain:
$350(\mathrm{~mm}): 14.5 \times 10^{3}(\mathrm{~mm})$


## 4 Financial mathematics

### 4.1 Money matters (MS-F1)

### 4.1.1 Interest and depreciation

Students are expected to apply percentage increase or decrease in various contexts, for example, calculating GST payable on a range of goods and services.

Percentage change involves increasing or decreasing a quantity as a percentage of the original amount of the quantity.

For example, percentage decrease involves the following two steps:
(1) Subtract the percentage decrease from $100 \%$.
(2) Multiply the percentage determined in (1) by the amount.

## Example: Applying a percentage decrease

This example shows how to use a calculator to decrease a quantity by a given percentage.
A shop has a sale where there is a $30 \%$ discount on sporting equipment.
Use the Tl-30X Plus MathPrint ${ }^{T M}$ to find the discounted price on a tennis racquet that normally costs $\$ 250$.
Teacher Note: When using a calculator to answer such questions, it is important that students anticipate an answer less than $\$ 250$.

## Keystrokes and solution:

Determine the percentage to be paid by subtracting the percentage discount from $100 \%$.
$(100-30) \%=70 \%$
To find the discounted price, calculate $70 \%$ of the marked price.
(100-30) $\%$ 250
175

Press $\square$ and enter 100 30 2nd [ $\%$ ]区 250 enter.

The discounted price of the tennis racquet is $\$ 175$
The GST is a federal tax applied to most goods and services in Australia.
It is calculated at the rate of $10 \%$ of the purchase price of the goods or services.
The price including the GST (the price after the GST is added) is described as 'price including GST'.
The price excluding the GST (the price before the GST is added) is described as 'price excluding GST'.

## Example: Calculating a price including GST

This example shows how to use a calculator to calculate the GST and GST inclusive prices for goods purchased in Australia, given the pre-GST price.

Use the TI-30X Plus MathPrint ${ }^{\text {TM }}$ to calculate the GST and the price including GST of a coffee machine whose listed price excluding GST is $\$ 760$.

Teacher Note: Remind students of the usefulness of the last answer feature when using the TI-30X Plus MathPrint ${ }^{T M}$.

## Keystrokes and solution:

GST is $10 \%$ of $\$ 760$.
Enter 10 and press [2nd [ $\%$ ] 区 760 enter.

GST is $\$ 76$.
Price including GST:
Enter 760 and press $\oplus$ [2nd [answer] enter.
$10 \% * 760$ 760+ans


The price including GST is $\$ 836$
(\$760 + \$76)

Students are expected to use technology to calculate simple interest for different rates and periods.
Simple interest is calculated only on the original amount borrowed or invested.
The simple interest earned or owed can be calculated using the formula $I=\operatorname{Prn}$ where:
$I$ is the interest earned or paid (in dollars);
$P$ is the principal (initial amount borrowed or invested) (in dollars);
$r$ is the interest rate per time period (usually expressed as a decimal);
$n$ is the number of time periods.
$A=P+I$ where $A$ is the amount owed or total to be paid.
Example: Exploring a simple interest investment
This example shows how to use a calculator to explore a simple interest investment.
$\$ 5000$ is invested at a simple interest rate of $8.25 \%$ per annum for a ten-year period.
Use the TI-30X Plus MathPrint ${ }^{T M}$ data editor and list formula feature to explore the growth in interest earned over this period.

Teacher Note: This example could be extended to explore simple interest graphs for different rates and periods.

## Keystrokes and solution:

$I=5000 \times 8.25 \% \times n$
Press data. Press data 4 to clear all lists.
Press data (1) 3. Select L1 and press enter.
Press $x$ xabed to paste $\boldsymbol{x}$, complete the sequence set-up as shown, scroll down to select SEQUENCE FILL and press enter.

These ten time period values should now be displayed in L1.


Press (1) to scroll across to the top of $\mathbf{L 2}$.
Press data (1) to select FORMULA and press enter.
Enter the list formula 5000 * $\mathbf{8 . 2 5 \%}$ * L1 to L2.
Enter 5000 and press $\boxtimes 8.25$ 2nd [ $\%$ ] $\boxtimes$.
Press data enter to paste $\mathbf{L 1}$ into the author line. Press enter.
L2 should now display the required interest amounts.
Press 2 nd $\odot$ to go to the bottom of a list.
Interest of $\$ 4125$ will be earned after 10 years.
The graph of the amount of simple interest earned is linear with slope (interest added each year) equal to $\$ 412.50$.



L2(1) $=412.5$

| $\begin{aligned} & \text { L4 } \\ & 8 \\ & 9 \\ & 10 \end{aligned}$ |  |
| :---: | :---: |
| L2(11) |  |

Students are expected to calculate the depreciation of an asset using the straight-line method.

Straight-line depreciation occurs when the value of an asset decreases by the same amount each time period.
The salvage value (current value) of the asset, $S$, is given by the formula $S=V_{0}-D n$ where
$V_{0}$ is the initial value of the asset.
$D$ is the depreciated amount per time period.
$n$ is the number of time periods.
Example: Exploring the straight-line depreciation of an asset
This example shows how to use a calculator to explore the straight-line depreciation of an asset.
Serena paid $\$ 23500$ for a used car.
It is thought that the car will depreciate in value by an average amount of $\$ 1950$ each year for a period of four years.

Use the TI-30X Plus MathPrint ${ }^{\text {TM }}$ data editor and list formulas feature to complete the following depreciation table for the first four years.

| Year | Depreciated value |
| :---: | :---: |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |

Teacher Note: It is a good idea to graph $S=V_{0}-D n$ and interpret the meaning of $V_{0}$ and $D$ in the context of straight-line depreciation.

## Keystrokes and solution:

$S=23500-1950 n$
Press data. Press data 4 to clear all lists.
In L1, enter $\mathbf{1}$ and press $\Theta$.
Repeat for 2, 3 and 4.
Press (1) to scroll across to the top of $\mathbf{L 2}$.
Press data (1) to select FORMULA and press enter.
Enter the list formula 23500-1950 * L1 to L2.
Enter 23500 and press1950 区.

Press data enter to paste $\mathbf{L 1}$ into the author line. Press enter.


| $\begin{aligned} & 40 \\ & \frac{4}{2} \\ & \frac{2}{3} \\ & 4 \\ & 4 \end{aligned}$ |  | 田------ |
| :---: | :---: | :---: |
| L2(4) 15700 |  |  |

The depreciated values should now be displayed in L2.

| Year | Depreciated value |
| :---: | :---: |
| 1 | $\$ 21550$ |
| 2 | $\$ 19600$ |
| 3 | $\$ 17650$ |
| 4 | $\$ 15700$ |

Students are expected to use a spreadsheet to calculate and graph compound interest as a recurrence relation involving repeated applications of simple interest.

Compound interest calculates the interest on the original amount plus any interest that is earned to that time.
Example: Exploring compound interest by repeated application of simple interest (1)
This example shows how to use a calculator to explore compound interest through repeated use of the simple interest formula.
$\$ 200$ is invested at $5 \%$ per annum, where the interest earned is added to the account each year.
Use the TI-30X Plus MathPrint ${ }^{\text {TM }}$ last answer feature to calculate how much money, $A$, is in the account after three years.

Teacher Note: Such an example shows how the TI-30X Plus MathPrint ${ }^{T M}$ last answer feature can be used to explore compound interest through repeated use of the simple interest formula.

## Keystrokes and solution:

After one year: $I=200 \times 5 \% \times 1$
Enter 200 and press $\begin{aligned} \text { 2nd [ } \% \text { ] } ⿴ 囗\end{aligned} 1$ enter.
The interest earned is $\$ 10$.
After one year, the amount of money, $A$, in the account is
$\$ 200+\$ 10$.
Enter 200 and press $\dagger$ [2nd [answer] enter.
After one year, $A=\$ 210$.
After two years: $I=210 \times 5 \% \times 1$
Repeat the calculation with $P=\$ 210$ to find the value of $I$.
Note the use of the last answer feature (third author line in the screenshot at right).


The interest earned is $\$ 10.50$.
After two years, the amount of money, $A$, in the account is $\$ 210+\$ 10.50$.

After two years, $A=\$ 220.50$.


After three years: $I=220.50 \times 5 \% \times 1$
Repeat the calculation with $P=\$ 220.50$ to find the value of $I$.

The interest earned is $\$ 11.03$.
After three years, the amount of money, $A$, in the account is $\$ 220.50+\$ 11.03$.

After three years, $A=\$ 231.53$.
Example: Exploring compound interest by repeated application of simple interest (2)
This example, following on from the previous example, shows how to use a calculator to help see a pattern to compound interest calculations thus leading towards developing a formula for compound interest.
\$200 is invested at 5\% per annum, where the interest earned is added to the account each year.
(a) Use the $\mathrm{TI}-30 \mathrm{X}$ Plus MathPrint ${ }^{\mathrm{TM}}$ stored operations feature to calculate how much money, $A$, is in the account after three years.
(b) Develop a formula for $A$, the amount of money in the account, after $n$ years.

Teacher Note: This example shows how to connect the calculation of the total value of a compound interest investment to repeated multiplication. Example: a rate of $5 \%$ per annum leads to repeated multiplication by 1.05 .

## Keystrokes and solution:

(a) Increasing a quantity by $5 \%$ is equivalent to multiplying that quantity by 1.05 .

Press 2nd [setop].
[If required, press clear to clear any previously stored operations.]

Press $\begin{array}{r}\text { and } \\ 1.05 \text { enter. } \\ \text { enter }\end{array}$
After one year:
Enter 200 and press 2 nd [ op ].
$A=\$ 200 \times 1.05=\$ 210$
After two years:
Press 2nd [op].

$$
\begin{aligned}
A & =\$ 200 \times 1.05 \times 1.05 \\
& =\$ 200 \times 1.05^{2} \\
& =\$ 220.50
\end{aligned}
$$

| After three years: |  |
| :---: | :---: |
| Press 2 Ld [ op ]. | $\mathrm{n}=2 \mathrm{~L}$ |
| $A=\$ 200 \times 1.05 \times 1.05 \times 1.05$ | $220.5 * 1.05$ |
| $=\$ 200 \times 1.05^{3}$ |  |
| $=\$ 231.53$ |  |

(b) $A=\$ 200 \times(1.05)^{n}$

### 4.1.2 Earning and managing money

Students are expected to calculate earnings based on commission.
A commission payment is an amount paid to a person based on how much they sell.
Often, a commission payment is calculated as a percentage of the total sales.

## Example: Calculating a commission

This example shows how to use a calculator to calculate a commission.
A real estate agent sold a house for $\$ 900000$.
Her rate of commission on the sale was $1.5 \%$.
Use the TI-30X Plus MathPrint ${ }^{\text {TM }}$ to find the commission from the sale.
Teacher Note: When using a calculator to answer such questions, it is important that students anticipate the order of magnitude of the answer. For example, $1 \%$. of $\$ 900000$ is $\$ 9000$.

## Keystrokes and solution:

Commission is $1.5 \%$ of $\$ 900000$.
Enter 1.5 and press [2nd [ $[\%$ ] 区 900000 enter.
The commission earned is $\$ 13500$.
$1.5 \% * 900000$
13500

Students are expected to calculate the amount of tax payable.
The tax payable is the amount of money owed in taxes.
Example: Calculating the tax payable
Ari has a taxable income of $\$ 39600$.
Use the Tl-30X Plus MathPrint ${ }^{T M}$ to find the amount of tax he will have to pay.
Teacher Note: The Australian resident tax rates can be sourced from the Australian Tax Office website. The address is https://www.ato.gov.au/rates/individual-income-tax-rates/.

## Keystrokes and solution:

The tax payable on this income is
$\mathrm{Nil}+(\$ 39600-\$ 18200) \times 0.19$.
Enter $\mathbf{0}$ and press $\ddagger \square 39600 \square 18200 \square \boxtimes 0.19$ enter.
The tax payable is $\$ 4066$.

## $0+(39600-18200) \stackrel{\text { DEG }}{-1}$ 4066

### 4.1.3 Budgeting and household expenses

Students are expected to plan for the purchase of a car, including sale price and loan repayments.
When buying a car with a car dealers' finance, the purchaser usually pays a deposit and then makes a number of monthly repayments.

The total cost of using car dealers' finance is greater than the cash price.

## Example: Purchasing a car

Hannah wishes to buy a car for $\$ 30000$.
Finance is available at $\$ 6000$ deposit and monthly repayments of $\$ 750$ for 5 years.
Use the TI-30X Plus MathPrint ${ }^{\text {TM }}$ last answer feature to find the amount of interest Hannah would pay.
Teacher Note: Students should understand that the amount of interest paid is the deposit plus the total repayments minus the sale price.

## Keystrokes and solution:

Total repayment is $\$ 750 \times 12 \times 5=\$ 45000$.

Enter $\mathbf{7 5 0}$ and press |  |
| ---: | :--- |
| ® enter |

Add the deposit to the total repayments.
Enter 6000 and press $⿴$ [2nd [answer] enter].
$\$ 6000+\$ 45000=\$ 51000$
Press [2nd [answer] $\square$ and enter 30000 enter.


Interest paid is $\$ 51000-\$ 30000=\$ 21000$.

### 4.2 Investments and loans (MS-F4)

### 4.2.1 Investments

Students are expected to calculate the future value, $F V$, or present value, $P V$, and the interest rate, $r$, of a compound interest investment using the formula $F V=P V(1+r)^{n}$.

Compound interest is calculated on the initial amount borrowed or invested plus any interest that has been charged or earned.

The future value, $F V$, of a compound interest investment (or loan) can be calculated using the formula $F V=P V(1+r)^{n}$ where:
$F V$ is the future value of the loan or amount (in dollars).
$P V$ is the present value of the loan or principal (in dollars).
$r$ is the interest rate per compounding time period expressed as a decimal.
$n$ is the number of compounding time periods.
Note that the present value of an investment, $P V$, can be calculated using the formula $P V=\frac{F V}{(1+r)^{n}}$.
The compound interest, $I$, earned or owed is given by the formula $I=F V-P V$.

## Example: Exploring a compound interest investment

This example shows how to use a calculator to explore a compound interest investment.
Use the TI-30X Plus MathPrint ${ }^{T M}$ data editor and list formulas feature to
(a) show the future value, $F V$, accumulating over 5 years if $\$ 1000$ is invested at an interest rate of $6 \%$ per annum, compounded annually.
(b) calculate, to the nearest cent, the future value, $F V$, of the investment accumulated after 5 years.
(c) show the amount of interest, $I$, accumulating over the 5 years.
(d) calculate the amount of interest, $I$, earned after 5 years.

Teacher Note: Such an example shows how the TI-30X Plus MathPrint ${ }^{T \mathrm{TM}}$ can be used to explore a compound interest investment for teaching and learning purposes. In the formula, the future value, $F V$, is the same as the amount, $A$, and the present value, $P V$, is the same as the principal, $P$.

## Keystrokes and solution:

(a) $F V=1000(1+6 \%)^{n}$

Press data. Press data 4 to clear all lists.
Press data (1) 3 . Select L1 and press enter.
Press $x_{\text {aboce }}^{v e d} d$ to paste $\boldsymbol{x}$, complete the sequence set-up as shown,


Press (1) to scroll across to the top of $\mathbf{L} 2$.
Press datal (1) to select FORMULA and press enter.
Enter the list formula 1000 * $(1+6 \%)^{\wedge}$ L1 to L2.
Enter 1000 and press 区 1 ( 6 [2nd [\%] $\square$.
Press data enter to paste $\mathbf{L 1}$ into the author line. Press enter.
L2 should now display the future value, $F V$, of the investment at the end of each year.

(b) Press 2nd $\odot$ to go to the bottom of a list and press $\Theta$.

The future value, $F V$ after 5 years is $\$ 1338.23$.

(c) Press (1) to scroll across to the top of L3.

Press data (1) to select FORMULA and press enter.
Enter the list formula L2 - 1000 to L3.
Press data 2 to paste L2 into the author line.
Press $\square$ and enter 1000. enter.
L3 should now display the interest accumulating over the 5 years

(d) The total amount of interest, $I$, earned after 5 years is $\$ 1338.23-\$ 1000=\$ 338.23$.


Students are expected to use technology to:

- compare the growth of simple interest investments (linear growth) and compound interest investments (exponential growth) numerically and graphically.
- investigate the effect of varying the interest rate, the term or the compounding period on the future value, $F V$, of an investment.
- compare and contrast different investment strategies.


## Example: Comparing the growth of a simple interest investment and a compound interest

 investmentThis example shows how to use a calculator to compare the growth of a simple interest investment and a compound interest investment.

Use the TI-30X Plus MathPrint ${ }^{T M}$ data editor and list formulas feature to compare a simple interest investment and a compound interest investment of $\$ 5000$ at $4 \%$ per annum over a period of 6 years.

Teacher Note: Simple interest increases by a constant amount each time period, resulting in a straight-line graph. Compound interest increases by a different amount each time period, resulting in an exponential curve.

## Keystrokes and solution：

Simple interest investment：
$F V=5000+(5000 \times 4 \% \times n)$
Press data．Press data 4 to clear all lists．
Press data（1）3．Select L1 and press enter ．
Press scroll down to select SEQUENCE FILL and press enter．

These six time period values should now be displayed in L1．

| EXPR IN $x: x$ | DEG |
| ---: | :--- |
| START $x: 1$ |  |
| ENND $x: 6$ |  |
| STEP SIZE：1 |  |
|  |  |

Press（1）to scroll across to the top of L2．
Press data（1）to select FORMULA and press enter．
Enter the list formula $5000+(5000 \times 4 \% \times L 1)$ to L2．
Enter 5000 and press $\ddagger$ Q 5000 区 4 ［2nd［\％］区．
Press data enter to paste $\mathbf{L 1}$ into the author line．Press $\square$ enter．
L2 should now display the future value，$F V$ ，of the simple interest investment at the end of each year．

Compound interest investment：
$F V=5000(1+4 \%)^{n}$
Press（1）to scroll across to the top of L3．
Press data（1）to select FORMULA and press enter．
Enter the list formula 5000 ＊$(\mathbf{+ 4 \%})^{\wedge}$ L1 to L3．

Press data enter to paste $\mathbf{L 1}$ into the author line．Press enter．
L3 should now display the future value，$F V$ ，of the compound investment at the end of each year．

After the first year the value of the compound interest investment is greater and the difference between the two investments increases over time．

After six years，the difference is $\$ 126.60$ ．


| 4 | ［19 DEG | ［1 |
| :---: | :---: | :---: |
| 1 | 5200 | 5200 |
| 2 | 5400 | 5408 |
| 3 | 5600 | 5624.32 |
| 4 | 5800 | 5849.293 |

L3（1）$=5200$


Tables of values can be used to solve certain problems involving compound interest．

## Example: Calculating the number of time periods for a compound interest investment

This example shows how to use a calculator to determine the number of time periods required to obtain a particular total amount for a compound interest investment.

Use the TI-30X Plus MathPrint ${ }^{T M}$ function feature to find the number of time periods required for an initial investment of $\$ 20000$ to increase to at least $\$ 25000$ given that it has an interest rate of $7 \%$ per annum compounding monthly.

Teacher Note: Such an example shows how the TI-30X Plus MathPrint ${ }^{T M}$ can be used to automate a 'guess and refine' numerical solution strategy. This example could also be solved using the TI-30X Plus MathPrint ${ }^{T M}$ function feature.

## Keystrokes and solution:

$$
F V=20000\left(1+\frac{7 \%}{12}\right)^{n}
$$

Press table 1 to access the function table. [If required, press
clear.
Enter 20000 and press 区 1 ( 1 圖 7 2nd [ $\%$ ] $\odot 12 \oplus \square$
x.

Press $x_{a b c a t}^{v z e}$ to paste $\boldsymbol{x}$ and press enter $\odot$.
Move the cursor to select $\boldsymbol{x}=$ ? and press enter (CALC) enter.
Enter 30, for example, and press enter.
Column two of the table should show the output 23812.81.
Enter 38, for example, and press enter.
Column two of the table should show the output 24947.03.
Enter 39, for example, and press enter.
Column two of the table should show the output 25092.56.
It would take 39 months for the investment to become at least
 \$25000.

### 4.2.2 Depreciation and loans

## Students are expected to:

- calculate the depreciation of an asset using the declining balance method.
- solve practical problems involving reducing balance loans, for example determining the total loan amount and monthly repayments.

Declining-balance depreciation occurs when the value of the asset decreases by a fixed percentage each time period.

The salvage value (current value) of the asset, $S$, is given by the formula $S=V_{0}(1-r)^{n}$ where $V_{0}$ is the initial value of the asset (purchase price).
$r$ is the rate of depreciation per time period.
$n$ is the number of time periods.

## Example: Exploring the declining-balance depreciation of an asset

This example shows how to use a calculator to calculate the declining-balance depreciation of an asset.
Coco paid $\$ 24000$ for a used car. It is thought that the car will depreciate in value by $15 \%$ each year for a period of four years.

Use the TI-30X Plus MathPrint ${ }^{T M}$ data editor and list formulas feature to complete the following depreciation table for the first four years.

| Year | Depreciated value |
| :---: | :---: |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |

Teacher Note: It is a good idea to graph $S=V_{0}(1-r)^{n}$ and interpret the meaning of $V_{0}$ and $r$ in the context of declining-balance depreciation.

## Keystrokes and solution:

$S=24000(1-0.15)^{n}$

$$
=24000(0.85)^{n}
$$

Press data. Press data 4 to clear all lists.
In L1, enter 1and press $\odot$. Repeat for 2, $\mathbf{3}$ and 4.


Press (1) to scroll across to the top of L2.
Press data (1) to select FORMULA and press enter.
Enter the list formula 24000 * ( $\mathbf{~ - ~ 1 5 \% ) ~ \wedge ~ L 1 ~ t o ~ L 2 . ~}$



Press data enter to paste L1 into the author line. Press enter.
The depreciated values should now be displayed in L2.

| Year | Depreciated value |
| :---: | :---: |
| 1 | $\$ 20400$ |
| 2 | $\$ 17340$ |
| 3 | $\$ 14739$ |
| 4 | $\$ 12528.15$ |

## Example: Calculating the percentage rate of declining-balance depreciation of an asset

 Hideki bought a 4WD vehicle four years ago for $\$ 36000$.Using declining-balance depreciation, a car dealer estimates the vehicle's present value to be $\$ 14750$.
Use the formula $S=V_{0}(1-r)^{n}$ and the TI-30X Plus MathPrint ${ }^{\text {TM }}$ to find the annual depreciation rate. Give your answer correct to the nearest whole number.

Teacher Note: The TI-30X Plus MathPrint ${ }^{\text {TM }}$ expression evaluation feature could be used here. However, the feature does not house the variables $S, V_{0}$ and $n$.

## Keystrokes and solution:

$$
\begin{aligned}
S & =V_{0}(1-r)^{n} \\
(1-r)^{n} & =\frac{S}{V_{0}} \\
1-r & =\sqrt[n]{\frac{S}{V_{0}}} \\
r & =1-\sqrt[n]{\frac{S}{V_{0}}}
\end{aligned}
$$

Substituting $n=4, S=14750$ and $V_{0}=36000$ gives $r=1-\sqrt[4]{\frac{14750}{36000}}$.

| Enter 1 and press $\square 4$ 2nd [ $\square^{\circ}$ ] 14750 圖 36000 enter . |  |
| :---: | :---: |
| $r=0.199940 \ldots$ | $1-\sqrt{\frac{1}{360}}$ |
| The rate of depreciation is $20 \%$, correct to the nearest whole | 0.199940328 | number.

### 4.3 Annuities (MS-F5)

Students are expected to:

- solve compound interest problems involving financial decisions, for example a home loan, a savings account, a car loan or an annuity.
- use technology to model an annuity as a recurrence relation and investigate numerically (or graphically) the effect of varying the amount and frequency of each contribution, the interest rate or the payment amount on the duration and/or future value of the annuity.

An annuity is a type of investment involving the regular contribution of money.
A familiar example of an annuity is a monthly loan repayment.
A recurrence relation uses the previous result (value) to generate the next result (value).
Annuities can be modelled as recurrence relations where a future value at the end of a year becomes the present value for the next year.

A loan can be modelled by the recurrence relation $A_{n+1}=A_{n}(1+r)-D$ where
$A_{n+1}$ is the value of the loan after $(n+1)$ payments.
$A_{n}$ is the value of the loan after $n$ payments.
$r$ is the rate of interest per compounding period expressed as a decimal.
$D$ is the payment made per compounding period.
Note that the recurrence relation for an investment is $A_{n+1}=A_{n}(1+r)+D$
Example: Modelling a loan expressed as a recurrence relation
This example shows how to use a calculator to model a loan using a recurrence relation.
Tamara and Paul have borrowed $\$ 100000$ at an interest rate of $7.5 \%$ per annum reducing balance with monthly repayments of $\$ 1200$ over approximately 10 years.

Use the TI-30X Plus MathPrint ${ }^{T M}$ stored operations feature to calculate the amount owing after two years. Give your answer correct to the nearest dollar.

Teacher Note: A similar approach can be employed to model the growth of an investment expressed as a recurrence relation.

Keystrokes and solution:
$A_{0}=100000, r=\frac{0.075}{12}=0.0625, D=1200$
Using $A_{n+1}=A_{n}(1+r)-D$ :
$A_{1}=100000(1+0.00625)-1200=99425$
$A_{2}=A_{1}(1+0.00625)-1200=98846.41$


Press 2 nd [set op].
[If required, press clear to clear any previously stored operations.]
Press $\boxtimes$ and enter $1.00625 \square 1200$ enter.
Enter 100000 and press 2 2nd [op].
Press 2nd [op] until the iteration counter shows $\mathbf{n}=\mathbf{2 4}$ (corresponding to the end of two years).

The amount owing after two years is $\$ 85161$ (correct to the nearest


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5 Statistical analysis
Press data to enter and edit lists (see page 21 for the data editor and list formulas feature).
Press 2nd [stat-reg/distr] to access the STAT-REG (statistics-regressions) menu.


The STAT-REG menu contains the following useful options for NSW Stage 6 Mathematics Standard.
StatVars displays a secondary menu of the last calculated statistical result variables. Use $\Theta$ and $\Theta$ to locate the desired variable and press enter to select it.

The statistical variables are defined in the following table.

| Variables |  |
| :--- | :--- |
| $\mathbf{n}$ | Number of $x$ or $(x, y)$ data points. |
| $\overline{\mathbf{x}}$ or $\overline{\mathbf{y}}$ | Mean of all $x$ or $y$ values. |
| $\mathbf{S}_{\mathbf{x}}$ or $\mathbf{S}_{\mathbf{y}}$ | Sample standard deviation of $x$ or $y$. |
| $\boldsymbol{\sigma}_{\mathbf{x}}$ or $\boldsymbol{\sigma}_{\mathbf{y}}$ | Population standard deviation of $x$ or $y$ |
| $\sum \mathbf{x}$ or $\sum \mathbf{x}^{\mathbf{2}}$ | Sum of all $x$ or $x^{2}$ values. |
| $\sum \mathbf{y}$ or $\sum \mathbf{y}^{\mathbf{2}}$ | Sum of all $y$ or $y^{2}$ values. |
| $\sum \mathbf{x y}$ | Sum of $(x \times y$ ) for all $x y$ pairs. |
| $\mathbf{a}$ | Linear regression slope. |
| $\mathbf{b}$ | Correlation coefficient. |
| $\mathbf{r}$ | Uses $a$ and $b$ to calculate a predicted $x$-value for an inputted $y$-value. |
| $\mathbf{x}^{\prime}$ | Uses $a$ and $b$ to calculate a predicted $y$-value for an inputted $x$-value. |
| $\mathbf{y}^{\prime}$ | Minimum or maximum of $x$-values. |
| MinX or MaxX $y$-intercept. |  |
| Q1 | Lower quartile (median of the elements between MinX and Med). |
| Med | Median of all data points. |
| Q3 | Upper quartile (median of the elements between Med and $\mathbf{M a x X}$ ). |
| MinY or MaxY | Minimum or maximum of $y$-values. |

1-VAR STATS analyses statistical data from one dataset with one measured variable, $x$. Frequency data may be included.

2-VAR STATS analyses paired statistical data from two datasets with two measured variables, the independent variable, $x$ and the dependent variable, $y$. Frequency data may be included. 2-VAR STATS also calculates a least-squares regression equation, displaying values for $\mathbf{a}$ (slope), $\mathbf{b}\left(y\right.$-intercept), $\mathbf{r}^{2}$ (if needed) and $\mathbf{r}$ (Pearson's correlation coefficient).

LinReg $\mathbf{a x}+\mathbf{b}$ calculates a least-squares regression equation, $y=a x+b$. It displays values for $\mathbf{a}$ (slope), $\mathbf{b}$ ( $y$ intercept), $\mathbf{r}^{2}$ (if needed) and $\mathbf{r}$ (Pearson's correlation coefficient).

Regressions store the regression information, along with the two-variable statistics for the data in StatVars.
A regression equation can be stored to $f(x)$ or $g(x)$.
To obtain one-variable or two-variable statistics for a dataset entered into L1, L2 or L3:
Press 2nd [stat-reg/distr].
Select 1-VAR STATS or 2-VAR STATS and press enter.
Select L1, L2 or L3 and the frequency (FREQ).
Press enter to display the menu of statistical variables with their values.

### 5.1 Data analysis (MS-S1)

### 5.1.1 Classifying and representing data (grouped and ungrouped)

Students are expected to describe and use appropriate data collection methods for populations or samples. One such sampling method is simple random sampling.
A random sample occurs when all members of a given population have an equal chance of being selected.
For any given population, a sample that is selected free of bias, using random numbers is called a simple random sample.

For example, a simple random sample (SRS) of 10 students (the sample) from a HSC Mathematics Standard 2 class of 28 students (the population) can be made by assigning each student a number, placing all 28 numbers in a hat and then selecting 10 numbers from the hat 'at random'.

Random number generators can help in generating sets of suitable random numbers.
The TI-30X Plus MathPrint ${ }^{\text {TM }}$ can be used to obtain data by sampling using a pseudo-random number generator.
Press 2nd [random] to access rand and randint.

## RANDOM <br> 1:rand <br> 2: randint (

The command rand generates a pseudo-random number between 0 and 1 . These pseudo-random numbers are generated by a formula.

To initiate this formula, the TI-30X Plus MathPrint ${ }^{T M}$ uses a starting value called a seed and then generates 'random' numbers based upon this seed. This process is known as seeding the calculator.

If two or more TI-30X Plus MathPrint ${ }^{\text {TM }}$ calculators start with the same seed value, they will generate the same sequence of 'random' values. Hence, if you wish, you can control the starting seed value.

To control a sequence of numbers, store an integer (seed value) $\geq 0$ to rand. The seed value changes every time a number is generated.

## Example: Seeding the TI-30X Plus MathPrint ${ }^{T M}$

This example shows how to seed the TI-30X Plus MathPrint ${ }^{T M}$.
By using the integer 4 , seed the TI-30X Plus MathPrint ${ }^{\text {TM }}$ to generate a 'random' number between 0 and 1 .
Teacher Note: It is important to decide whether you want students generating the same set of 'random' numbers or different sets of 'random' numbers. It is mostly desirable to have each student generating a different set of 'random' numbers. To achieve this, assign each student a different seed value and follow the instructions below. For example, students could enter their mobile phone number as a seed value. Note that the leading zero is not required.

## Keystrokes and solution:

Enter 4 and press sto $\rightarrow$ 2nd [random] [1] enter.
To generate a 'random' number between 0 and 1 :
Press [2nd [random] 1 enter.
 rand

Press enter to generate further 'random' numbers between 0 and 1 .

The command randint( generates a 'random' integer between two integers, $a$ and $b$, where $a \leq$ randint $\leq b$. The syntax involves the use of a comma (press 2nd [,]) to separate the two integers. For example, randint ( $\mathbf{1 , 6}$ ) will generate a 'random' integer between 1 and 6 inclusive.

## Example: Generating 'random' integers between 1 and 6 inclusive

This example shows how to use a calculator to generate 'random' integers between 1 and 6 inclusive.
Generate some 'random' integers on the TI-30X Plus MathPrint ${ }^{\text {TM }}$ between 1 and 6 inclusive.
Teacher Note: Generating 'random' integers between 1 and 6 inclusive simulates the rolling of an unbiased sixsided die and can be used to play games such as Greedy Pig.

## Keystrokes and solution:

Press 2nd [random] 2 and enter 1 2nd [,] $6 \square$ enter.
Press enter to generate further 'random' integers between 1 and 6.


## Example: Generating a simple random sample (SRS)

This example shows how to use a calculator to generate a simple random sample.
Rachael wishes to determine the opinion of her year level's 80 students about whether they should have the valedictory dinner before or after the end-of-year exams.

Describe how Rachael could select a simple random sample (SRS) of 20 students with a TI-30X Plus MathPrintTM to obtain results which are likely to represent the views of the entire year level.

Teacher Note: In the event of obtaining repeat 'random' integers in the sample, 25 'random' integers can be generated with any repeat 'random' integers discarded. In this example, 25 'random' integers are generated.

## Keystrokes and solution:

Press data. Press data 4 to clear all lists.
Press data (1) 3. Select L1 and press enter.
Press 2nd [random] 2 and enter 1 2nd [,] $80 \square$.
Complete the sequence set-up as shown, scroll down to select SEQUENCE FILL and press enter.

The generated random sample should now be displayed in L1.
To help detect the presence of any repeat values, the 'random' integers in L1 can be ordered from smallest to largest (or largest to smallest).

Press data © 1 to select SORT Sm-Lg.
Complete the sorting set-up as shown, scroll down to select SORT and press enter.

The following students were selected in the sample:
$2,5,13,14,18,19,22,24,25,28$,
$30,43,46,47,58,62,64,66,67,68$
Considering repeats, 77 was not required.

## EXPR IN $x$ :randint $(1,80)$ START $\chi: 1$ END $x: 25$ <br> STEP SIZE:1 <br> SEQUENCE FILL



SORT SMALL-LARGI SORT LIST: L1 L2 L3 $\rightarrow$ LIST: L1 L2 L3

SORT


Students are expected to organise and display data into appropriate tabular representations, for example, cumulative frequency distribution tables.

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## Example: Calculating cumulative frequencies

This example shows how to use a calculator to generate cumulative frequencies from a frequency table.
A researcher interviewed commuters, asking how often they caught a bus in the last week.
The following table shows the results of the survey.

| Number of days | Number of commuters (frequency) | Cumulative frequency |
| :---: | :---: | :---: |
| 0 | 8 |  |
| 1 | 16 |  |
| 2 | 22 |  |
| 3 | 14 |  |
| 4 | 20 |  |
| 5 | 32 |  |
| 6 | 4 |  |
| 7 | 4 |  |

(a) Use the TI-30X Plus MathPrint ${ }^{T M}$ last answer feature to calculate the cumulative frequencies.
(b) How many people caught a bus on four days or less?

Teacher Note: Students should realise that for the first data value, the cumulative frequency is the same as the frequency. For subsequent values, they should realise to add the frequency for that value to the previous total. This example shows how the last answer feature preserves continuity in calculations.

## Keystrokes and solution:

Enter 8 and press $\square 16$ enter.
$8+16=24$
Press $\dagger$ and enter 22 enter.
$24+22=46$
Continuing the same process:

## 8+16 <br> ans +22 ans +14


$112+8=120$
$46+16=60$
$60+20=80$
$80+32=112$
$120+4=124$
The cumulative frequency column can now be filled in.
(b) The relevant row from the table is:
4

$$
20
$$

$$
60+20=80
$$

80 commuters caught the bus on four days or less.

### 5.1.2 Summary statistics

## Students are expected to:

- calculate measures of central tendency, including the arithmetic mean and the median and use them to compare datasets in real-world contexts.
- calculate measures of spread including the range, interquartile range (IQR) and standard deviation
- investigate and describe the effect of outliers on summary statistics (for example, use of $Q_{1}-1.5 \times I Q R$ and $Q_{3}+1.5 \times I Q R$ as criteria), on the mean and median.
- complete a five-number summary for different datasets.


## Example: Comparing the central tendency and spread of two datasets

This example shows how to use a calculator to compare data displays using mean and median to describe and interpret centre (location) and quartiles, interquartile range and standard deviation to describe and interpret spread.

Two companies, $A$ and $B$, produce packets of chips which are labelled as having a weight of 50 grams. A random sample of 10 packets is taken from each company. Each packet is weighed and the results, in grams, are as follows.

Company A: $50.0,50.6,50.0,50.4,49.2,49.0,51.4,50.1,47.4,51.9$
Company B: $51.0,50.9,51.1,51.5,51.3,50.2,50.6,50.0,50.5,50.9$
(a) For each company's data, use the TI-30X Plus MathPrint ${ }^{\text {TM }}$ to calculate the
(i) sample mean weight.
(ii) sample standard deviation. Give your answer correct to two decimal places.
(iii) five-number summary ([minimum, Q1, median, Q3, maximum]).
(b) For Company A , are the weights of any of the packets in the sample considered outliers by the ' $1.5 \times I Q R$ ' rule?
(c) What, if anything, can be concluded about the manufacturing processes of the two companies?

Teacher Note: Numerical comparisons should be made in conjunction with graphical comparisons of two datasets, for example, with parallel box plots.

## Keystrokes and solution:

Press data. Press data 4 to clear all lists.
Enter the Company A data in L1. Start by entering 50.0 and pressing $\Theta$ (or enter).

Press (1) to scroll across to the top of $\mathbf{L 2}$.
Enter the Company B data in L2. Start by entering $\mathbf{5 1 . 0}$ and pressing $\odot$ (or enter).

Press [2nd [stat-reg/distr] to access the statistics menu.
Press 2 to access 1-VAR STATS. Select L1 and press enter.
Press $\odot$ enter (CALC) to calculate the results for Company A.


## 1-VAR STATS



FREQ: ONI L1 L2 L3
CALC
(a) (i) Company A
$\bar{x}=50$ (grams)
(a) (ii) Company A
$s=1.27$ (grams) (correct to two decimal places)
(a) (iii) Company A

The five-number summary is $[47.4,49.2,50.05,50.6,51.9]$.

Press 2nd [stat-reg/distr] to access the statistics menu.
Press 2 to access 1-VAR STATS. Select L2 and press enter.
Press $\odot$ enter (CALC) to calculate the results for Company B.

(a) (i) Company B
$\bar{x}=50.8$ (grams)
1-Var:L2,1
1: n=10
$2: \bar{x}=50.8$
$3 \downarrow S x=0.47375568$
(a) (ii) Company B
$s=0.47$ (grams) (correct to two decimal places)
(a) (iii) Company B

The five-number summary is [50, 50.5, 50.9, 51.1, 51.5].

1-Var:L2, 1
7 个minX $=50$
8:01 $=50.5$
94Med $=50.9$


Press 2nd [stat-reg/distr] 2 to access 1-VAR STATS.
Ensure L1 is selected and press enter.
Press $\odot$ enter (CALC) to again access the results for Company A.

Press 8 (or scroll up or down and select Q1) and press enter
$\square 1.5$ ® $\square$ 2nd [stat-reg/distr] $\square$.
Scroll up or down to select Q3 and press enter $\square$ 2nd
[stat-reg/distr] 18 enter.
The home screen should show Q1-1.5 * (Q3-Q1).
Press $\Theta \odot$ to select Q1-1.5 * (Q3- Q1) and press
enter. Press 2 nd (1), edit the new author line to form
Q3 + 1.5 * (Q3-Q1) and press enter.
(b) Company A
$Q_{1}-1.5 \times I Q R=47.1$ (grams)
$Q_{3}+1.5 \times I Q R=52.7$ (grams)
None of the packets in the sample are considered outliers by the ' $1.5 \times I Q R$ ' rule .
(c) Company A produces packets of chips with a weight centred closer to 50 grams than Company B, but with greater variation (using the sample standard deviation and/or the IQR) in the weights. Company B produces packets of chips with a weight centred slightly greater than 50 grams whereas some of the packets produced by Company $A$ are less than 50 grams.

## Example: Effect of outliers on the mean and median

This example shows how to use a calculator to investigate and recognise the effect of outliers on the mean and median.

A group of 10 students had their hand span measured and recorded to the nearest cm .
The measurements are as follows: $16,15,18,20,15,15,14,17,23,17$
(a) Use the TI-30X Plus MathPrint ${ }^{T M}$ to calculate the mean and the median.
(b) Confirm that 23 is an outlier.

Remove 23 from the dataset.
(c) Use the TI-30X Plus MathPrint ${ }^{\text {TM }}$ to calculate the new mean and new median of the dataset. Give the new mean correct to two decimal places.
(d) Describe the effect the outlier had on the
(i) mean.
(ii) median.

Teacher Note: It is important that students understand how the mean and median are calculated when examining the effects of outliers.

## Keystrokes and solution:

(a) Press data. Press data 4 to clear all lists.

Enter the hand span data in L1. Start by entering 16 and pressing $\Theta$ (or enter).

Press [2nd [stat-reg/distr] to access the statistics menu.
Press 2 to access 1-VAR STATS. Select L1 and press enter.
Press $\odot$ enter (CALC) to calculate the results.
$\bar{x}=17(\mathrm{~cm})$ and the median is $16.5(\mathrm{~cm})$.

(b) Scroll up or down to select Q3 and press enter.

Press $\square$ and enter 1.5 区 $\square$ 2nd [stat-reg/distr] 1 .


Scroll up or down to select Q3 and press enter.
Press - 2nd [stat-reg/distr] 180 enter.
The home screen should show Q3+1.5 * $\mathbf{( Q 3 - Q 1 )}$.
$Q_{3}+1.5 \times I Q R=22.5(\mathrm{~cm})$
$23>22.5$ and hence 23 is an outlier by this definition.
(c) Press data, scroll down L1 to select 23 and press deletee.

Press 2nd [stat-reg/distr] 2, ensure L1 is selected and press enter $\odot$ enter (CALC) to calculate the new results.
$\bar{x}=16.33(\mathrm{~cm})$ and the median is $16(\mathrm{~cm})$.


1-Var: L1,1
8个Q1=15
9: Med=16
$\downarrow$ Q $=17.5$
(d) (i) Removing the outlier had a small effect on the mean, decreasing it from 17 to 16.33 .

A decrease of 0.67.
(d) (ii) Removing the outlier had a small effect on the median, decreasing it from 16.5 to 16 .

A decrease of 0.5 (a smaller decrease than on the mean).

### 5.2 Relative frequency and probability (MS-S2)

Students are expected to:

- solve problems involving simulations or trials of experiments in a variety of contexts.
- perform simulations of experiments using technology.
- use relative frequency as an estimate of probability.
- recognise that an increasing number of trials produces relative frequencies that gradually become closer in value to the theoretical probability.


## Example: Simulating 50 trials of rolling two unbiased six-sided dice

This example shows how to use a calculator to simulate 50 trials of rolling two unbiased six-sided dice.
Use the TI-30X Plus MathPrint ${ }^{T M}$ data editor and list formulas feature to
(a) simulate 50 trials of rolling two unbiased six-sided dice, taking the result from each die in each trial and adding the two results together to obtain a score.
(b) find the sample mean score, $\bar{x}$, for these 50 trials.

Teacher Note: Such simulations encourage comparison of experimental probabilities and results with theoretical probabilities and considerations. Entering the formula randint $(\mathbf{1 , 6})+\operatorname{randint}(\mathbf{1 , 6})$ on the $\mathrm{TI}-30 \mathrm{X}$ Plus MathPrint ${ }^{T M}$ home screen simulates the rolling of two unbiased six-sided dice. Press enter to perform further trials.

Keystrokes and solution:
(a) Press data. Press data 4 to clear all lists.

Press data (1) 3. Select L1 and press enter.
Press 2nd [random] 2 and enter 1 2nd [,] $6 \square$.
EXPR IN $x:$ randint $(1,6)$
Complete the sequence set-up as shown, scroll down to select SEQUENCE FILL and press enter.

The results for the first die should now be displayed in L1.
Press (1) to scroll across to the top of L2.
Press data (1) 3. Select L2 and press enter.
Scroll down to select SEQUENCE FILL and press enter.


The results for the second die should now be displayed in L2.
Press (1) to scroll across to the top of L3.
Press data (1) to select FORMULA and press enter.
Enter the list formula $\mathbf{L 1} \mathbf{+} \mathbf{L} \mathbf{2}$ to $\mathbf{L 3}$.


Press data enter to paste $\mathbf{L 1}$ into the author line.
Press $\square$ and press data 2 to paste $\mathbf{L}$ 2 into the author line. Press enter.


L3 should now show the scores in each of the 50 trials.
(b) To find the sum of L3, press data (1) 4. Select L3 and press enter. Press enter (CALC).

The sum is 347 .
Store the sum as $\boldsymbol{x}$, select DONE and press enter.
Press 2nd [quit] to return to the home screen.
Press $x_{a b c d}^{y z t}$ to paste $\boldsymbol{x}$. Press $\div$ and enter 50.

$$
\bar{x}=6.94
$$



DONE


Note: Use of 回 gives $\bar{x}=\frac{347}{50}$.
Alternatively:
Press 2nd [stat-reg/distr] to access the statistics menu.
Press 2 to access 1-VAR STATS. Select L3 and press enter. Press $\odot$ enter (CALC).
$\bar{x}=6.94$

## Example: Theoretical outcome of an experiment

This example shows how to use a calculator to find the theoretical outcome (for example, mean score) of an experiment.

Consider the following experiment.
Two unbiased six-sided dice are rolled, the result from each die noted and the two results added together to obtain a score.
(a) By listing the sample space, verify the theoretical probabilities shown in the following table.

| Score | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Probability | $\frac{1}{36}$ | $\frac{2}{36}$ | $\frac{3}{36}$ | $\frac{4}{36}$ | $\frac{5}{36}$ | $\frac{6}{36}$ | $\frac{5}{36}$ | $\frac{4}{36}$ | $\frac{3}{36}$ | $\frac{2}{36}$ | $\frac{1}{36}$ |

(b) Use the $\mathrm{TI}-30 \mathrm{X}$ Plus MathPrint ${ }^{\mathrm{TM}}$ data editor and list formulas feature and one-variable statistics feature to find the theoretical mean score of the experiment.
(c) Compare your part (b) answer with the sample mean score, $\bar{x}$, obtained from simulating 50 trials of rolling two unbiased six-sided dice, taking the result from each die in each trial and adding the two results together to obtain a score.

Teacher Note: Activities like the simulation conducted in the previous example and the theoretical situation considered in this example should help students recognise the difference between relative frequencies and theoretical probabilities. It is also important to conduct activities that demonstrate that increasing the number of trials produces relative frequencies that gradually become closer in value to the theoretical probability.

## Keystrokes and solution:

(a) For example, a score a 3 can be obtained by rolling $\{1,2\}$ or $\{2,1\}$.

There are 36 total possible outcomes so the probability of obtaining a 3 is $\frac{2}{36}\left(=\frac{1}{18}\right)$.
(b) Press data. Press data 4 to clear all lists.

Press data (1) 3. Select L1 and press enter.
Press $x_{a b c a t}^{y z e d}$ to paste $\boldsymbol{x}$, complete the sequence set-up as shown, scroll down to select SEQUENCE FILL and press
 enter.

These eleven scores should now be displayed in L1.
Press (1) to scroll across to the top of L2.
Enter the probabilities to L2.


To enter the first probability, enter $\mathbf{1}$ and press 圖.
Note: If $\square$ is used in this example, all probabilities will be in decimal form.

Enter 36 and press enter. Repeat this for the other ten probabilities.

L2 should now display the eleven probabilities.
Press 2nd [stat-reg/distr] to access the statistics menu.
Press 2 to access 1 -VAR STATS.
Select L1 for DATA, select L2 for FREQ and press enter.
Press enter (CALC).


The theoretical mean score is 7 .
(c) Answers for $\bar{x}$ may vary.

For example, $E(X)=7$ and $\bar{x}=6.94$ are similar values.
Students are expected to calculate the expected frequency of an event occurring using $n p$ where $n$ represents the number of times an experiment is repeated, and on each of those times the probability that the event occurs is $p$.

Expected frequency is the number of times that a particular event should occur.

## Example: Expected frequencies

This example shows how to use a calculator to calculate the expected frequency of an event occurring.
Consider the following experiment.
Two unbiased six-sided dice are rolled, the result from each die noted and the two results added together to obtain a score.

The sample space and associated probabilities are shown in the following table.

| Score | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Probability | $\frac{1}{36}$ | $\frac{2}{36}$ | $\frac{3}{36}$ | $\frac{4}{36}$ | $\frac{5}{36}$ | $\frac{6}{36}$ | $\frac{5}{36}$ | $\frac{4}{36}$ | $\frac{3}{36}$ | $\frac{2}{36}$ | $\frac{1}{36}$ |

900 trials of this experiment are to be conducted.
Use the TI-30X Plus MathPrint ${ }^{\text {TM }}$ data editor and list formulas feature to calculate the expected frequency of each event in the sample space occurring.
Teacher Note: Remind students that the expected frequency may not match the results from an experiment (simulation). For example, if a coin is tossed 200 times, the expected number of tails is 100 . However, tossing a coin 200 times may not result in obtaining exactly 100 tails. The larger the number of trials, the closer the experimental frequency should be to the expected frequency. Also remind students that the expected frequency may not be a whole number.

Keystrokes and solution:
Press data. Press data 4 to clear all lists.
Press data (1) 3 . Select L1 and press enter.
Press scroll down to select SEQUENCE FILL and press enter.
These eleven scores should now be displayed in L1.
Press (1) to scroll across to the top of L2.


SEQUENCE FILL


Enter 36 and press enter. Repeat this for the other ten probabilities.
L2 should now display the eleven probabilities.
Press (1) to scroll across to the top of $\mathbf{L 3}$.
Press data (1) to select FORMULA and press 1 .
Enter the list formula $\mathbf{9 0 0}$ * L2 to L3.

| $\begin{aligned} & \text { 適 } \\ & 2 \\ & 3 \\ & 4 \\ & 5 \\ & \hline \end{aligned}$ |  | 田 |
| :---: | :---: | :---: |
| CL3 $=900$ : L 2 |  |  |

Enter 900 and press | . Press data 2 to paste $\mathbf{L 2}$ into the |
| :---: | author line. Press enter.

| L3 should now display the expected frequencies. |
| :--- |
| $\qquad$Score Expected frequency <br> 2 25 <br> 3 50 <br> 4 75 <br> 5 100 <br> 6 125 <br> 7 150 <br> 8 125 <br> 9 100 <br> 10 75 <br> 11 50 <br> 12 25 |



### 5.3 Bivariate data analysis (MS-S4)

Students are expected to:

- calculate and interpret Pearson's correlation coefficient $(r)$ using technology to quantify the strength of a linear association of a sample.
- fit a least -squares regression line to a dataset using technology and to interpret the intercept and gradient of the fitted line.
- make predictions by either interpolation or extrapolation and to recognise the limitations of interpolation and extrapolation.


## Example: Pearson's correlation coefficient and least-squares regression

This example shows how to use a calculator to calculate Pearson's correlation coefficient and to interpret its value. It also shows how to calculate and use a least-squares regression line.

The length measurements (correct to the nearest cm ) of the femur bone and humerus bone of a particular species of fossil are shown in the following table.

| Femur length $(x)$ | 60 | 57 | 65 | 39 | 75 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Humerus length $(y)$ | 71 | 64 | 73 | 42 | 85 |

(a) Use the TI-30X Plus MathPrint ${ }^{\text {TM }}$ to determine Pearson's correlation coefficient, giving your answer correct to two decimal places. Comment on the direction and strength of association.
(b) Use the TI-30X Plus MathPrint ${ }^{T M}$ to determine the least-squares regression line. Give your answer in the form $y=a x+b$, where $a, b$ are expressed correct to two decimal places.
(c) Interpret the gradient of the least-squares regression line found in part (b).
(d) Use the least-squares regression line found in part (b) to estimate the length of a humerus bone of this species of fossil whose femur length is 48 cm . Give your answer correct to the nearest centimetre.

Teacher Note: Given reliable (unbiased) data, a line of best fit can reasonably be used for interpolation. Students should be made aware of the dangers of using a line of best fit for extrapolation.

## Keystrokes and solution:

(a) Press data. Press data 4 to clear all lists.

Enter the femur bone lengths in L1. Start by entering $\mathbf{6 0}$ and pressing $\Theta$ (enter).

Press (1) to scroll across to the top of L2.
Enter the humerus bone lengths in L2. Start by entering 71 and
 pressing $\odot$ (or enter).

## METHOD 1:

Press 2nd [stat-reg/distr] to access the statistics menu.
Press 4 to access LinReg $\mathbf{a x}+\mathrm{b}$.
Select the options as shown at right and press enter (CALC).
$r=0.99$ (correct to two decimal places)
This reflects a (very) strong positive association.




## METHOD 2:

Press [2nd [stat-reg/distr] to access the statistics menu.
Press 3 to access 2-VAR STATS.


Select L1 for $\boldsymbol{x}$ DATA and select L2 for $\boldsymbol{y}$ DATA.
Press $\Theta$ to select CALC and press enter. Scroll up or down.
$r=0.99$ (correct to two decimal places)

(b) Press 2nd [stat-reg/distr] to access the statistics menu.

Either by using LinReg $a x+b$ or 2-VAR STATS, we obtain:
$a=1.196 \ldots$ and $b=-3.856 \ldots$


The least-squares regression line is $y=1.20 x-3.86$ (correct to two decimal places).
(c) The humerus length increases by 1.2 cm (correct to one decimal place) for each increase of 1 cm in femur length.v

(d)


## METHOD 1:

With the least-squares regression equation stored as $f(x)$, press
table 22 and enter 48. Press $\square$ enter.
Correct to the nearest centimetre, the humerus bone length is estimated to be 54 cm .

## METHOD 2:

Press 2nd [stat-reg/distr] to access the statistics menu.
In StatVars, scroll up or down to locate $\mathbf{y}^{\prime}$ ( and press enter. Enter 48 and press $\square$ enter.


Correct to the nearest centimetre, the humerus bone length is estimated to be 54 cm .

### 5.4 The normal distribution (MS-S5)

Students are expected to calculate the $z$-score corresponding to a particular value in a dataset and use calculated $z$ - scores to compare scores from different datasets.

If a dataset has a mean $\bar{x}$ and a standard deviation $s$, then any score $x$ has a $z$-score given by $z=\frac{x-\bar{x}}{s}$.
A $z$-score provides a measure of how extreme a particular score is relative to the mean.
It is the number of standard deviations a score lies above or below the mean.
The set of $z$-scores for data arising from a random variable that is normally distributed has a mean of 0 and a standard deviation of 1 .

## Example: Calculating $z$-scores from a dataset

This example shows how to use a calculator to calculate $z$-scores from a dataset.
The weights of strawberries are known to have a mean of 14.3 grams and a standard deviation of 0.8 grams.
Five strawberries were randomly sampled (labelled S1 to S5) from a batch.
Their weights (in grams) are shown in the following table.

| S1 | S2 | S3 | S4 | S5 |
| :---: | :---: | :---: | :---: | :---: |
| 12.9 | 15.1 | 16.2 | 13.7 | 14.5 |

(a) Use the TI-30X Plus MathPrint ${ }^{T M}$ data editor and list formulas feature to convert each of the sample weights to $z$-scores.
(b) Relative to the mean weight, rank the strawberries from the most extreme to the least extreme.

Teacher Note：Remind students that the larger the $z$－score（ignoring the positive or negative），the further away it is from the centre of the data．This makes comparisons easier to make．

## Keystrokes and solution：

（a）Store the mean as $\boldsymbol{a}$ ．
Enter 14.3 and press $s t o \rightarrow$ and press $x_{a b c a t}^{y z e n t i l}$ until $a$ appears， then press enter

Press data data 4 to clear all lists．
$14.3 \rightarrow a{ }^{\text {066 }} 14.3$

Enter 12.9 and press $\Theta$ ．Repeat for 15．1，16．2， 13.7 and 14．5．

The five weights should now be displayed in L1．
Press（1）to scroll across to the top of L2．Press data（1） 3 ．
Select L2 and press enter．
Use the sequence feature to enter $\mathbf{0 . 8}$ five times in L2．
Complete the sequence set－up as shown at right，scroll down to select SEQUENCE FILL and press enter．

These five values should now be displayed in L2．

| EXPR IN $x: 0.8$ |  |
| :--- | :--- |
| START $x: 1$ |  |
| END $x: 5$ |  |
| STEP SIZE：1 |  |
|  |  |
|  | SEQUENCE FILL |

Press（1）to scroll across to the top of L3．
Press data（1）to select FORMULA and press 1 ．
Enter the list formula（ $\mathbf{L} 1 \mathbf{- a}$ ）／L2 to L3．
Press data enter to paste $\mathbf{L 1}$ into the author line．
Press $\square$ and press $x_{a b c a d}^{x z e d}$ until $\boldsymbol{a}$ appears．
Press $\square$－and press data 2 to paste $\mathbf{L 2}$ into the author line．Press enter．

L3 should now display the five $z$－scores．
S 1 is $z=-1.75 . \quad \mathrm{S} 2$ is $z=1$ ．
S 3 is $z=2.375 . \quad \mathrm{S} 4$ is $z=-0.75$ ．
S 5 is $z=0.25$ ．


| 1 | 荫。 | ${ }^{\text {DEG }}$ | 田 |
| :---: | :---: | :---: | :---: |
| 12.9 |  |  | $\frac{-1.75}{1.75}$ |
| 16.2 | 0.8 |  | 2.375 |
| 13.7 | 0.8 |  | －0．75 |


| L | DEG |  |
| :---: | :---: | :---: |
| 16.2 | 0.8 | 2.375 |
| 13.7 | 0.8 | －0．75 |
| 14.5 | 0.8 | 0.25 |
| L3（6）${ }^{\text {a }}$ |  |  |

b）Ranking from most to least extreme：$S 3, S 1, S 2, S 4, S 5$
Students are expected to use $z$－scores to identify probabilities of events less or more extreme than a given event and to use technology to determine probabilities．

In a normal distribution：
$68 \%$ of scores have a $z$－score between -1 and 1 ．
$95 \%$ of scores have a $z$－score between -2 and 2 ．
$99.7 \%$ of scores have a $z$-score between -3 and 3 .
Press [2nd [stat-reg/distr] (1) to display the DISTR (distributions) menu which has the following distribution functions for NSW Stage 6 Mathematics Standard 2:

```
STAT-REG DEGISTR
1:Normalpdf
2:Normalcdf
3\downarrowinvNormal
```


## Normalcdf

Calculates the normal distribution probability between LOWERbnd and UPPERbnd for a given mean mu and standard deviation sigma. The defaults are $\mathbf{m u}=\mathbf{0}$ and sigma $=\mathbf{1}$ with
LOWERbnd $=\mathbf{- 1 E 9 9}$ and UPPERbnd $=$ 1E99. These bounds represent $-\infty$ to $\infty$.

## Example: Normal distribution

This example shows how to use a calculator to calculate a normal distribution probability.
Random variable $X$ has a normal distribution with mean $\mu=28$ and standard deviation $\sigma=1.7$.
Use the TI-30X Plus MathPrint ${ }^{\text {TM }}$ to find $P(24.6 \leq X \leq 31.4)$.
Give your answer correct to four decimal places.
Teacher Note: A rough sketch of a normal curve can be extremely useful. The symmetry of the normal distribution can be used to simplify, clarify or visualise the question.

## Keystrokes and solution:

$X$ has a normal distribution with mean $\mu=28$ and standard deviation $\sigma=1.7$.

Press [2nd [stat-reg/distr] (1) to access the probability distributions menu.

STAT-REG ${ }^{\text {DEE }}$ DISTR
1:Normalpdf

Press 2 to select Normalcdf.
Enter the required values for $\mu$ and $\sigma$ and press $\Theta$. Enter 24.6 for the lower bound and enter 31.4 for the upper bound.

Press $\Theta$ to select CALC and press enter.
So $P(24.6 \leq X \leq 31.4)=0.9545$ (correct to two decimal places).

```
Normaicdf
VALUEE=0.9544998759714
STORE: NOD x yztab c d
SOLVE AGAIN
                                    QUIT
```

Press $\odot(1)$ to select QUIT and press enter.

## 6 Error and messages

When the TI-30X Plus MathPrint ${ }^{T M}$ detects an error, the screen will display the error type or a message. To correct an error, press clear to clear the error screen. The cursor will display at or near the error where it can be corrected.

To close the error screen without correcting the expression, press [2nd [quit] to return to the home screen. Examples of errors and messages that you may encounter are outlined below.

## Argument

This error is returned when:

- a function does not have the correct number of arguments.
- the lower limit is greater than the upper limit in a summation or a product function.


## Bounds: Enter LOWER $\leq$ UPPER

This error is returned when the input for the lower bound is greater than the upper bound for the Normalcdf distribution.

## Break

This error is returned when the on key is pressed to stop the evaluation of an expression.

## Calculate 1-Var, 2-Var Stat or a regression

This message is returned when no statistics or regression calculation has been stored.

## Change mode to DEC

This error is returned when the mode is set to BIN, HEX or OCT and the following applications are accessed.
[expr-eval] table [convert] [stat-reg/distr] data
These applications are available in DEC mode only.

## Dimension mismatch

This error is returned if:

- the dimensions of the lists used in a data formula are not the same length for the operation.
- a calculation of 2-VAR STATS is attempted when the data lists are not of equal length.


## Domain

This error is returned when an argument is not in the function domain.

- For $x \sqrt{y}: x=0$ or $y<0$ and $x$ is not an odd integer.
- For $y^{x}: x, y=0$.
- For $\sqrt{x}: x<0$.
- For log, ln or logBASE: $x \leq 0$.
- For tan: $x=90^{\circ},-90^{\circ}, 270^{\circ},-270^{\circ}, 450^{\circ}$ etc. and equivalent for radian mode.
- For $\sin ^{-1}$ or $\cos ^{-1}:|x|>1$.
- For ${ }^{n} C_{r}$ and ${ }^{n} P_{r}: n$ or $r$ are not integers $\geq 0$.
- For $x!: x$ is not an integer between 0 and 69 .


## Enter $0 \leq$ area $\leq 1$

This error is returned when you enter an invalid area value in invNormal for a distribution.

## Enter sigma > 0

This error is returned when the input for sigma in a distribution is required.

## Expression is too long

This error is returned when an entry exceeds the digits limits. For example, pasting an expression entry with a constant that exceeds the limit. A checkerboard cursor may display when limits are reached in each MathPrint ${ }^{T M}$ feature.

## Formula

This error is returned in data when the formula:

- does not contain a list name (L1, L2 or L3).
- for a list contains its own list name. For example, a formula for L1 contains L1.

Frequency: Enter FREQ $\geq 0$
This error is returned when at least one element in a list selected for FREQ is a negative real number in 1-VAR or 2-VAR STATS.

Input must be non-negative integer
This error is returned when an input is not the expected number type. For example, in the distribution argument TRIALS and $\boldsymbol{x}$ in Binomialpdf.

## Input must be Real

This error is returned when an input requires a real number.

## Invalid data type

This error is returned when the argument of a command or function is the incorrect data type. For example, the error will be displayed for $\boldsymbol{\operatorname { s i n }}(\boldsymbol{i})$ where the argument(s) must be real number(s).

List Dimension $1 \leq \operatorname{dim}($ list $) \leq 50$
This error is returned when, in data:

- the SUM LIST function is executed on an empty list.
- a sequence is created with a length of 0 or $>50$.


## Mean: Enter mu > 0

This error is returned when an invalid value is input for the mean in Poissonpdf or Poissoncdf.

## Memory limit reached

This error is returned when a calculation contains a circular reference such as two functions referencing each other, or a very long calculation.
[2nd] [set op]: Operation is not defined
This error is returned when an operation has not been defined in 2nd [set op] and 2nd [op] is pressed.
Operation set! [2nd] [op] pastes to home screen
This message is returned when an operation is stored (set) from 2nd [set op] editor. Press any key to continue.

## Overflow

This error is returned when a calculation or value is beyond the range of the $\mathrm{Tl}-30 \mathrm{X}$ Plus MathPrint ${ }^{\mathrm{TM}}$.
Probability: Enter $0 \leq p \leq 1$
This error is returned when an input for the probability in distributions is invalid.

## Statistics

This error is returned when a statistical or regression function is invalid. For example, when a calculation in 1-VAR or 2-VAR STATS is attempted with no defined data points.

Step size must not be 0
This error is returned when, in data, the STEP SIZE input is set to 0 in the SEQUENCE FILL function.

## Syntax

This error is returned when an expression contains misplaced functions, arguments, parentheses or commas.

```
TRIALS: Enter \(0 \leq n \leq 49\)
```

This error is returned in Binompdf and Binomcdf when the number of trials is out of range, $0 \leq n \leq 49$ in the case of ALL.

