

2021 May Mathematics: Applications and Interpretation SL TZ1 Examination Paper

Paper 1 Question 2

[Maximum mark: 4]

Deb used a thermometer to record the maximum daily temperature over ten consecutive days. Her results, in degrees Celsius ($^{\circ}\text{C}$), are shown below.

14, 15, 14, 11, 10, 9, 14, 15, 16, 13

For this data set, find the value of

- (a) the mode. [1]
 - (b) the mean. [2]
 - (c) the standard deviation. [1]
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Paper 1 Question 4

[Maximum mark: 7]

The price of gas at Leon's gas station is \$1.50 per litre. If a customer buys a minimum of 10 litres, a discount of \$5 is applied.

This can be modelled by the following function, L , which gives the total cost when buying a minimum of 10 litres at Leon's gas station.

$$L(x) = 1.50x - 5, \quad x \geq 10$$

where x is the number of litres of gas that a customer buys.

- (a) Find the total cost of buying 40 litres of gas at Leon's gas station. [2]
- (b) Find $L^{-1}(70)$. [2]

The price of gas at Erica's gas station is \$1.30 per litre. A customer must buy a minimum of 10 litres of gas. The total cost at Erica's gas station is cheaper than Leon's gas station when $x > k$.

- (c) Find the minimum value of k . [3]
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Paper 1 Question 6

[Maximum mark: 5]

Arriane has geese on her farm. She claims the mean weight of eggs from her black geese is less than the mean weight of eggs from her white geese.

She recorded the weights of eggs, in grams, from a random selection of geese. The data is shown in the table.

Weights of eggs from black geese	136	134	142	141	128	126
Weights of eggs from white geese	135	138	141	140	136	134

In order to test her claim, Arriane performs a t -test at a 10% level of significance. It is assumed that the weights of eggs are normally distributed and the samples have equal variances.

- (a) State, in words, the null hypothesis. [1]
 - (b) Calculate the p -value for this test. [2]
 - (c) State whether the result of the test supports Arriane's claim. Justify your reasoning. [2]
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Paper 1 Question 7 (partial)

[Maximum mark: 6]

Professor Wei observed that students have difficulty remembering the information presented in his lectures.

He modelled the percentage of information retained, R , by the function $R(t) = 100e^{-pt}$, $t \geq 0$, where t is the number of days after the lecture.

He found that 1 day after a lecture, students had forgotten 50% of the information presented.

- (a) Find the value of p . [2]
 - (b) Use this model to find the percentage of information retained by his students 36 hours after Professor Wei's lecture. [2]
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Paper 1 Question 8

[Maximum mark: 8]

Charlie and Daniella each began a fitness programme. On day one, they both ran 500m. On each subsequent day, Charlie ran 100m more than the previous day whereas Daniella increased her distance by 2% of the distance ran on the previous day.

(a) Calculate how far

(i) Charlie ran on day 20 of his fitness programme.

(ii) Daniella ran on day 20 of her fitness programme. [5]

On day n of the fitness programmes Daniella runs more than Charlie for the first time.

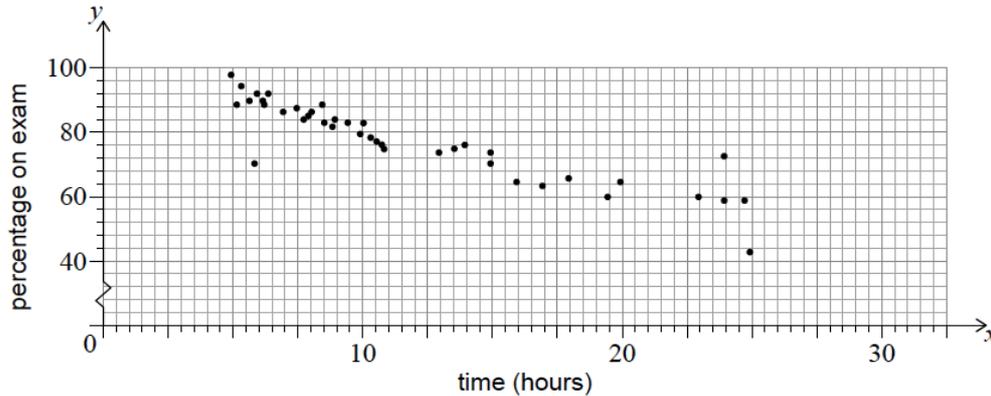
(b) Find the value of n . [3]

Paper 2 Question 1 (partial)

Mackenzie, a member of the sample, took 25 hours to read the novel. Jason believes Mackenzie's time is not an outlier.

- (d) Determine whether Jason is correct. Support your reasoning. [4]

For each student interviewed, Jason recorded the time taken to read *The Old Man and the Sea* (x), measured in hours, and paired this with their percentage score on the final exam (y). These data are represented on the scatter diagram.



- (e) Describe the correlation. [1]

Jason correctly calculates the equation of the regression line y on x for these students to be

$$y = -1.54x + 98.8.$$

He uses the equation to estimate the percentage score on the final exam for a student who read the book in 1.5 hours.

- (f) Find the percentage score calculated by Jason. [2]
 (g) State whether it is valid to use the regression line y on x for Jason's estimate. Give a reason for your answer. [2]

Jason found a website that rated the 'top 50' classic books. He randomly chose eight of these classic books and recorded the number of pages. For example, Book H is rated 44th and has 281 pages. These data are shown in the table.

Book	A	B	C	D	E	F	G	H
Number of pages (n)	4215	863	585	1225	366	209	624	281
Top 50 rating (t)	1	2	5	7	13	22	40	44

Jason intends to analyse the data using Spearman's rank correlation coefficient, r_s .

- (h) Copy and complete the information in the following table. [2]

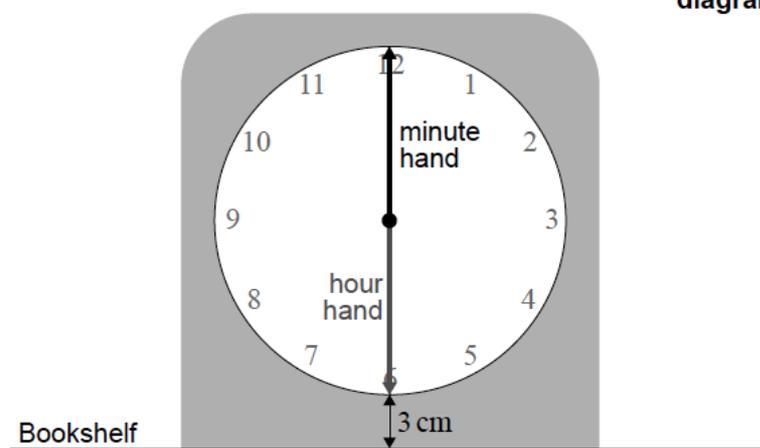
Book	A	B	C	D	E	F	G	H
Rank – Number of pages	1							
Rank – Top 50 Rating	1							

- (i) (i) Calculate the value of r_s .
 (ii) Interpret your result. [3]

Paper 2 Question 2 (partial)

A **second** clock is illustrated in the diagram below. The clock face has radius 10 cm with minute and hour hands both of length 10 cm. The time shown is 6:00 am. The bottom of the clock face is located 3 cm above a horizontal bookshelf.

diagram not to scale



- (f) Write down the height of the endpoint of the minute hand above the bookshelf at 6:00 am. [1]

The height, h centimetres, of the endpoint of the minute hand above the bookshelf is modelled by the function

$$h(\theta) = 10 \cos \theta + 13, \theta \geq 0,$$

where θ is the angle rotated by the minute hand from 6:00 am.

- (g) Find the value of h when $\theta = 160^\circ$. [2]

The height, g centimetres, of the endpoint of the **hour hand** above the bookshelf is modelled by the function

$$g(\theta) = -10 \cos \left(\frac{\theta}{12} \right) + 13, \theta \geq 0,$$

where θ is the angle in degrees rotated by the minute hand from 6:00 am.

- (h) Write down the amplitude of $g(\theta)$. [1]

The endpoints of the minute hand and hour hand meet when $\theta = k$.

- (i) Find the smallest possible value of k . [2]

Paper 2 Question 3 (partial)

[Maximum mark: 19]

Give your answers in parts (a), (d)(i), (e) and (f) to the nearest dollar.

Daisy invested 37 000 Australian dollars (AUD) in a fixed deposit account with an annual interest rate of 6.4% compounded **quarterly**.

(a) Calculate the value of Daisy's investment after 2 years. [3]

After m months, the amount of money in the fixed deposit account has appreciated to more than 50 000 AUD.

(b) Find the minimum value of m , where $m \in \mathbb{N}$. [4]

Daisy is saving to purchase a new apartment. The price of the apartment is 200 000 AUD.

Daisy makes an initial payment of 25% and takes out a loan to pay the rest.

(c) Write down the amount of the loan. [1]

The loan is for 10 years, compounded monthly, with equal monthly payments of 1700 AUD made by Daisy at the end of each month.

(d) For this loan, find

(i) the amount of interest paid by Daisy.

(ii) the annual interest rate of the loan. [5]

After 5 years of paying off this loan, Daisy decides to pay the **remainder** in one final payment.

(e) Find the amount of Daisy's final payment. [3]

(f) Find how much money Daisy saved by making one final payment after 5 years. [3]

Paper 2 Question 4

[Maximum mark: 13]

The stopping distances for bicycles travelling at 20 km h^{-1} are assumed to follow a normal distribution with mean 6.76 m and standard deviation 0.12 m .

- (a) Under this assumption, find, correct to four decimal places, the probability that a bicycle chosen at random travelling at 20 km h^{-1} manages to stop
- (i) in less than 6.5 m .
 - (ii) in more than 7 m .

[3]

1000 randomly selected bicycles are tested and their stopping distances when travelling at 20 km h^{-1} are measured.

- (b) Find, correct to four significant figures, the expected number of bicycles tested that stop between
- (i) 6.5 m and 6.75 m .
 - (ii) 6.75 m and 7 m .

[3]

The measured stopping distances of the 1000 bicycles are given in the table.

Measured stopping distance	Number of bicycles
Less than 6.5 m	12
Between 6.5 m and 6.75 m	428
Between 6.75 m and 7 m	527
More than 7 m	33

It is decided to perform a χ^2 goodness of fit test at the 5% level of significance to decide whether the stopping distances of bicycles travelling at 20 km h^{-1} can be modelled by a normal distribution with mean 6.76 m and standard deviation 0.12 m .

- (c) State the null and alternative hypotheses. [2]
- (d) Find the p -value for the test. [3]
- (e) State the conclusion of the test. Give a reason for your answer. [2]