# TI in Focus: AP® Calculus

2021 AP® Calculus Exam: AB-2
Particle Motion

Stephen Kokoska
Professor Emeritus, Bloomsburg University
Former AP® Calculus Chief Reader



Texas Instruments

# Outline

(1) Position of a particle at time $t$

(2) Displacement, or net distance traveled

(3) Total distance traveled

(4) Examples

## Background

(a) Fundamental Theorem of Calculus, Part 2:

If $f$ is continuous on $[a, b]$ and $F$ is any antiderivative of $f$, then,

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

(b) $F$ is an antiderivative of $f$: $F' = f$.

$$\int_a^b F'(x)\,dx = F(b) - F(a)$$

(c) **Net Change Theorem**

The definite integral of a rate of change $(F')$ may be interpreted as the net change in the original function $F$.

$$\int_a^b F'(x)\,dx = F(b) - F(a)$$

## A Closer Look

(1) Another way to interpret the definite integral $\int_a^b F'(x)\,dx$: this represents an *accumulation* of the change in $F$ over the interval $[a, b]$.

(2) We can rearrange the terms in the Net Change Theorem to provide an alternate interpretation and practical approach for solving many problems.

$$\underbrace{F(b)}_{\text{End amount}} = \underbrace{F(a)}_{\text{Start amount}} + \underbrace{\int_a^b F'(x)\,dx}_{\text{Net change}}$$

(3) This principle has many uses and can be applied to *all* rates of change in the natural and social sciences: water flowing over a dam or in a pipe, the concentration of a chemical, the mass of a segment of a rod with given linear density, rate of population growth, marginal cost.

## Particle Motion Problems

Suppose a particle moves along a horizontal line so that its position at time $t$ is given by $s(t)$, and therefore, its velocity at time $t$ is given by $v(t) = s'(t)$.

(1) Suppose the position of the particle at time $t = t_1$ is $s(t_1) = s_1$.

The position of the particle at time $t = t_2$ is

$$s(t_2) = s(t_1) + \int_{t_1}^{t_2} v(t)\,dt$$

(2) The net change in position, or displacement, of the particle over the time period from $t_1$ to $t_2$:

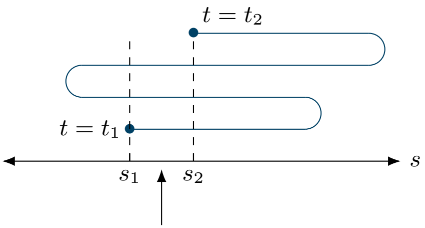
$$\int_{t_1}^{t_2} v(t)\,dt$$

(3) The total distance traveled by the particle from time $t = t_1$ to $t = t_2$ is:

$$\int_{t_1}^{t_2} |v(t)|\,dt$$

## Illustrations

Net change in position, or displacement.

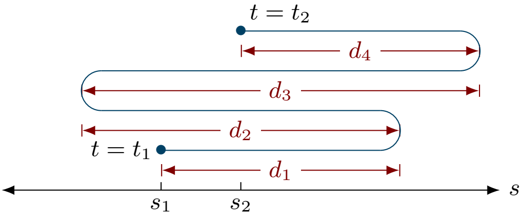



$$\int_{t_1}^{t_2} v(t)\,dt$$

$$s_2 = s_1 + \int_{t_1}^{t_2} v(t)\,dt$$

## Illustrations

Total distance traveled




$$\int_{t_1}^{t_2} |v(t)|\, dt = d_1 + d_2 + d_3 + d_4$$

### Example 1 Displacement versus Total Distance Traveled

A particle moves along a horizontal line so that its velocity at time $t$, $t \geq 0$ is given by $v(t) = t\sin\left(\sqrt{\dfrac{t^3}{3}}\right)$, where $v$ is measured in meters per second.

(a) Find the displacement of the particle during the time interval $[1, 6]$.

(b) Find the total distance traveled during this same time period.

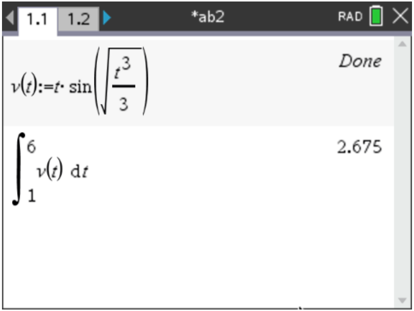
### Solution

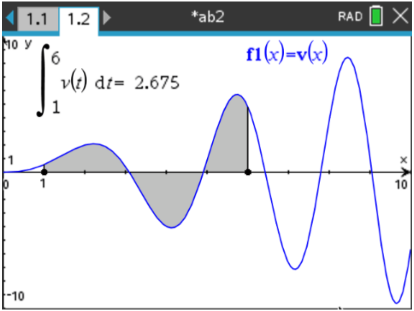
$$s(6) - s(1) = \int_1^6 v(t)\,dt \qquad \text{Net Change Theorem}$$

$$= 2.675 \qquad \text{Technology}$$

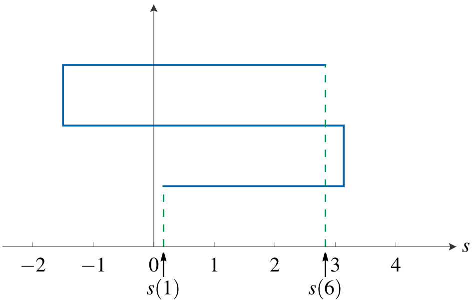
The particle's net change in position over the time interval $[1, 6]$ is 2.675 meters to the right.

# Example 1 (Continued)



## Example 1 (Continued)



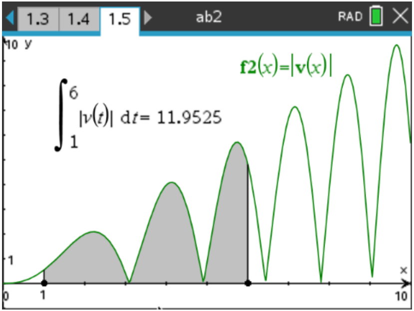

$s(6) - s(1) = 2.675$

## Example 1 (Continued)

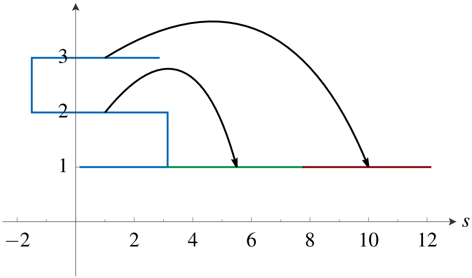
In order to find the total distance traveled, we need to integrate $v(t)$.

Analytically, we need to find the intervals on which $v(t) \geq 0$ and those on which $v(t) \leq 0$.

$$\int_1^6 |v(t)|\, dt = 11.953$$

## Example 1 (Continued)




$$\int_1^6 |v(t)|\, dt = 11.925$$

## Example 2 Particle Motion, Technology Required

A particle moves along a horizontal line. For $0 \leq t \leq 10$, the velocity of the particle is given by

$$v(t) = 2\cos\left(\frac{t^2}{10}\right) \cdot \ln(t^2 + 1)$$

The position of the particle is given by $s(t)$ and it is known that $s(0) = -4$.

(a) Write an expression involving an integral that gives the position $s(t)$. Use this expression to find the position of the particle at time $t = 4$.

(b) Find the first time the speed of the particle is 5.

(c) Find all the times in the interval $0 \leq t \leq 10$ at which the particle changes direction.

## Example 2 (Continued)

(a) The position of the particle at time $t$ is

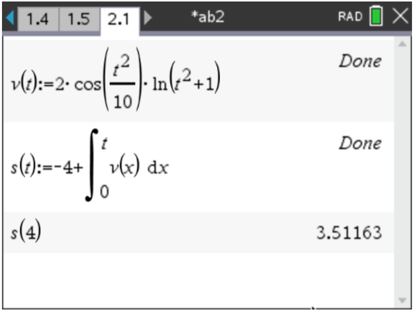
$$s(t) = s(0) + \int_0^t v(x)\,dx = -4 + \int_0^t v(t)\,dx$$

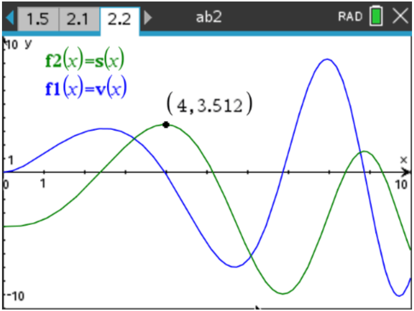
Net Change Theorem

The position of the particle at time $t = 4$ is

$$s(4) = -4 + \int_0^4 v(t)\,dx = 3.512$$

Technology



## Example 2 (Continued)

(b) The speed of the particle is the absolute value of the velocity.

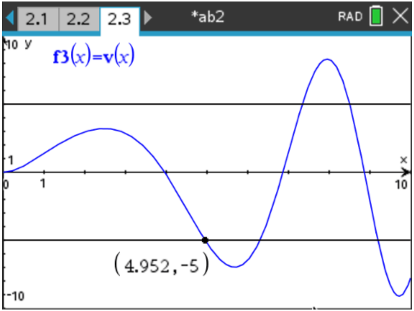
Solve the expression $|v(t)| = 5$ for $0 \leq t \leq 10$, and select the minimum value.

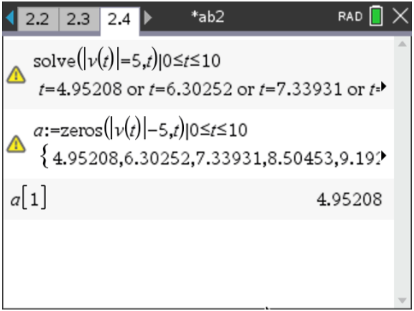
Use technology to graph $v$ and the horizontal lines at $5$ and $-5$.

(Or, graph $|v|$ and the horizontal line at 5.)

The graphs suggest there are several times at which the speed of the particle is 5.

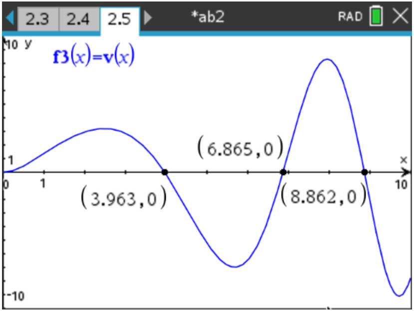
Use technology to find the first time the speed is 5.



## Example 2 (Continued)

(c) The particle changes direction when $v(t)$ changes from positive to negative, or negative to positive.




Can you find these values with the numeric solver?

Can you find these values analytically?

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