TI in Focus: AP® Calculus

2021 AP® Calculus Exam: AB-2

Technology Solutions and Problem Extensions

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Outline

2. A particle, P, is moving along the x-axis. The velocity of particle P at time t is given by $v_P(t) = \sin\left(t^{1.5}\right)$ for $0 \le t \le \pi$. At time $t = 0$, particle P is at position $x = 5$.

   A second particle, Q, also moves along the x-axis. The velocity of particle Q at time t is given by $v_Q(t) = (t - 1.8) \cdot 1.25^t$ for $0 \le t \le \pi$. At time $t = 0$, particle Q is at position $x = 10$.

   (a) Find the positions of particles P and Q at time $t = 1$.

   (b) Are particles P and Q moving toward each other or away from each other at time $t = 1$? Explain your reasoning.

   (c) Find the acceleration of particle Q at time $t = 1$. Is the speed of particle Q increasing or decreasing at time $t = 1$? Explain your reasoning.

   (d) Find the total distance traveled by particle P over the time interval $0 \le t \le \pi$.

(a) Find the positions of particles P and Q at time $t = 1$.

$x_P(1) = 5 + \int_0^1 v_P(t)\,dt = 5.370660$

At time $t = 1$, the position of particle P is $x = 5.371$ (or 5.370).

$x_Q(1) = 10 + \int_0^1 v_Q(t)\,dt = 8.564355$

At time $t = 1$, the position of particle Q is $x = 8.564$.

One definite integral	**1 point**
One position	**1 point**
The other position	**1 point**

(b) Are particles P and Q moving toward each other or away from each other at time $t = 1$? Explain your reasoning.

$v_P(1) = \sin\left(1^{1.5}\right) = 0.841471 > 0$ At time $t = 1$, particle P is moving to the right.	Direction of motion for one particle **1 point**
$v_Q(1) = (1 - 1.8) \cdot 1.25^1 = -1 < 0$ At time $t = 1$, particle Q is moving to the left. At time $t = 1$, $x_P(1) < x_Q(1)$, so particle P is to the left of particle Q. Thus, at time $t = 1$, particles P and Q are moving toward each other.	Answer with explanation **1 point**

(c) Find the acceleration of particle Q at time $t = 1$. Is the speed of particle Q increasing or decreasing at time $t = 1$? Explain your reasoning.

$a_Q(1) = v_Q'(1) = 1.026856$ The acceleration of particle Q is 1.027 (or 1.026) at time $t = 1$.	Setup and acceleration	**1 point**
$v_Q(1) = -1 < 0$ and $a_Q(1) > 0$ The speed of particle Q is decreasing at time $t = 1$ because the velocity and acceleration have opposite signs.	Speed decreasing with reason	**1 point**

(d) Find the total distance traveled by particle P over the time interval $0 \le t \le \pi$.

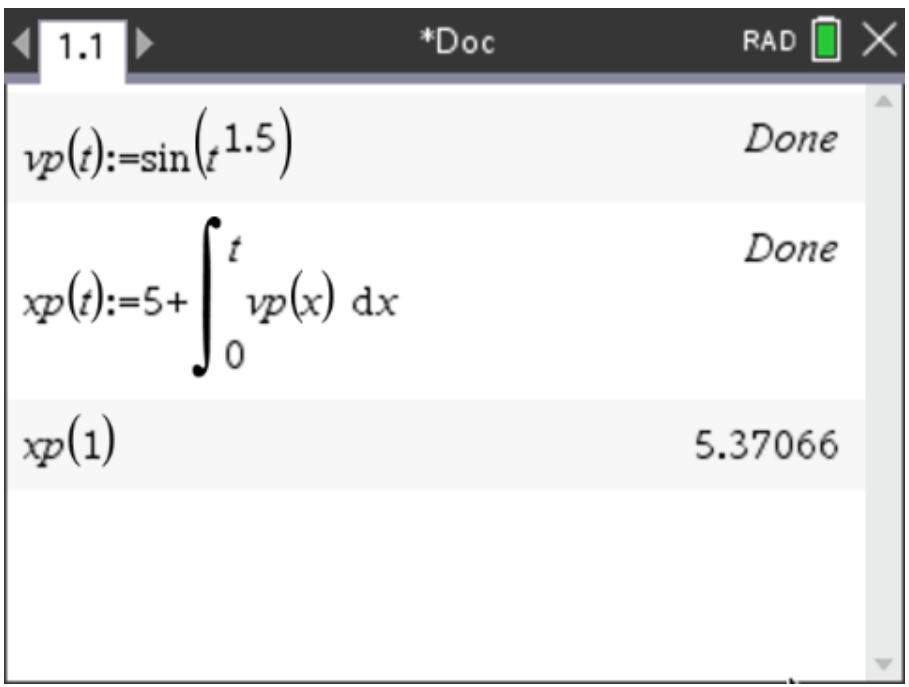
$\int_0^{\pi} \lvert v_P(t) \rvert \, dt = 1.93148$	Definite integral	**1 point**
Over the time interval $0 \le t \le \pi$, the total distance traveled by particle P is 1.931.	Answer	**1 point**

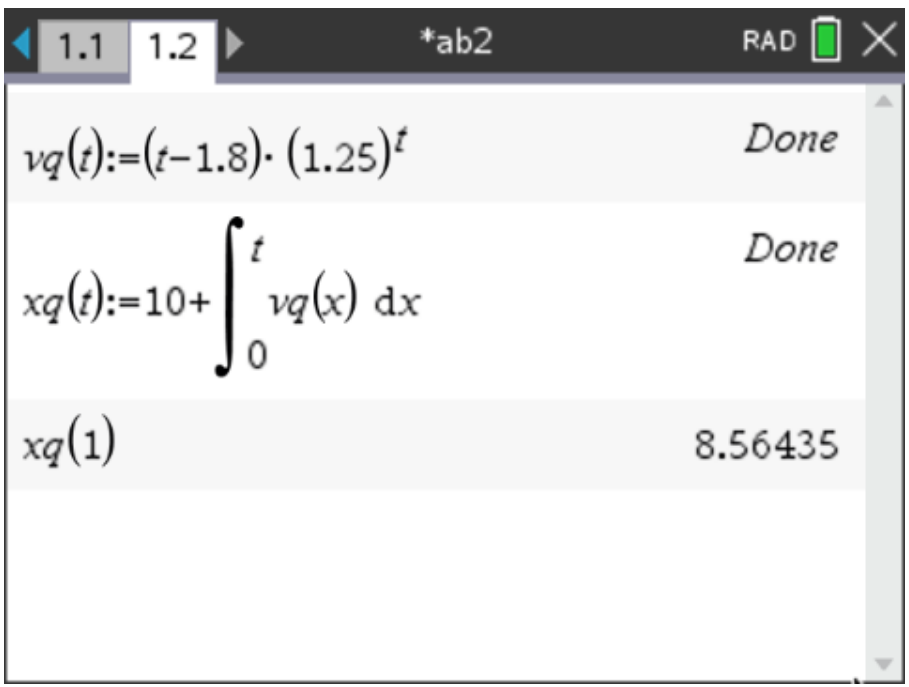
Part (a)

Two applications of the Net Change Theorem (or the FTC).

$$x_P(1) = x_P(0) + \int_0^1 v_P(t)\,dt = 5 + \int_0^1 v_P(t)\,dt = 5.371$$

$$x_Q(1) = x_Q(0) + \int_0^1 v_Q(t)\,dt = 10 + \int_0^1 v_Q(t)\,dt = 8.564$$





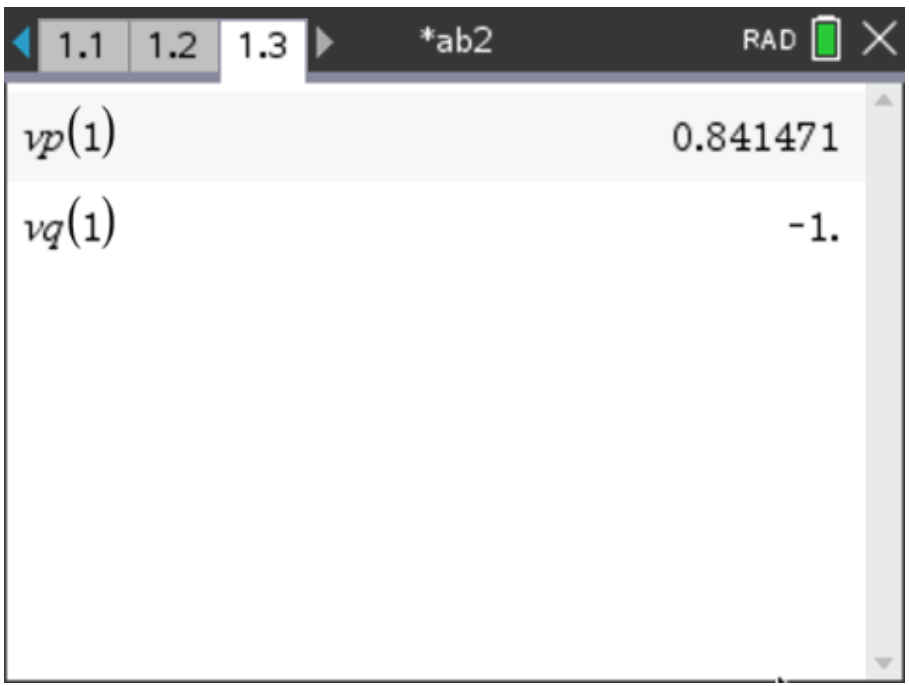
Part (b)

Consider the sign of the velocity of each particle at time $t = 1$.

$v_P(1) = \sin(t^{1.5}) = 0.841 > 0 \qquad v_Q(1) = (1 - 1.8) \cdot 1.25^1 = -1 < 0$

From part (a): at time $t = 1$, $x_P(1) < x_Q(1)$, particle P to the left of particle Q.

Particle P moving to the right; Particle Q moving to the left.
Therefore, particles P and Q are moving toward each other.



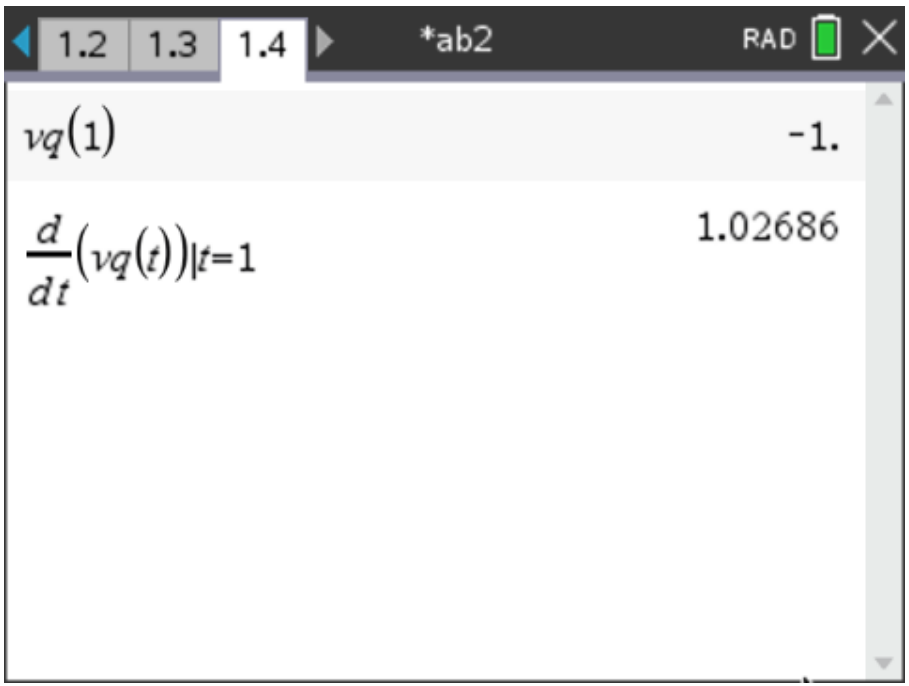
Part (c)

Acceleration is the derivative of the velocity with respect to time.

$a_Q(1) = v'_Q(1) = 1.027$

$a_Q(1) = 1.027 > 0 \quad \text{and} \quad v_Q(1) = -1 < 0$

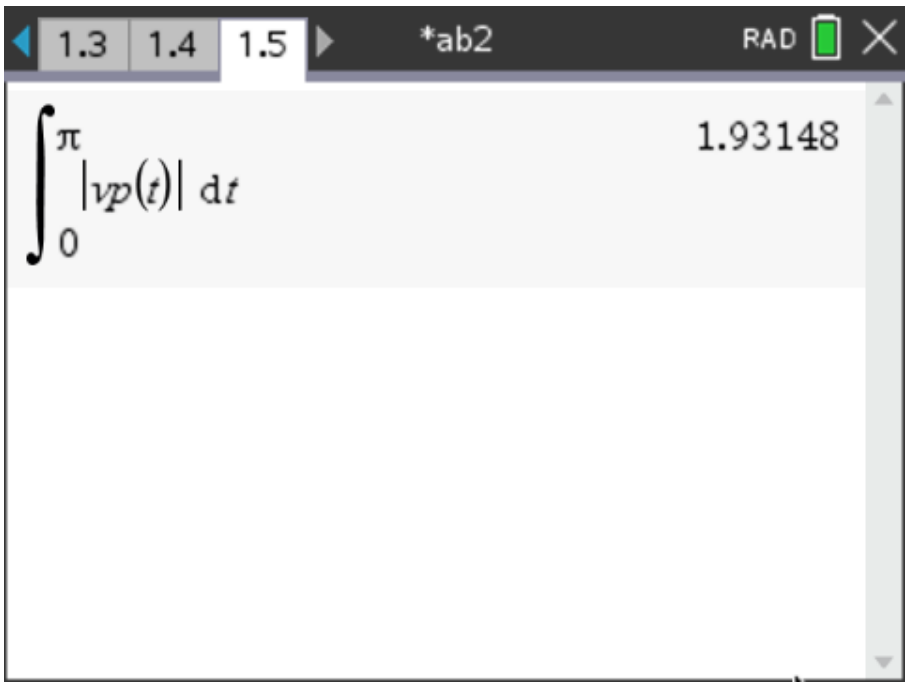
Therefore, the speed of the particle is decreasing at time $t = 1$.

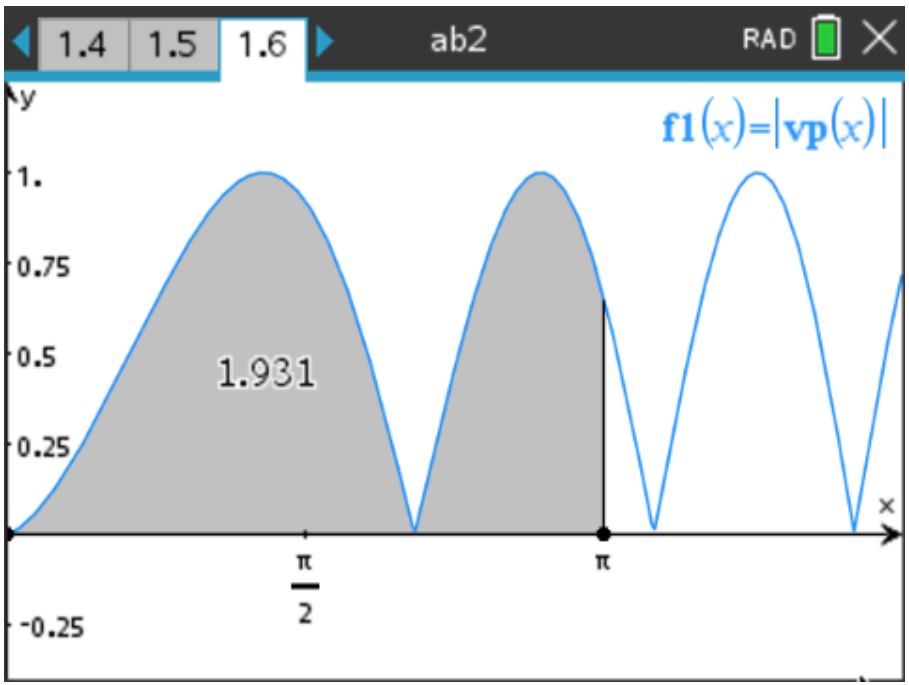


Part (d)

Total distance traveled by particle P over the time interval $[0, \pi]$.

$$\int_0^{\pi} |v_P(t)|\, dt = 1.931$$

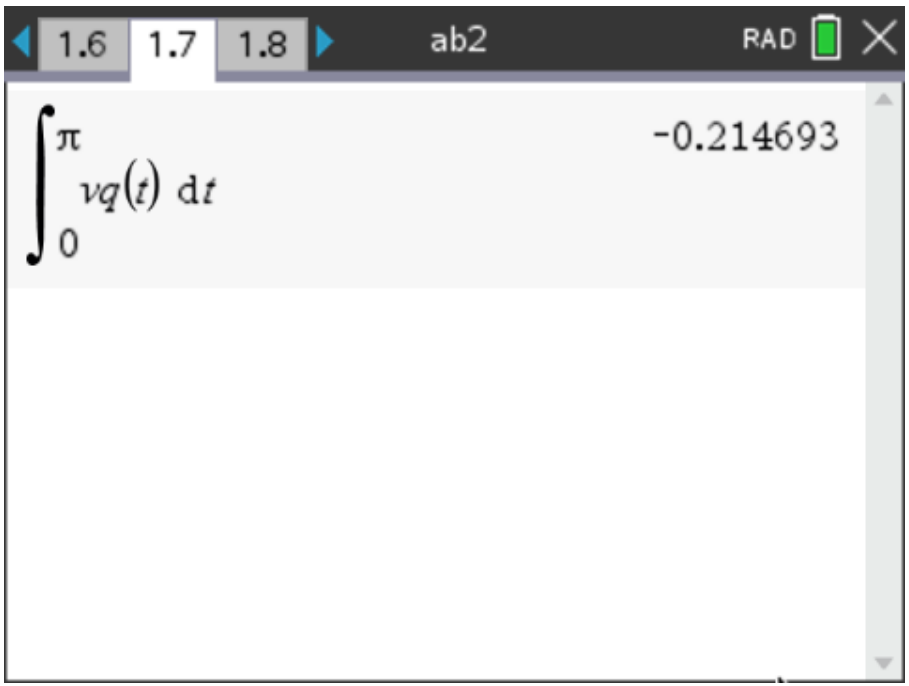


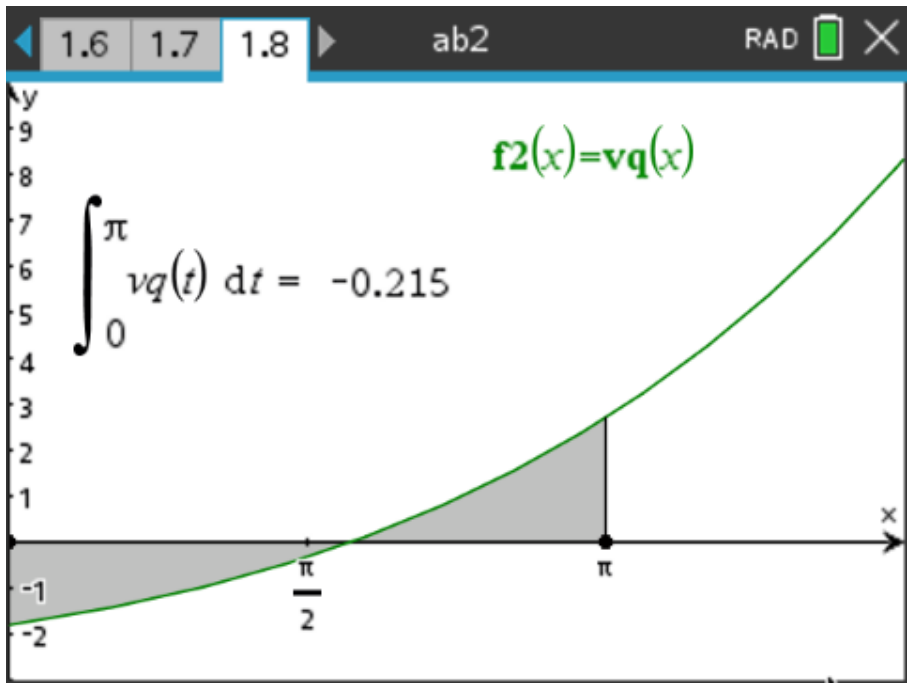


(e) Find the net distance traveled by the particle Q over the time interval $0 \le t \le \pi$.

Solution

$$\int_0^{\pi} v_Q(t)\,dt = -0.215$$

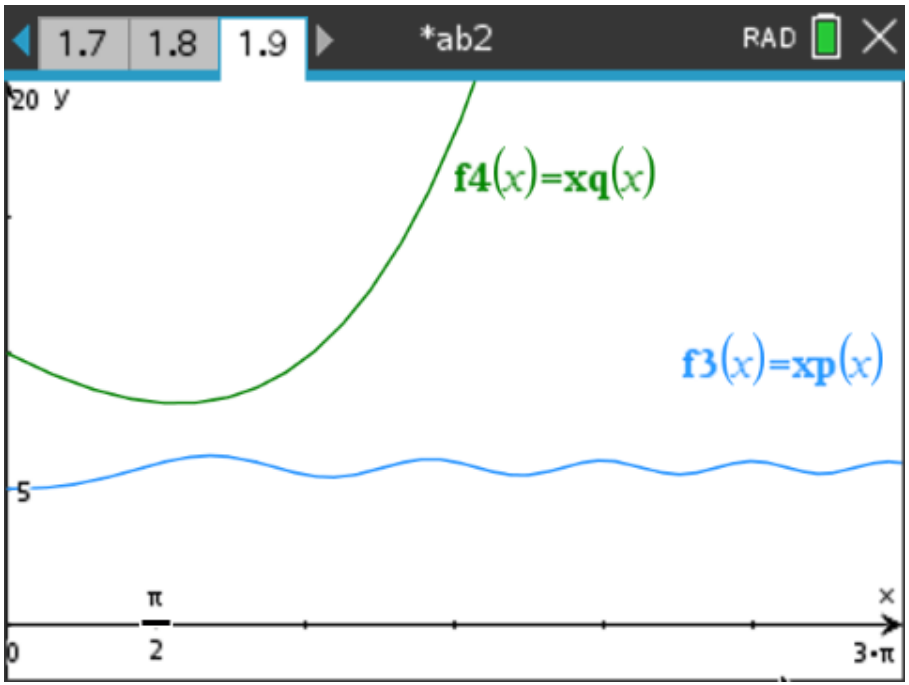




(f) Find the first time t at which the particles are at the same position.

Solution

Consider a graph of the position functions.



$$v_P(t) = \sin(t^{1.5})$$

$$v_Q(t) = (t - 1.8) \cdot 1.25^t = 0 \quad \Rightarrow \quad t = 1.8$$

Solution (Continued)

$v_Q(t) = (t - 1.8) \cdot 1.25^t$

For $0 \leq t < 1.8 : \ v_Q(t) < 0$ and for $t > 1.8 : \ v_Q(t) > 0$

Therefore, particle Q is farthest to the left when $t = 1.8$: $x_Q(1.8) = 8.139 > 7$

Particles P and Q are never at the same position at the same time.

Example 1 Particle Motion

A particle moves along a horizontal line. For $t \geq 0$ the velocity of the particle is given by $v(t) = e^{-(t/10)\cos(t^2/4)} - 1$, and the position of the particle is given by $s(t)$, a continuous, differentiable function. It is known that $s(0) = 4$.

(a) Find the position of the particle at time $t = 5$.

(b) How many times does the particle change direction on the interval $[0, 2\pi]$? Justify your answer.

(c) For $0 \leq t \leq 5$, find all the values of t for which the speed of the particle is 0.2.

(d) Is the speed of the particle increasing or decreasing at time $t = 5$? Justify your answer.

(e) Find the displacement of the particle and the total distance traveled by the particle over the time interval $[0, 2\pi]$.

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