TI in Focus: AP® Calculus

2020 Mock AP® Calculus Exam
BC-1: Solutions, Concepts, and Scoring Guidelines

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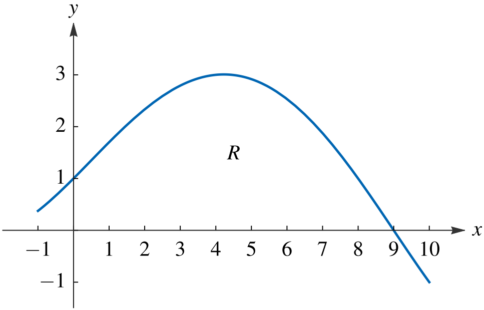
BC 1

The graph of g', the derivative of a twice-differentiable function g is shown in the figure. The graph has exactly one horizontal tangent line in the interval -1 to 10, at $x = 4.2$.



Graph of g'

R is the region in the first quadrant bounded by the graph of g' and the x-axis from $x = 0$ to $x = 9$. It is known that $g(0) = -7$, $g(9) = 12$, and $\int_0^9 g(x)\,dx = 27.6$.

(a) Find all values of x in the interval $-1 < x < 10$, if any, at which g has a critical point. Classify each critical point as the location of a relative minimum, relative maximum, or neither, Justify your answers.

Key Concepts

Definition

A **critical number**, or **critical point**, of a function f is a number c in the domain of f such that $f'(c) = 0$ or $f'(c)$ does not exist.

Note:

The word *number* makes it clear that these are x-values and not points in the plane.

The term *critical point* is consistent with AP Calculus language in questions and solutions.

Background

- Fermat's Theorem: If f has a local maximum or minimum at c, then c must be a critical number of f.
- Not every critical number indicates a local maximum or a minimum.
- We need a method to determine whether f has a local maximum, or a local minimum, or neither, at a critical number c.

The First Derivative Test

Suppose that c is a critical number of a continuous function f.

(a) If f' changes from positive to negative at c, then f has a local maximum at c.

(b) If f' changes from negative to positive at c, then f has a local minimum at c.

(c) If f' is positive to both the left and right of c, or negative to both the left and right of c, then f has neither a local maximum nor a local minimum at c.

Solution

$g'(x) = 0$: $\quad x = 9$

$g'(x)$ DNE: none

Therefore, g has a critical point at $x = 9$.

At $x = 9$, g has a relative maximum because $g'(x)$ changes from positive to negative there.

Scoring Guidelines

(a) $g'(x) = 0$: $x = 9$

$g'(x)$ DNE: none

g has a critical point at $x = 9$.

At $x = 9$, g has a relative maximum because $g'(x)$ changes from positive to negative there.

$$2: \begin{cases} 1 : \text{critical point at } x = 9 \\ 1 : \text{relative maximum with} \\ \quad\;\; \text{justification} \end{cases}$$

Scoring Notes

- The response must identify $x = 9$, and no other x values.
- An ordered pair may be presented, but if so, then the y-coordinate must be correct.
- The following responses earn the first point.

  $x = 9$

  $(9, 12)$
- The following responses do not earn the first point.

  $(9, 0)$

  $x = 0,\ 4.2,\ 9$
- The justification must discuss g' changing from positive to negative at $x = 9$.
- The point cannot be earned for simply stating g' changes sign, or changes, at $x = 9$.
- References to *the slope* or *the derivative* are too ambiguous, and do not earn the point.

(b) How many points of inflection does the graph of g have on the interval $-1 < x < 10$? Give a reason for your answer.

Key Concepts

Concavity Test

(a) If $f''(x) > 0$ for all x in I, then the graph of f is concave up on I.

(b) If $f''(x) < 0$ for all x in I, then the graph of f is concave down on I.

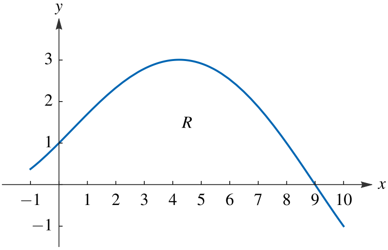
Definition

A point P on the graph of f is called an **inflection point** if f is continuous there and the graph changes from concave up to concave down or concave down to concave up at P.

A Closer Look

1. $f''(x)$ can change sign only when $f''(x) = 0$ or when $f''(x)$ DNE.
2. A sign chart can be used to help determine where $f''(x)$ changes sign.
3. $f''(x)$ changes sign where f' changes from increasing to decreasing, or decreasing to increasing.

Solution



Graph of g'

g' changes from increasing to decreasing at $x = 4.2$.

There is one point of inflection on the graph of g.
It occurs at the point where $x = 4.2$.

Scoring Guidelines

(b) The graph of g has a point of inflection where g' changes from increasing to decreasing or from decreasing to increasing.

g' changes from increasing to decreasing at $x = 4.2$.

Therefore the graph of g has one point of inflection at the point where $x = 4.2$.

$$2: \begin{cases} 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$$

Scoring Notes

- We are looking for the application of the relationship between the increasing and decreasing behavior of the first derivative and the inflection points on the graph of g.
- The first point is earned for *one*, with some relevant mathematical discussion.
- The reason point is earned for correctly appealing to the increasing and then decreasing behavior of g'.
- g'' changes sign there: earns the reason point.

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