TI in Focus: AP® Calculus

2019 AP® Calculus Exam: AB-6

The Squeeze Theorem

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Outline

(1) The Squeeze Theorem

(2) Application

(3) Examples

Properties of Limits

Theorem

If $f(x) \leq g(x)$ for all x near a, except possibly at a, and the limits of f and g both exist as x approaches a, then

$$\lim_{x \to a} f(x) \leq \lim_{x \to a} g(x)$$

The Squeeze Theorem

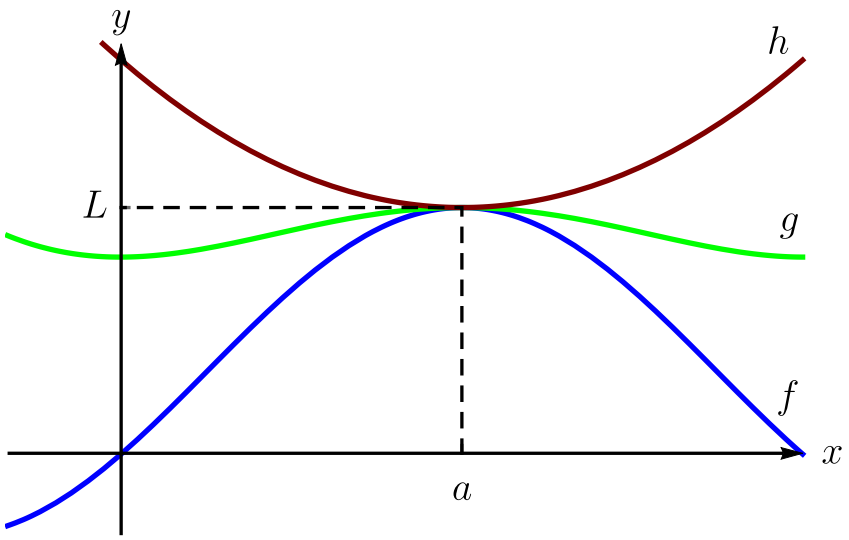
If $f(x) \leq g(x) \leq h(x)$ for x near a, except possibly at a, and

$$\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L$$

then

$$\lim_{x \to a} g(x) = L$$

Squeeze Theorem Illustration



If $g(x)$ is squeezed between $f(x)$ and $h(x)$ near a, and if f and h have the same limit L at a, then g is *squeezed* to the same limit L at a.

Example 1 Squeeze Me

Show that $\lim_{x \to 0} x^2 \sin\left(\frac{1}{x}\right) = 0$

Solution

Direct substitution doesn't work: $\sin\left(\frac{1}{x}\right)$ is undefined at $x = 0$

The Product Law for Limits cannot be used: $\lim_{x \to 0} \sin\left(\frac{1}{x}\right)$ DNE

Try the Squeeze Theorem:

Search for a function f less than $g(x) = x^2 \sin\left(\frac{1}{x}\right)$;

And a function h greater than g as x approaches 0.

Solution (Continued)

$$-1 \leq \sin\left(\frac{1}{x}\right) \leq 1$$

Sine of any number is between -1 and 1

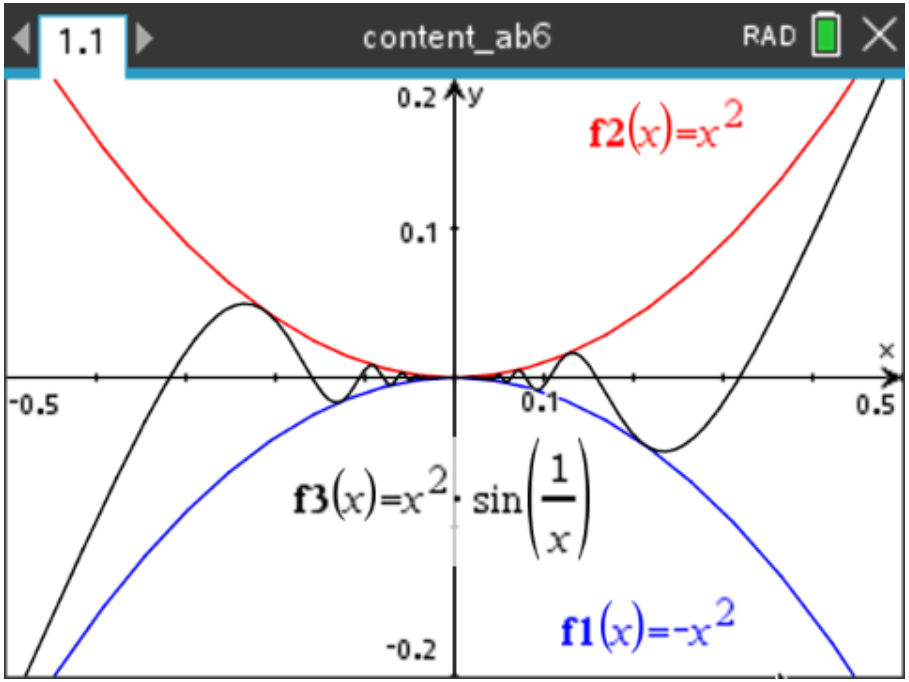
$$-x^2 \leq x^2 \sin\left(\frac{1}{x}\right) \leq x^2$$

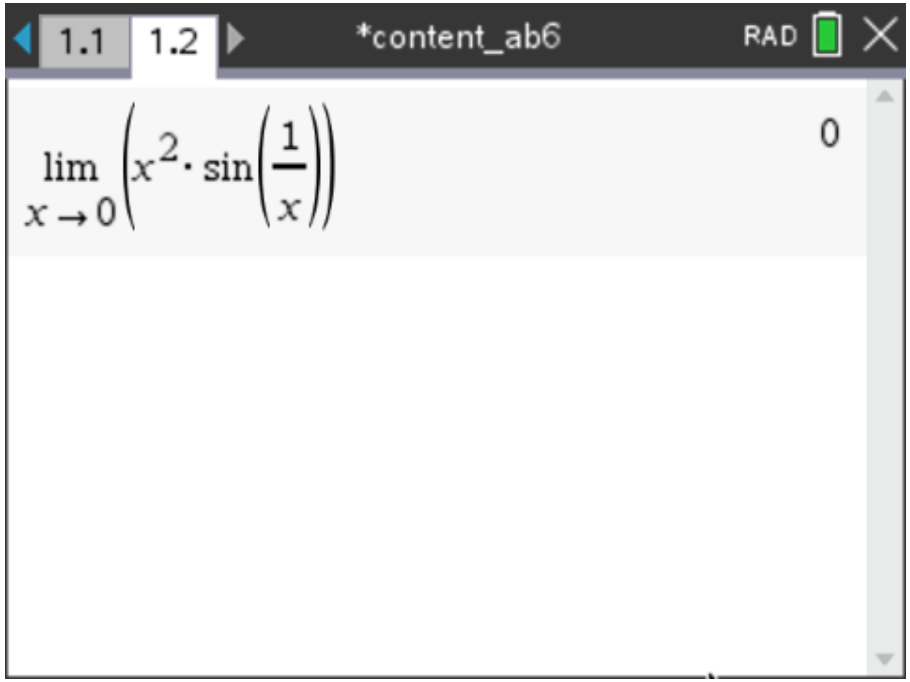
Multiply all parts of inequality by a positive number

$$\lim_{x \to 0}(-x^2) = \lim_{x \to 0} x^2 = 0$$

Therefore, $\lim_{x \to 0} x^2 \sin\left(\frac{1}{x}\right) = 0$

Illustration





Example 2 Applications

(a) Let $f(x) = -x^2$, $g(x) = x^2 \cos(20\pi x)$, and $h(x) = x^2$.
Use the Squeeze Theorem to show that $\lim_{x \to 0} g(x) = 0$.
Use technology to illustrate this result.

(b) If $3x - 5 \leq f(x) \leq x^2 - 2x + 2$ for $x \geq 0$, find $\lim_{x \to 3} f(x)$.

(c) Find $\lim_{x \to 0^+} \sqrt{x}\, e^{\sin(\pi/x)}$.

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