

TI in Focus: AP[®] Calculus

2019 AP[®] Calculus Exam: AB-5

Technology Solutions and Problem Extensions

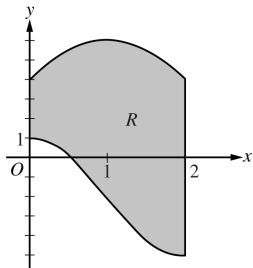
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Outline

- (1) Free Response Question
- (2) Scoring Guidelines
- (3) Solutions in greater detail
- (4) Solutions using technology
- (5) Problem Extensions



5. Let R be the region enclosed by the graphs of $g(x) = -2 + 3\cos\left(\frac{\pi}{2}x\right)$ and $h(x) = 6 - 2(x - 1)^2$, the y -axis, and the vertical line $x = 2$, as shown in the figure above.

(a) Find the area of R .

(b) Region R is the base of a solid. For the solid, at each x the cross section perpendicular to the x -axis has

area $A(x) = \frac{1}{x+3}$. Find the volume of the solid.

(c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 6$.

$$\begin{aligned}
 \text{(a)} \quad \int_0^2 (h(x) - g(x)) \, dx &= \int_0^2 \left((6 - 2(x-1)^2) - (-2 + 3\cos(\frac{\pi}{2}x)) \right) dx \\
 &= \left[\left(6x - \frac{2}{3}(x-1)^3 \right) - \left(-2x + \frac{6}{\pi}\sin(\frac{\pi}{2}x) \right) \right]_{x=0}^{x=2} \\
 &= \left(\left(12 - \frac{2}{3} \right) - (-4 + 0) \right) - \left(\left(0 + \frac{2}{3} \right) - (0 + 0) \right) \\
 &= 12 - \frac{2}{3} + 4 - \frac{2}{3} = \frac{44}{3}
 \end{aligned}$$

The area of R is $\frac{44}{3}$.

$$4 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{antiderivative of } 3\cos\left(\frac{\pi}{2}x\right) \\ 1 : \text{antiderivative of} \\ \quad \text{remaining terms} \\ 1 : \text{answer} \end{cases}$$

$$\begin{aligned}
 \text{(b)} \quad \int_0^2 A(x) \, dx &= \int_0^2 \frac{1}{x+3} \, dx \\
 &= [\ln(x+3)]_{x=0}^{x=2} = \ln 5 - \ln 3
 \end{aligned}$$

The volume of the solid is $\ln 5 - \ln 3$.

$$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$$

$$\text{(c)} \quad \pi \int_0^2 ((6 - g(x))^2 - (6 - h(x))^2) \, dx$$

$$3 : \begin{cases} 1 : \text{limits and constant} \\ 1 : \text{form of integrand} \\ 1 : \text{integrand} \end{cases}$$

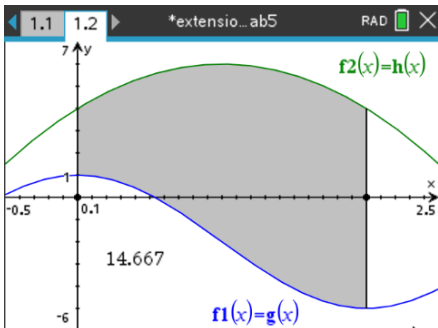
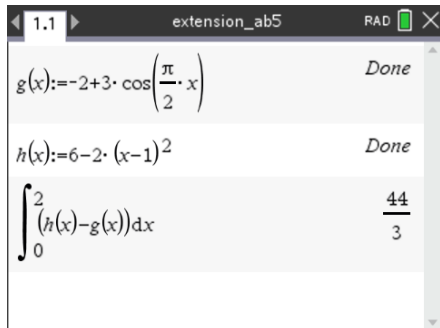


Part (a)

Solution

$$\begin{aligned}\int_0^2 (y_H - y_L) dx &= \int_0^2 [h(x) - g(x)] dx \\&= \int_0^2 \left[(6 - (2(x - 1))^2) - \left(-2 + 3 \cos \left(\frac{\pi}{2} x \right) \right) \right] dx \\&= \left[\left(6x - \frac{2}{3}(x - 1)^3 \right) - \left(-2x + \frac{6}{\pi} \sin \left(\frac{\pi}{2} x \right) \right) \right]_0^2 \\&= \left[\left(12 - \frac{2}{3} \right) - (-4 + 0) \right] - \left[\left(0 + \frac{2}{3} \right) - (0 + 0) \right] \\&= 12 - \frac{2}{3} + 4 - \frac{2}{3} = \frac{44}{3}\end{aligned}$$

Technology Solution



Example 1 More with R

Let R be the region described in the Free Response Question.

- (a) Find the value a such that the line $y = a$ divides the region R into two regions of equal area.
- (b) Find the value b such that the line $x = b$ divides the region R into two regions of equal area.
- (c) Find the value m such that the line $y = mx + 1$ divides the region R into two regions of equal area.
- (d) Find the point of intersection, (A, B) , of the graphs $y = g(x)$ and $y = h(x)$ in the fourth quadrant. Find the area of the region bounded by the graphs of $y = g(x)$, $y = h(x)$, the y -axis, and the line $x = A$.

Part (b)

Solution

$$\int_0^2 A(x) \, dx = \int_0^2 \frac{1}{x+3} \, dx$$

$$u = x + 3 \quad u(0) = 3$$

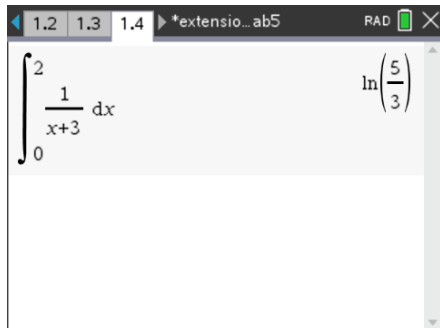
$$du = dx \quad u(2) = 5$$

$$= \int_3^5 \frac{1}{u} \, du$$

$$= \ln |u| \Big|_3^5$$

$$= \ln 5 - \ln 3 = \ln(5/3)$$

Technology Solution



A TI-84 Plus CE calculator screen showing the definite integral of $\frac{1}{x+3}$ from 0 to 2. The integral is displayed on the left, and the result, $\ln\left(\frac{5}{3}\right)$, is displayed on the right. The calculator interface includes a top menu bar with options 1.2, 1.3, and 1.4, a title bar with the text '*extensio...ab5', and a status bar with 'RAD' and a battery icon.

$$\int_0^2 \frac{1}{x+3} dx = \ln\left(\frac{5}{3}\right)$$

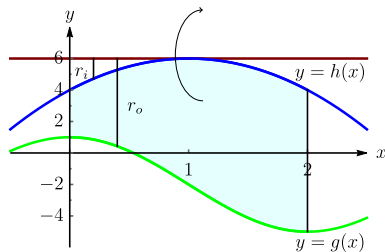
Example 2 Various Cross-Sections

Region R is the base of a solid.

- (a) For solid S_1 , the cross-sections perpendicular to the x -axis are squares. Find the volume of S_1 .
- (b) For solid S_2 , the cross-sections perpendicular to the x -axis are semicircle with diameters extending from $y = g(x)$ to $y = h(x)$. Find the volume of S_2 .
- (c) For solid S_3 , the cross-sections perpendicular to the x -axis are equilateral triangles. Find the volume of S_3 .

Part (c)

Solution



$$V = \pi \int_0^2 [r_o^2 - r_i^2] dx = \pi \int_0^2 [(6 - g(x))^2 - (6 - h(x))^2] dx$$

Part (c)

Technology Solution

A TI-84 Plus calculator screen showing the evaluation of a definite integral. The top of the screen has a navigation bar with buttons for 1.3, 1.4, and 1.5, and a text field containing '*extensio...ab5'. To the right of the navigation bar, it says 'RAD' with a green icon and a close button. The main display area shows the integral expression:
$$\pi \cdot \int_0^2 \left((6-g(x))^2 - (6-h(x))^2 \right) dx$$
 and the result:
$$\frac{677 \cdot \pi}{5}$$

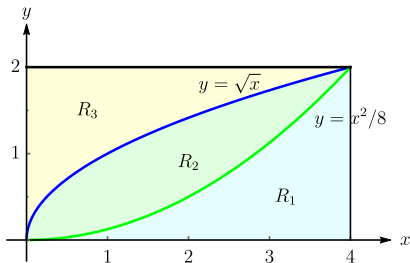
Example 3 Even More with R

Let R be the region described in the Free Response Question.

- (a) Let S_1 be the solid generated when R is rotated about the horizontal line $y = -6$. Find the volume of S_1 .
- (b) Let S_2 be the solid generated when R is rotated about the horizontal line $y = k$ ($k > 6$). Find the value of k such that the volume of S_2 is $\frac{3461\pi}{15}$.

Example 4 Calculus Regions

Three regions are defined in the figure. Find the volume generated by rotating the given region about the specified line.



- | | |
|----------------------------------|----------------------------------|
| (a) R_1 about the x -axis | (b) R_1 about the line $y = 2$ |
| (c) R_1 about the line $x = 4$ | (d) R_1 about the y -axis |
| (e) R_3 about the x -axis | (f) R_3 about the y -axis |
| (g) R_3 about the line $x = 4$ | (h) R_3 about the line $y = 2$ |
| (i) R_2 about the x -axis | (j) R_2 about the y -axis |
| (k) R_2 about the line $y = 2$ | (l) R_2 about the line $x = 4$ |

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