TI in Focus: AP® Calculus

2019 AP® Calculus Exam: AB-5
Scoring Guidelines

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Outline

(1) Free Response Question

(2) Scoring Guidelines

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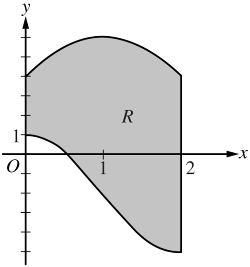

5. Let R be the region enclosed by the graphs of $g(x) = -2 + 3\cos\left(\frac{\pi}{2}x\right)$ and $h(x) = 6 - 2(x-1)^2$, the y-axis, and the vertical line $x = 2$, as shown in the figure above.

   (a) Find the area of R.

   (b) Region R is the base of a solid. For the solid, at each x the cross section perpendicular to the x-axis has area $A(x) = \frac{1}{x+3}$. Find the volume of the solid.

   (c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 6$.

(a) $$\int_0^2 (h(x) - g(x))\,dx = \int_0^2 \left(\left(6 - 2(x-1)^2\right) - \left(-2 + 3\cos\left(\frac{\pi}{2}x\right)\right)\right) dx$$

$$= \left[\left(6x - \frac{2}{3}(x-1)^3\right) - \left(-2x + \frac{6}{\pi}\sin\left(\frac{\pi}{2}x\right)\right)\right]_{x=0}^{x=2}$$

$$= \left(\left(12 - \frac{2}{3}\right) - (-4+0)\right) - \left(\left(0 + \frac{2}{3}\right) - (0+0)\right)$$

$$= 12 - \frac{2}{3} + 4 - \frac{2}{3} = \frac{44}{3}$$

The area of R is $\frac{44}{3}$.

4 :
- 1 : integrand
- 1 : antiderivative of $3\cos\left(\frac{\pi}{2}x\right)$
- 1 : antiderivative of remaining terms
- 1 : answer

(b) $$\int_0^2 A(x)\,dx = \int_0^2 \frac{1}{x+3}\,dx$$

$$= \left[\ln(x+3)\right]_{x=0}^{x=2} = \ln 5 - \ln 3$$

The volume of the solid is $\ln 5 - \ln 3$.

2 :
- 1 : integral
- 1 : answer

(c) $$\pi\int_0^2 \left((6 - g(x))^2 - (6 - h(x))^2\right) dx$$

3 :
- 1 : limits and constant
- 1 : form of integrand
- 1 : integrand

Student Performance

Part (a)

- Most students understood the necessary concept, but algebra and arithmetic errors.
- Errors in finding the antiderivative of a composite function: $3\cos(kx)$.
- Some students: copy errors, misidentified $g(x)$ and $h(x)$, expanded the quadratic.
- Some students reversed the integrand which led to a negative value for area.

Part(b)

- Some students did know how to use $A(x)$.
- Some students included an incorrect constant multiple: π, $\frac{44}{3}$
- Some students presented incorrect integrands: $[h(x) - g(x)]A(x)$, $A(x)^2$

Student Performance

Part(c)

- Some students omitted the constant π or made a presentation error involving π.
- Some students did not use $y = 6$ as the axis of revolution.
- Some students presented functions of the variable y in the integrand.
- General presentation errors if explicit expressions used, many involving parentheses.

General Comments

(1) The functions g and h are defined in the problem. Students can, and should, use $g(x)$ and $h(x)$ where appropriate.

(2) The function A is also defined in part (b). Students can use $A(x)$ where appropriate.

(3) If unambiguous, $\cos \frac{\pi}{2}x$ is read as $\cos\left(\frac{\pi}{2}x\right)$.

(4) Missing dx allowed (not in my class).

(5) Bald answers do not earn points.

(6) There were some common simplified expressions for $h(x) - g(x)$ and $g(x) - h(x)$.

Part (a)

- Bald answers or bald integrands: 0 - 0 - 0 - 0

  Examples: $\frac{44}{3}$; $\quad h(x) - g(x)$
- Limits of integration are considered in the 4th point (the answer point).
- Special case: limits of integration $x = 1$ to $x = 2$

Part (a) 1: integrand

(1) Point is earned for $h(x) - g(x)$ or $g(x) - h(x)$ in the context of integration.

(2) Reversal earns the point, but must be resolved to earn the 4th point.

(3) Copy errors: missing or incorrect constants, missing 3.
Not eligible for 4th point.

(4) Simplification or arithmetic errors in the integrand: eligible for 2nd and 3rd points, but not 4th.

Part (a) 1: integrand

Notes

(1) Presents $h(x) + g(x)$: Only eligible for 2nd and 3rd points; 0 - ? - ? - 0

(2) Presents an integrand with other functions: 0 - 0 - 0 - 0

Part (a) 1: antiderivative of $3\cos\left(\frac{\pi}{2}x\right)$

(1) If no copy error, point is earned for the correct antiderivative.

(2) Only copy error read if missing 3; $\cos\left(\frac{\pi}{2}x\right)$.
Not eligible for 4th point.

(3) If u-substitution, must see $u = \frac{\pi}{2}x$.

Part (a) 1: antiderivative of remaining terms

(1) Read for the antiderivative of all remaining terms

(2) If u-substitution, must see $u = x - 1$ and $\frac{1}{3}u^3$.

Part (a) 1: answer

Eligibility:

- First point for the integrand.
- And one of the 2nd or 3rd points (antiderivatives).
- And no copy errors, simplification or arithmetic errors in the integrand.

Special cases:

- Responses with no integral symbol: eligible for all 4 points.
- Responses with no antiderivative of $3\cos\left(\frac{\pi}{2}x\right)$:

  Must see antiderivative of all remaining terms. If eligible, answer must be 44/3.

Part (b) 1: integral

- Integrand must be $A(x)$ and must use 0 and 2 as limits.

Part (b) 1: answer

- Must be correct.

Notes

(1) Missing dx allowed.

(2) Bald answer: 0 - 0

(3) The limits may appear late.

(4) Integral point is banked.

(5) Redefining $A(x)$ does not earn the point. For example,

$$\int_0^2 A(x)\,dx = \int_0^2 \frac{1}{h(x) - g(x)}\,dx$$

Part (b) Examples

(1) $\ln 5 - \ln 3; \quad \ln(5/3)$ 0 - 0

(2) $\displaystyle\int A(x)\,dx; \quad \int \frac{1}{x+3}\,dx$ 0 - 0

(3) $\displaystyle\int A(x)\,dx = \ln 5 - \ln 3$ 0 - 1

(4) $\displaystyle u = x+3, \quad \int_0^2 \frac{1}{u}\,du = \ln 5 - \ln 3$ 0 - 1

(5) $\displaystyle u = x+3, \quad \int_3^5 \frac{1}{u}\,du = \ln 5 - \ln 3$ 1 - 1

Part (b) Examples

(1) $k\int_0^2 A(x)\,dx, \quad k \neq 1$ 1 - 0

(2) $k \pm \int_0^2 A(x)\,dx, \quad k \neq 0$ 1 - 0

(3) $\ln(x+3) = \ln 5 - \ln 3$ 0 - 0

(4) $\ln(x+3)\Big|_0^2 = \ln 5 - \ln 3$ 1 - 1

Part (c) 1: limits and constant

(1) No bald answers.

(2) Must be correct limits and constant.

(3) If there are any constants added or subtracted to the integral: point is not earned.

Common error: $k \pm \pi \int_0^2 [g(x)^2 - h(x)^2] \, dx$

(4) Parentheses issues.

Part (c) 1: form of integrand

(1) Only acceptable form: difference of squares with correct or consistent axis of rotation.

Forms that earned the point (expected integrand):

$[k - g(x)]^2 - [k - h(x)]^2$ or $[g(x) - k]^2 - [h(x) - k]^2$

Forms that earned the point (expected integrand multiplied by -1):

$[k - h(x)]^2 - [k - g(x)]^2$ or $[h(x) - k]^2 - [g(x) - k]^2$

(2) If one term is completely correct and the other has a single missing parenthesis: 2nd point earned.

Not eligible for the 3rd point.

Part (c) Special Cases

(1) Explicitly see $6+$ and $6-$:

$[6-g(x)]^2 - [6+h(x)]^2$ or $[6+g(x)]^2 - [6-h(x)]^2$

(Or one of these multiplied by -1)

Earns the second point but is not eligible for the 3rd point.

(2) Response begins with a simplified integrand:

Simplification must be correct (or $\times -1$) to earn 2nd point.

Correct simplified integrand: $\left[-8+3\cos\left(\frac{\pi}{2}x\right)\right]^2 - \left[-2(x-1)^2\right]^2$

Correct simplified integrand $\times -1$: $\left[-3\cos\left(\frac{\pi}{2}x\right)+8\right]^2 - \left[2(x-1)^2\right]^2$

Part (c) 1: integrand

Must be the correct integrand.

Reversed integrand does not earn this point.

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