TI in Focus: AP® Calculus

2019 AP® Calculus Exam: AB-5
Technology Solutions and Problem Extensions

Stephen Kokoska
Professor, Bloomsburg University
Former AP® Calculus Chief Reader

Outline

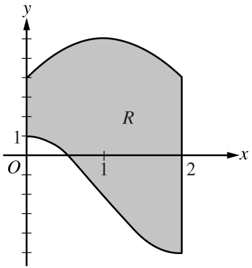
(1) Free Response Question

(2) Scoring Guidelines

(3) Solutions in greater detail

(4) Solutions using technology

(5) Problem Extensions



5. Let R be the region enclosed by the graphs of $g(x) = -2 + 3\cos\left(\frac{\pi}{2}x\right)$ and $h(x) = 6 - 2(x-1)^2$, the y-axis, and the vertical line $x = 2$, as shown in the figure above.

(a) Find the area of R.

(b) Region R is the base of a solid. For the solid, at each x the cross section perpendicular to the x-axis has area $A(x) = \dfrac{1}{x+3}$. Find the volume of the solid.

(c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 6$.

(a) $\int_0^2 (h(x) - g(x))\, dx = \int_0^2 \left(\left(6 - 2(x-1)^2\right) - \left(-2 + 3\cos\left(\frac{\pi}{2}x\right)\right)\right) dx$

$$= \left[\left(6x - \frac{2}{3}(x-1)^3\right) - \left(-2x + \frac{6}{\pi}\sin\left(\frac{\pi}{2}x\right)\right)\right]_{x=0}^{x=2}$$

$$= \left(\left(12 - \frac{2}{3}\right) - (-4 + 0)\right) - \left(\left(0 + \frac{2}{3}\right) - (0 + 0)\right)$$

$$= 12 - \frac{2}{3} + 4 - \frac{2}{3} = \frac{44}{3}$$

The area of R is $\frac{44}{3}$.

$4: \begin{cases} 1: \text{integrand} \\ 1: \text{antiderivative of } 3\cos\left(\frac{\pi}{2}x\right) \\ 1: \text{antiderivative of remaining terms} \\ 1: \text{answer} \end{cases}$

(b) $\int_0^2 A(x)\, dx = \int_0^2 \frac{1}{x+3}\, dx$

$$= \left[\ln(x+3)\right]_{x=0}^{x=2} = \ln 5 - \ln 3$$

The volume of the solid is $\ln 5 - \ln 3$.

$2: \begin{cases} 1: \text{integral} \\ 1: \text{answer} \end{cases}$

(c) $\pi\int_0^2 \left((6 - g(x))^2 - (6 - h(x))^2\right) dx$

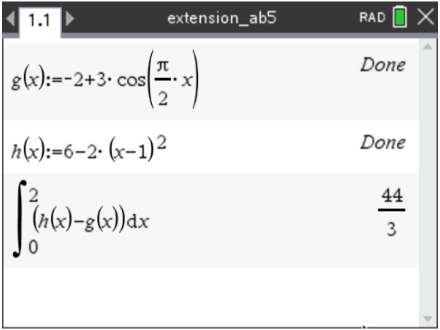
$3: \begin{cases} 1: \text{limits and constant} \\ 1: \text{form of integrand} \\ 1: \text{integrand} \end{cases}$

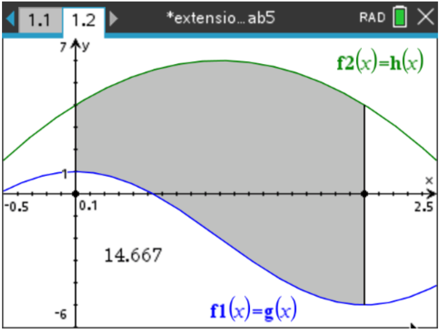
Part (a)

Solution

$$\begin{aligned}
\int_0^2 (y_{\mathsf{H}} - y_{\mathsf{L}})\,dx &= \int_0^2 [h(x) - g(x)]\,dx \\
&= \int_0^2 \left[(6 - (2(x-1)^2) - \left(-2 + 3\cos\left(\frac{\pi}{2}x\right)\right)\right]\,dx \\
&= \left[\left(6x - \frac{2}{3}(x-1)^3\right) - \left(-2x + \frac{6}{\pi}\sin\left(\frac{\pi}{2}x\right)\right)\right]_0^2 \\
&= \left[\left(12 - \frac{2}{3}\right) - (-4+0)\right] - \left[\left(0 + \frac{2}{3}\right) - (0+0)\right] \\
&= 12 - \frac{2}{3} + 4 - \frac{2}{3} = \frac{44}{3}
\end{aligned}$$

Technology Solution





Example 1 More with R

Let R be the region described in the Free Response Question.

(a) Find the value a such that the line $y = a$ divides the region R into two regions of equal area.

(b) Find the value b such that the line $x = b$ divides the region R into two regions of equal area.

(c) Find the value m such that the line $y = mx + 1$ divides the region R into two regions of equal area.

(d) Find the point of intersection, (A, B), of the graphs $y = g(x)$ and $y = h(x)$ in the fourth quadrant. Find the area of the region bounded by the graphs of $y = g(x)$, $y = h(x)$, the y-axis, and the line $x = A$.

Part (b)

Solution

$$\int_0^2 A(x)\,dx = \int_0^2 \frac{1}{x+3}\,dx$$

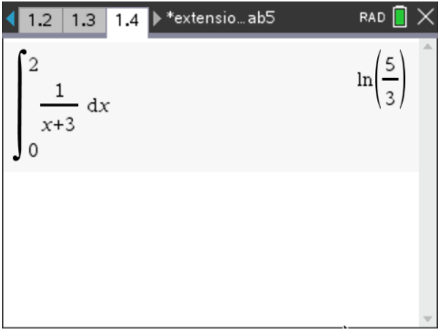
$$\begin{aligned} u &= x+3 \qquad & u(0) &= 3 \\ du &= dx \qquad & u(2) &= 5 \end{aligned}$$

$$= \int_3^5 \frac{1}{u}\,du$$

$$= \ln|u|\Big]_3^5$$

$$= \ln 5 - \ln 3 = \ln(5/3)$$

Technology Solution



Example 2 Various Cross-Sections

Region R is the base of a solid.

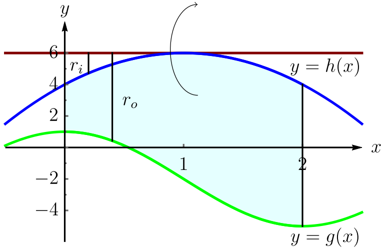
(a) For solid S_1, the cross-sections perpendicular to the x-axis are squares. Find the volume of S_1.

(b) For solid S_2, the cross-sections perpendicular to the x-axis are semicircle with diameters extending from $y = g(x)$ to $y = h(x)$. Find the volume of S_2.

(c) For solid S_3, the cross-sections perpendicular to the x-axis are equilateral triangles. Find the volume of S_3.

Part (c)

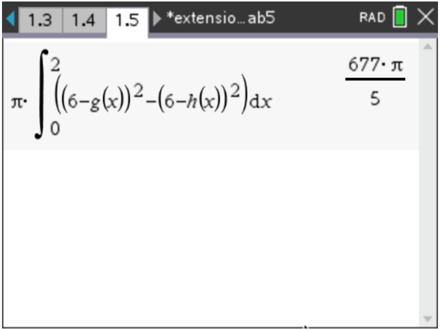
Solution



$$\mathsf{V} = \pi \int_0^2 [r_o^2 - r_i^2]\, dx = \pi \int_0^2 \left[(6 - g(x))^2 - (6 - h(x))^2\right]\, dx$$

Part (c)

Technology Solution



Example 3 Even More with R

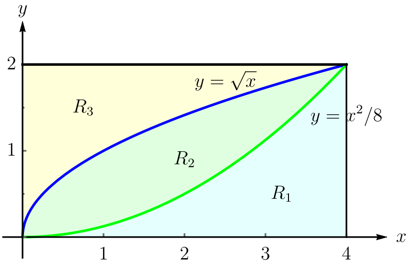
Let R be the region described in the Free Response Question.

(a) Let S_1 be the solid generated when R is rotated about the horizontal line $y = -6$. Find the volume of S_1.

(b) Let S_2 be the solid generated when R is rotated about the horizontal line $y = k$ $(k > 6)$. Find the value of k such that the volume of S_2 is $\frac{3461\pi}{15}$.

Example 4 Calculus Regions

Three regions are defined in the figure. Find the volume generated by rotating the given region about the specified line.



(a) R_1 about the x-axis

(b) R_1 about the line $y = 2$

(c) R_1 about the line $x = 4$

(d) R_1 about the y-axis

(e) R_3 about the x-axis

(f) R_3 about the y-axis

(g) R_3 about the line $x = 4$

(h) R_3 about the line $y = 2$

(i) R_2 about the x-axis

(j) R_2 about the y-axis

(k) R_2 about the line $y = 2$

(l) R_2 about the line $x = 4$

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