TI in Focus: AP® Calculus

2019 AP® Calculus Exam: AB-3/BC-3
Scoring Guidelines

Stephen Kokoska
Professor, Bloomsburg University
Former AP® Calculus Chief Reader

*AP is a registered trademark of the College Board, which was not involved in the production of, and does not endorse, this product.

Texas Instruments

Outline

(1) Free Response Question

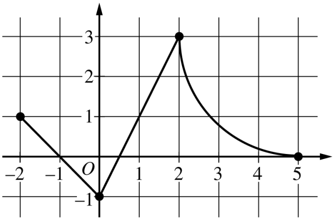
(2) Scoring Guidelines

(3) Student performance

(4) Interpretation

(5) Common errors

(6) Specific scoring examples



Graph of f

3. The continuous function f is defined on the closed interval $-6 \leq x \leq 5$. The figure above shows a portion of the graph of f, consisting of two line segments and a quarter of a circle centered at the point $(5, 3)$. It is known that the point $(3, 3 - \sqrt{5})$ is on the graph of f.

(a) If $\int_{-6}^{5} f(x)\, dx = 7$, find the value of $\int_{-6}^{-2} f(x)\, dx$. Show the work that leads to your answer.

(b) Evaluate $\int_{3}^{5} (2f'(x) + 4)\, dx$.

Source: 2019 AP Calculus AB and BC Free-Response Questions
Copyright © 2019 The College Board.
Reproduced with permission. http://apcentral.collegeboard.com.

(c) The function g is given by $g(x) = \int_{-2}^{x} f(t)\, dt$. Find the absolute maximum value of g on the interval $-2 \leq x \leq 5$. Justify your answer.

(d) Find $\lim_{x \to 1} \dfrac{10^x - 3f'(x)}{f(x) - \arctan x}$.

Source: 2019 AP Calculus AB and BC Free-Response Questions
Copyright © 2019 The College Board.
Reproduced with permission. http://apcentral.collegeboard.com.

(a) $\int_{-6}^{5} f(x)\,dx = \int_{-6}^{-2} f(x)\,dx + \int_{-2}^{5} f(x)\,dx$

$$\Rightarrow 7 = \int_{-6}^{-2} f(x)\,dx + 2 + \left(9 - \frac{9\pi}{4}\right)$$

$$\Rightarrow \int_{-6}^{-2} f(x)\,dx = 7 - \left(11 - \frac{9\pi}{4}\right) = \frac{9\pi}{4} - 4$$

$3 : \begin{cases} 1 : \int_{-6}^{5} f(x)\,dx = \int_{-6}^{-2} f(x)\,dx + \int_{-2}^{5} f(x)\,dx \\ 1 : \int_{-2}^{5} f(x)\,dx \\ 1 : \text{answer} \end{cases}$

(b)
$$\begin{aligned} \int_{3}^{5} (2f'(x) + 4)\,dx &= 2\int_{3}^{5} f'(x)\,dx + \int_{3}^{5} 4\,dx \\ &= 2(f(5) - f(3)) + 4(5 - 3) \\ &= 2\left(0 - \left(3 - \sqrt{5}\right)\right) + 8 \\ &= 2\left(-3 + \sqrt{5}\right) + 8 = 2 + 2\sqrt{5} \end{aligned}$$

— OR —

$$\begin{aligned} \int_{3}^{5} (2f'(x) + 4)\,dx &= \left[2f(x) + 4x\right]_{x=3}^{x=5} \\ &= (2f(5) + 20) - (2f(3) + 12) \\ &= (2 \cdot 0 + 20) - \left(2\left(3 - \sqrt{5}\right) + 12\right) \\ &= 2 + 2\sqrt{5} \end{aligned}$$

$2 : \begin{cases} 1 : \text{Fundamental Theorem of Calculus} \\ 1 : \text{answer} \end{cases}$

Source: 2019 AP Calculus AB and BC Free-Response Questions
Copyright © 2019 The College Board.
Reproduced with permission. http://apcentral.collegeboard.com.

(c) $g'(x) = f(x) = 0 \Rightarrow x = -1,\ x = \frac{1}{2},\ x = 5$

x	$g(x)$
-2	0
-1	$\frac{1}{2}$
$\frac{1}{2}$	$-\frac{1}{4}$
5	$11 - \frac{9\pi}{4}$

On the interval $-2 \le x \le 5$, the absolute maximum value of g is $g(5) = 11 - \frac{9\pi}{4}$.

$$3 : \begin{cases} 1 : g'(x) = f(x) \\ 1 : \text{identifies } x = -1 \text{ as a candidate} \\ 1 : \text{answer with justification} \end{cases}$$

(d) $$\lim_{x \to 1} \frac{10^x - 3f'(x)}{f(x) - \arctan x} = \frac{10^1 - 3f'(1)}{f(1) - \arctan 1} = \frac{10 - 3 \cdot 2}{1 - \arctan 1} = \frac{4}{1 - \frac{\pi}{4}}$$

1 : answer

Source: 2019 AP Calculus AB and BC Free-Response Questions
Copyright © 2019 The College Board.
Reproduced with permission. http://apcentral.collegeboard.com.

Student Performance

Part (a)

- Most students showed knowledge of the necessary property of definite integrals.
- Most students were able to show that the definite integral could be evaluated by finding area.
- Some students had trouble finding $\displaystyle\int_2^5 f(x)\,dx$:
  area of a square minus the area of a quarter circle.
- Some students tried to find an analytical expression for $f(x)$.

Part(b)

- Many students had trouble applying the FTC to $\displaystyle\int_3^5 f'(x)\,dx$
- Some students did not know how to handle the coefficient 2 or the constant 4.
- Unfortunately, there were may non-calculus errors.

Student Performance

Part (c)

- Vague or incomplete justifications; communication issues.
- Some responses presented an area argument, but without justification.
- Many students showed evidence of the FTC: $f = g'$.
  But there were some misconceptions:
- Many students used a local rather than global argument.
- Some students did not address the critical point $x = 1/2$.

Part(d)

- Some students incorrectly applied L'Hospital's Rule.
- Some students presented non-calculus errors.
- Some students did not evaluate $\arctan 1$ correctly (didn't need to).

Part (a) 1: $\int_{-6}^{5} f(x)\,dx = \int_{-6}^{-2} f(x)\,dx + \int_{-2}^{5} f(x)\,dx$

A conceptual point: evidence of the correct property of integrals needed to solve this problem.

(1) $\displaystyle\int_{-6}^{5} f(x)\,dx = \int_{-6}^{-2} f(x)\,dx + \int_{-2}^{5} f(x)\,dx$

(2) $\displaystyle 7 = \int_{-6}^{-2} f(x)\,dx + \int_{-2}^{5} f(x)\,dx$

(3) $\displaystyle\int_{-6}^{5} f(x)\,dx - \int_{-2}^{5} f(x)\,dx$

(4) $\displaystyle 7 - \int_{-2}^{5} f(x)\,dx$

(5) $\displaystyle\int_{-6}^{5} f(x) = \int_{-6}^{-2} f(x) + \int_{-2}^{5} f(x)$

(6) $\displaystyle\int_{-6}^{5} = \int_{-6}^{-2} + \int_{-2}^{5}$

Part (a) $\int_{-6}^{5} f(x)\,dx = \int_{-6}^{-2} f(x)\,dx + \int_{-2}^{5} f(x)\,dx$

The first point may be embedded within a final answer.

$$7-\left(2+\left(9-\frac{9\pi}{4}\right)\right)$$ 1 - 1 - 1

$$7-\left(11-\frac{9\pi}{4}\right)$$ 1 - 1 - 1

$7-(A)$ where A is an incorrect declared value of $\displaystyle\int_{-2}^{5} f(x)\,dx$ 1 - 0 - e

This response may or may not be eligible for the third point.

Part (a) 1: $\int_{-2}^{5} f(x)\,dx$

(1) This point is earned for the correct value only of $\displaystyle\int_{-2}^{5} f(x)\,dx$

(2) Answer does not need to be simplified; any simplification must be correct.

$$\int_{-2}^{5} f(x)\,dx = 11 - \frac{9\pi}{4} = \frac{44 - 9\pi}{4}$$

(3) We expect to see work leading to this result.
The work may appear on or near the graph.

Part (a) 1: answer

(1) Connect the property of integrals and the value of $\int_{-2}^{5} f(x)\,dx$

(2) Our answer or consistent answer with eligible declared value of $\int_{-2}^{5} f(x)\,dx$

(3) Eligible: work that conveys the student is finding our area.
Adding the area of a quarter circle: not eligible.

Other issues

(1) Reversal: can be corrected.

(2) Linkage errors.

(3) Decimal presentations.

(4) Analytic solutions.

Part (b) 1: Fundamental Theorem of Calculus

Need two items to earn this point

(1) Antiderivative of $f'(x)$

(2) Use of the limits of integration

Both must be present to earn the first point.
Work may or may not include integral notation or a general antiderivative.

Examples

(1) $2f(x) + 4x\Big|_3^5$ e - e

(2) $2f(x) + 4x\Big|_3^5 = (2f(5) + 4 \cdot 5) - (2f(3) + 4 \cdot 3)$ 1 - e

(3) $(2f(5) + 4 \cdot 5) - (2f(3) + 4 \cdot 3)$ 1 - e

Part (b) 1: Fundamental Theorem of Calculus

Errors in other parts of the integrand considered in second point.

Each response earns 1 - 0

$$2f(5) - 2f(3)$$

$$(2f(5) + 4) - (2f(3) + 4)$$

$$(f(5) + 20) - (f(3) + 12)$$

Part (b) 1: answer

Earned for our answer only.
No simplification required, but any simplification must be correct.

Part (b) Two points together

Note: $f(5) = 0$ and $f(3) = 3 - \sqrt{5}$ given.

Some responses are minimal. Must be convinced the student has done $\int f'(x)\,dx = f(x)$ and handled the limits of integration correctly.

Examples

(1)	$20 - (2f(3) + 12)$	1 - e
(2)	$20 - (6 - 2\sqrt{5} + 12)$	1 - 1
(3)	$20 - (18 - 2\sqrt{5})$	0 - 1
(4)	$9 - (2f(3) + 12)$	0 - 0
(5)	$20 - (6 - 2\sqrt{5}) + 12$	1 - 0

Part (c) 1: $g'(x) = f(x)$

Make the connection $g'(x) = f(x)$

(1) $g'(x) = f(x)$ 1 - e - e

(2) $\dfrac{d}{dx}\displaystyle\int_{-2}^{x} f(t)\,dt = f(x)$ 1 - e - e

(3) $F(x) = g(x)$: Not the connection we are looking for.

(4) $g'(x) = f(x) \cdot x'$ e - e - e

(5) $g'(x) = f(t); \quad g'(t) = f(x)$ 1 - e - e

(6) The graph of f is the derivative of g

Part (c) 1: identifies $x = -1$ as a candidate

(1) Independent of first point: 0 - 1 - 0 possible

(2) $x = -1$ may be identified as a candidate in a table.

(3) Identification may be embedded in a paragraph response.

(4) Earned without regard to other candidates; too many or too few.

(5) No reasoning necessary.

Part (c) 1: answer with justification

(1) Eligibility:
   (a) First two points must be earned.
   (b) Must declare the maximum value as $11 - \dfrac{9\pi}{4}$.

(2) A justification need only evaluate $g(-1)$ and $g(5)$.
But must include reasoning for excluding $g(1/2)$.

(3) Justification does not need a reason for excluding $g(-2)$.

(4) Read with imported values from part (a) greater than $1/2$.
But absolute maximum must occur at the right endpoint.

(5) A table is justification: all values must be correct.

(6) Extra values in the table must be correct.

(7) Alternate solution: accumulated area from the left endpoint, $x = -2$.

Part (d) 1: answer

(1) Earned for our answer only.

(2) Answer does not need to be simplified; $\arctan 1$ does not need to be evaluated.

(3) Responses do not need limit notation.

(4) Hanging limit notation: $\lim_{x\to 1} = \dfrac{4}{1-\arctan 1}$

(5) $\lim_{x\to 1} \dfrac{4}{1-\arctan 1}$ not incorrect, but not the answer.

(6) $\lim_{x\to 1} \dfrac{4}{1-\arctan 1} = \dfrac{4}{1-\arctan 1}$

education.ti.com