TI in Focus: AP® Calculus

2019 AP® Calculus Exam: AB-2

Particle Motion Analysis

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Outline

Expression	Translation
The particle is at position $s = b$ at time $t = a$.	This gives us the position of the particle at a specific time, $s(a) = b$.
The particle is at rest.	This means the velocity is 0, $v(t) = 0$.
The particle is moving to the right.	The velocity is greater than 0, $v(t) > 0$.
The particle is moving to the left.	The velocity is less than 0, $v(t) < 0$.
The average velocity of the particle on the interval $[a, b]$.	$\frac{1}{b-a}\int_a^b v(t)\,dt = \frac{1}{b-a}\Big[s(t)\Big]_a^b = \frac{s(b)-s(a)}{b-a}$
The instantaneous velocity of the particle at time $t = a$.	$v(a) = s'(a)$
The acceleration of the particle at time $t = b$	$a(b) = v'(b) = s''(b)$

Expression	Translation
The velocity of the particle is increasing.	The derivative of the velocity is positive, $a(t) = v'(t) > 0$.
The velocity of the particle is decreasing.	The derivative of the velocity is negative, $a(t) = v'(t) < 0$.
The speed of the particle	Speed is the absolute value of the velocity, $\lvert v(t) \rvert$
The speed of the particle is increasing or the particle is speeding up	The velocity and acceleration have the same sign, that is, either $v(t) > 0$ and $a(t) > 0$ or $v(t) < 0$ and $a(t) < 0$. Or, $\frac{d}{dt}\lvert v(t) \rvert > 0$
The speed of the particle is decreasing or the particle is slowing down	The velocity and acceleration have different signs, that is, either $v(t) > 0$ and $a(t) < 0$ or $v(t) < 0$ and $a(t) > 0$. Or, $\frac{d}{dt}\lvert v(t) \rvert < 0$

Expression	Translation
The total distance traveled by the particle over the interval $[a, b]$.	$\int_a^b \lvert v(t) \rvert \, dt$
The displacement of the particle, or the net distance traveled by the particle, over the time interval $[a, b]$	$\int_a^b v(t) \, dt$
The position of the particle at time $t = b$	$s(b) = s(a) + \int_a^b v(t) \, dt$
	Note that often $a = 0$ and, therefore, $s(a)$ is the initial position.

Expression	Translation
The particle changes direction	This occurs where the velocity of the particle changes from positive to negative or negative to positive.
The particle is farthest to the left (right)	Find the minimum (maximum) value of $s(t)$ over an interval. Find the values of t where $s'(t) = v(t) = 0$, the critical values of s. Consider the position of the particle at the critical values and, if the interval is closed, the endpoints of the interval.

Example 1 Comprehensive Particle Motion

A particle moves along a horizontal line. For $0 \leq t \leq 10$, the velocity of the particle is given by $v(t) = 20te^{-t/2} \sin\left(\frac{t^2}{7}\right)$ where $v(t)$ is measured in miles per hour and t is measured in hours. The position of the particle at time t is given by $s(t)$ and it is know that $s(0) = -7$.

(a) Find all the values of t in the interval $3 \leq t \leq 6$ for which the speed of the particle is 5.

(b) Write an expression involving an integral that gives the position $s(t)$. Use this expression to find the position of the particle at time $t = 6$.

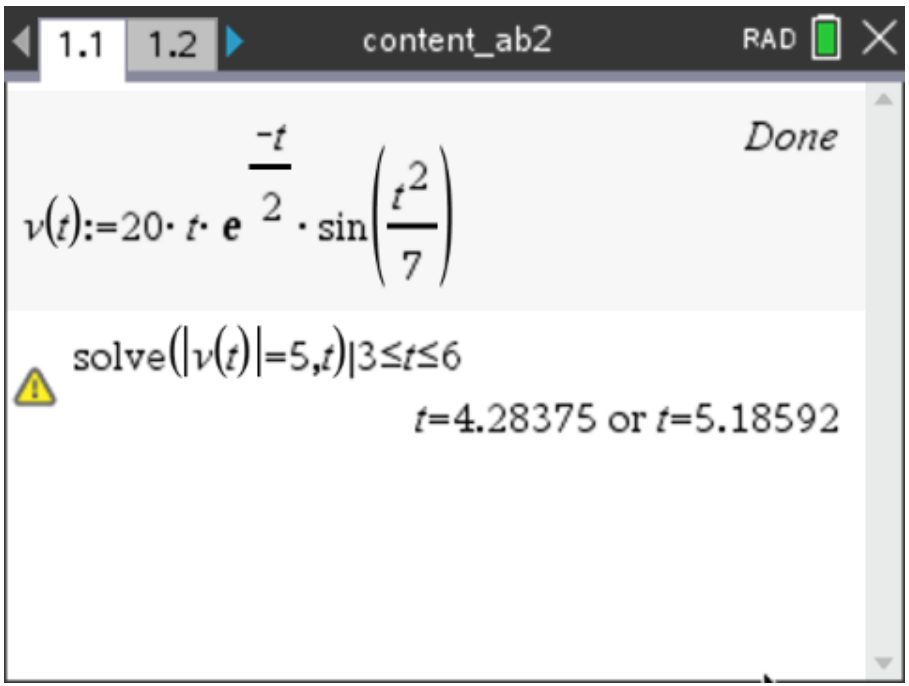
(c) Find all times t in the interval $0 < t < 10$ at which the particle changes direction.

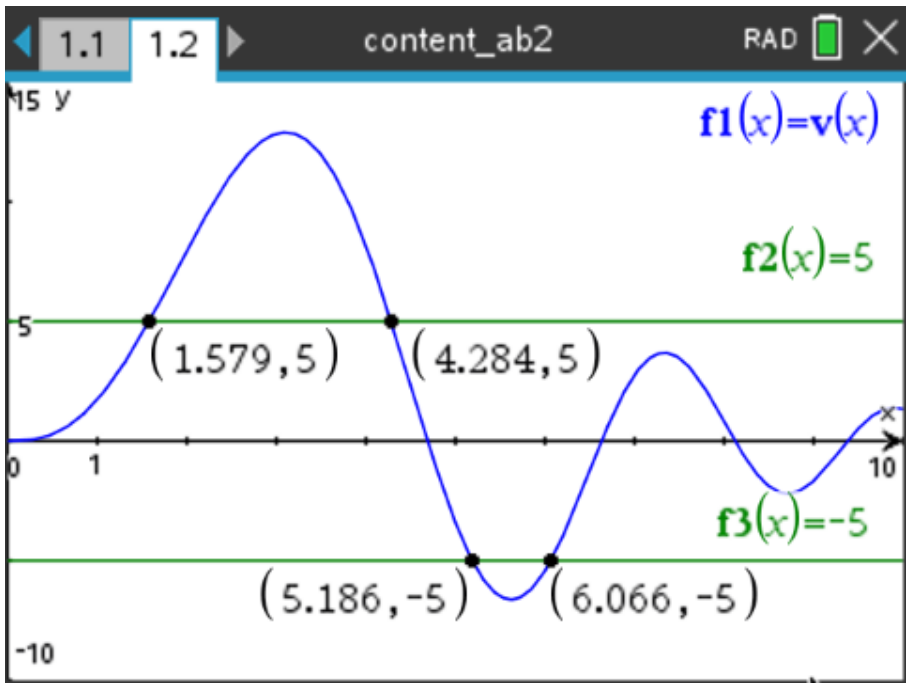
(d) Find the total distance traveled by the particle over the time interval $6 \leq t \leq 9$.

Solution

(a) Speed of the particle is the absolute value of the velocity.

Solve the expression $|v(t)| = 5$





The speed of the particle is 5 at $t = 4.284$ and $t = 5.186$.

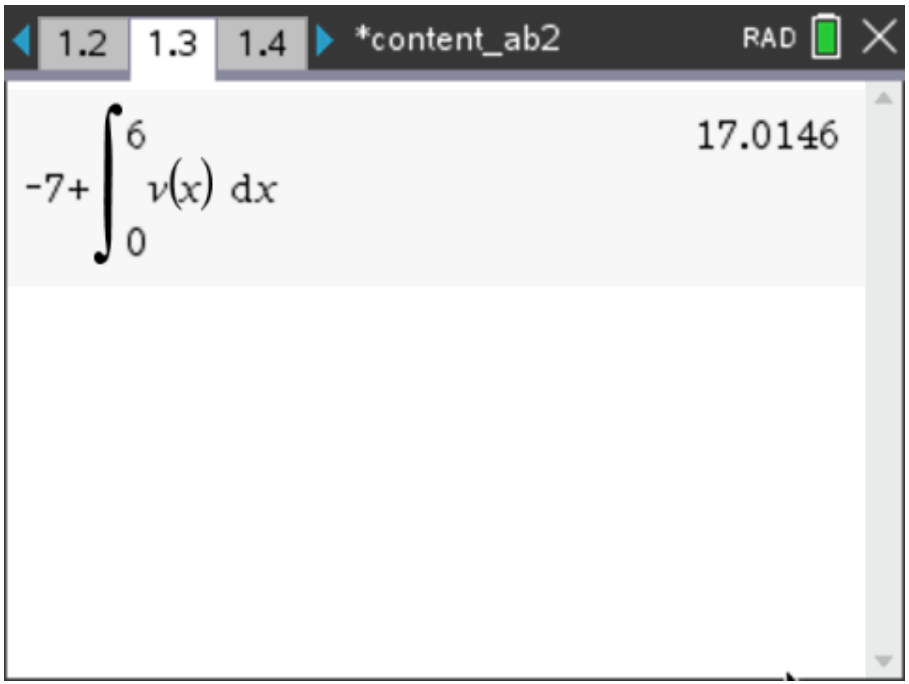
Solution

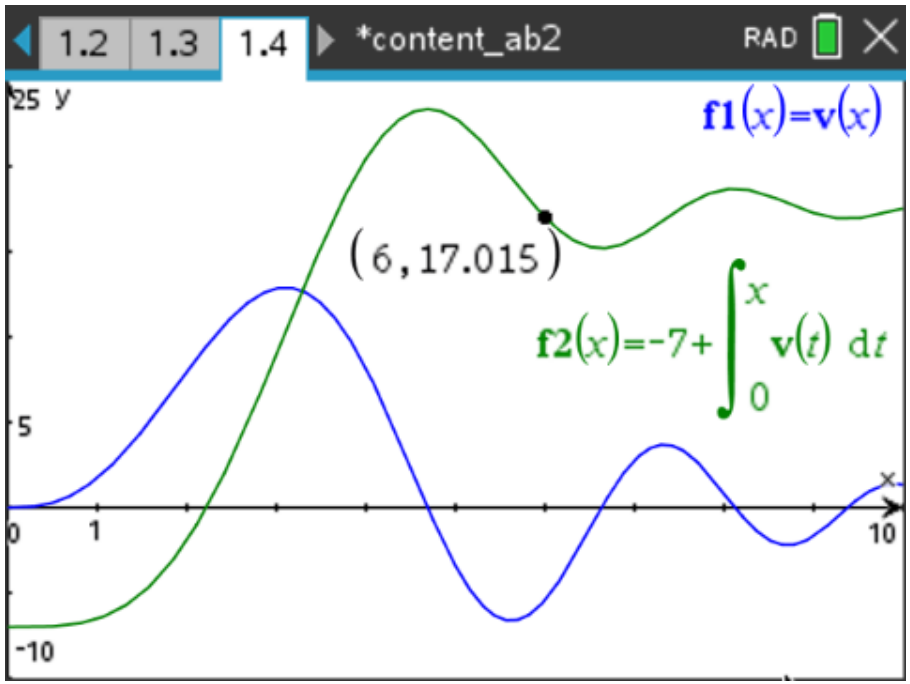
(b) The position of the particle at time t is given by

$$s(t) = s(0) + \int_0^t v(x)\,dx = -7 + \int_0^t v(x)\,dx$$

The position of the particle at time $t = 6$:

$$s(6) = -7 + \int_0^6 v(x)\,dx = 17.015$$

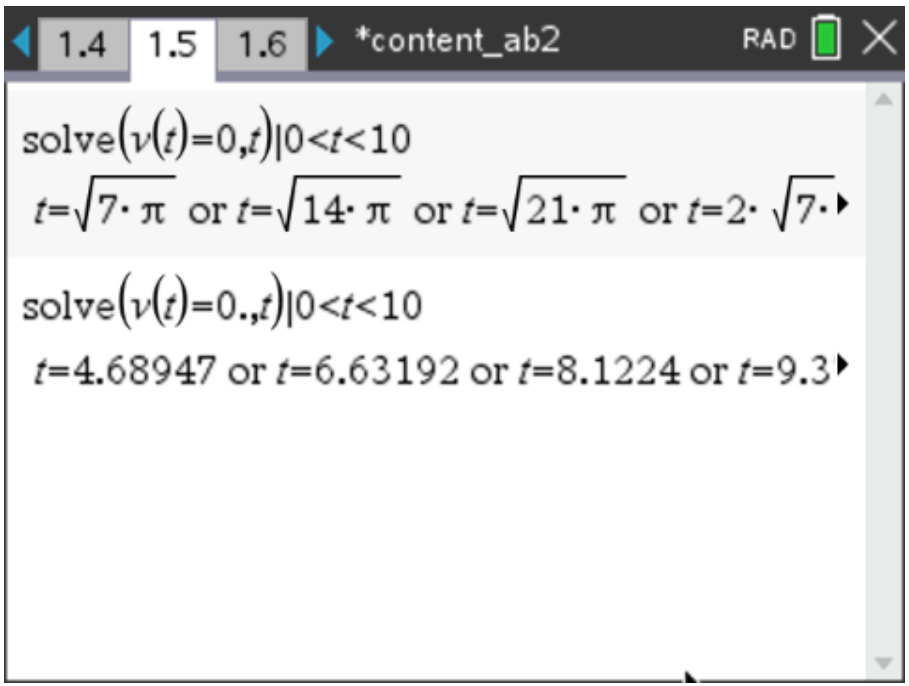


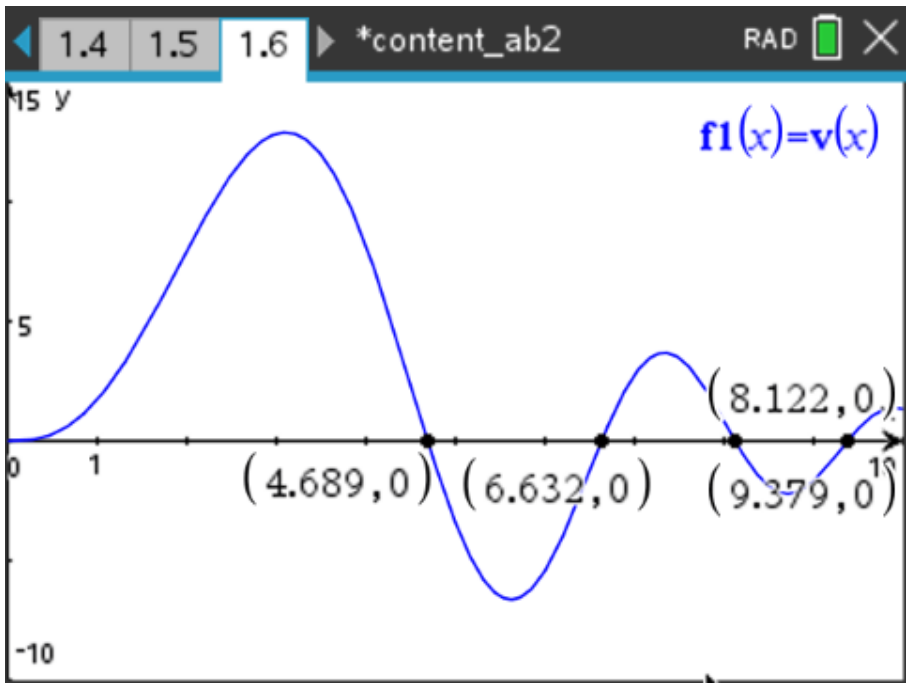


Solution

(c) Consider the graph of v.

Find the exact values on a calculator page (analytically).





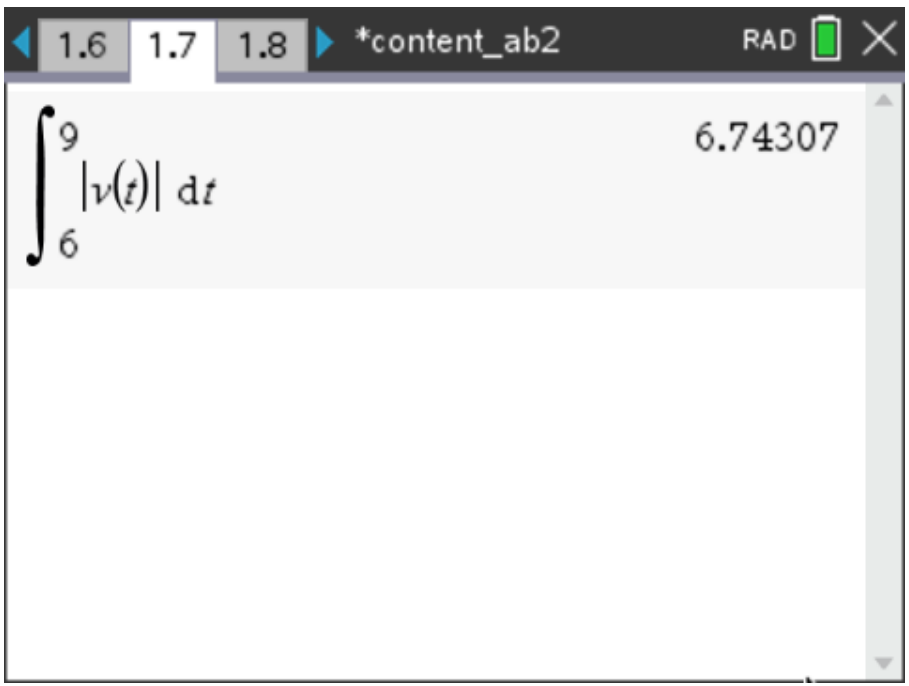
$v(t) = 0$ at $t = \sqrt{7\pi},\ \sqrt{14\pi},\ \sqrt{21\pi},\ 2\sqrt{7\pi}$

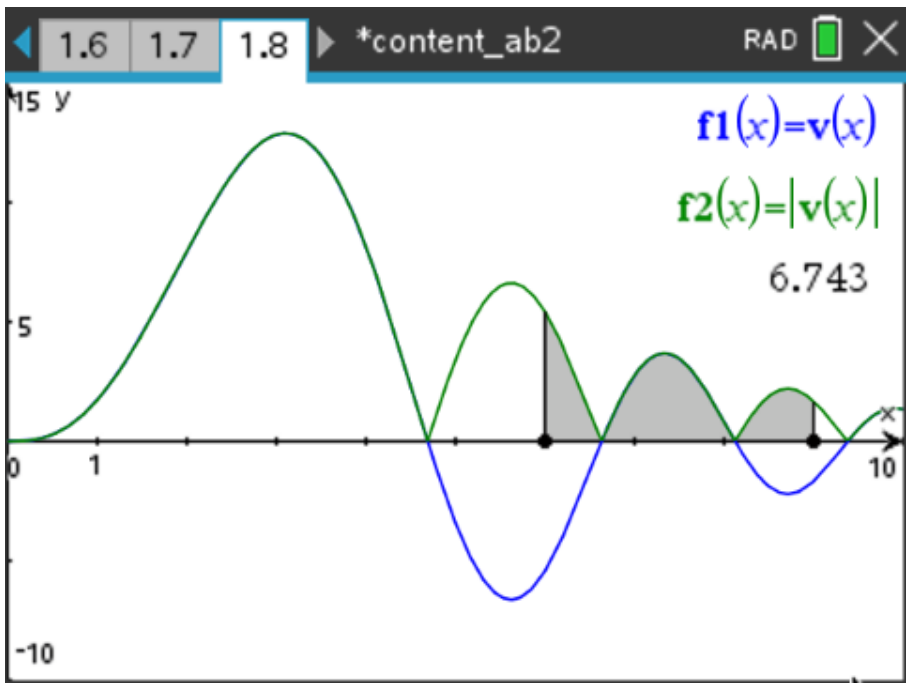
The particle changes direction at each of these times.

Solution

(d) The total distance traveled by the particle over the interval $6 \leq t \leq 9$:

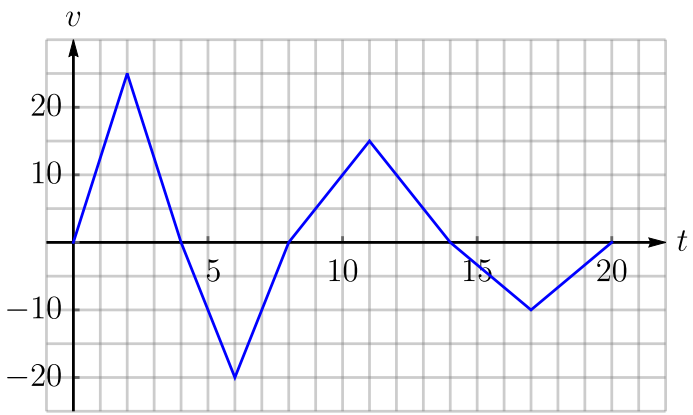
$$\int_6^9 |v(t)|\, dt = 6.743$$





Example 2 SlingShot Ride

The SlingShot ride at Cedar Point is a reverse bungee ride that uses specially designed springs to propel two-person capsules attached to cables. The ride bounces up and down until it eventually comes to rest. The velocity of the SlingShot, for $0 \leq t \leq 20$, is modeled by the function shown in the figure, where t is measured in seconds and v is measured in meters per second. At time $t = 0$ the SlingShot is 20 m above the ground.



(a) At what times in the interval $0 < t < 20$, if any, does the SlingShot change direction? Give a reason for your answer.

(b) At what time t in the interval $0 \leq t \leq 20$ is the SlingShot highest? How high is the ride at that time?

(c) Find the total distance the SlingShot travels during the time interval $0 \leq t \leq 20$.

Example 3 Particle Motion Analysis

For $0 \leq t \leq 12$, a particle moves along a horizontal line. The velocity of the particle at time t is given by $v(t) = t^3 - 19t^2 + 100t - 132$. The particle is at position $s = 40$ at time $t = 0$.

(a) For $0 \leq t \leq 12$, when in the particle moving to the left?

(b) Find the displacement of the particle from time $t = 0$ to time $t = 4$. Find the total distance traveled by the particle from time $t = 4$ to time $t = 10$.

(c) For $0 \leq t \leq 12$, find the time interval(s) for which the speed of the particle is at least 20.

(d) Find the position of the particle at time $t = 8$.

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