TI in Focus: AP® Calculus

2019 AP® Calculus Exam: AB-2
Scoring Guidelines

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Outline

(1) Free Response Question

(2) Scoring Guidelines

(3) Student performance

(4) Interpretation

(5) Common errors

(6) Specific scoring examples

t (hours)	0	0.3	1.7	2.8	4
$v_P(t)$ (meters per hour)	0	55	–29	55	48

2. The velocity of a particle, P, moving along the x-axis is given by the differentiable function v_P, where $v_P(t)$ is measured in meters per hour and t is measured in hours. Selected values of $v_P(t)$ are shown in the table above. Particle P is at the origin at time $t = 0$.

(a) Justify why there must be at least one time t, for $0.3 \le t \le 2.8$, at which $v_P'(t)$, the acceleration of particle P, equals 0 meters per hour per hour.

(b) Use a trapezoidal sum with the three subintervals [0, 0.3], [0.3, 1.7], and [1.7, 2.8] to approximate the value of $\int_0^{2.8} v_P(t)\, dt$.

(c) A second particle, Q, also moves along the x-axis so that its velocity for $0 \leq t \leq 4$ is given by $v_Q(t) = 45\sqrt{t}\cos\left(0.063t^2\right)$ meters per hour. Find the time interval during which the velocity of particle Q is at least 60 meters per hour. Find the distance traveled by particle Q during the interval when the velocity of particle Q is at least 60 meters per hour.

(d) At time $t = 0$, particle Q is at position $x = -90$. Using the result from part (b) and the function v_Q from part (c), approximate the distance between particles P and Q at time $t = 2.8$.

(a) v_P is differentiable $\Rightarrow$ v_P is continuous on $0.3 \le t \le 2.8$.

$$\frac{v_P(2.8) - v_P(0.3)}{2.8 - 0.3} = \frac{55 - 55}{2.5} = 0$$

By the Mean Value Theorem, there is a value c, $0.3 < c < 2.8$, such that $v_P{}'(c) = 0$.

— OR —

v_P is differentiable $\Rightarrow$ v_P is continuous on $0.3 \le t \le 2.8$.

By the Extreme Value Theorem, v_p has a minimum on $[0.3, 2.8]$.

$v_P(0.3) = 55 > -29 = v_P(1.7)$ and $v_P(1.7) = -29 < 55 = v_P(2.8)$.

Thus, v_P has a minimum on the interval $(0.3, 2.8)$.

Because v_P is differentiable, $v_P{}'(t)$ must equal 0 at this minimum.

$$2 : \begin{cases} 1 : v_P(2.8) - v_P(0.3) = 0 \\ 1 : \text{justification, using} \\ \quad\ \text{Mean Value Theorem} \end{cases}$$

— OR —

$$2 : \begin{cases} 1 : v_P(0.3) > v_P(1.7) \\ \quad\ \text{and } v_P(1.7) < v_P(2.8) \\ 1 : \text{justification, using} \\ \quad\ \text{Extreme Value Theorem} \end{cases}$$

(b)
$$\begin{aligned}\int_0^{2.8} v_P(t)\,dt &\approx 0.3\left(\frac{v_P(0) + v_P(0.3)}{2}\right) + 1.4\left(\frac{v_P(0.3) + v_P(1.7)}{2}\right) \\ &\quad + 1.1\left(\frac{v_P(1.7) + v_P(2.8)}{2}\right) \\ &= 0.3\left(\frac{0 + 55}{2}\right) + 1.4\left(\frac{55 + (-29)}{2}\right) + 1.1\left(\frac{-29 + 55}{2}\right) \\ &= 40.75\end{aligned}$$

1 : answer, using trapezoidal sum

(c) $v_Q(t) = 60 \Rightarrow t = A = 1.866181$ or $t = B = 3.519174$

$v_Q(t) \geq 60$ for $A \leq t \leq B$

$$\int_A^B |v_Q(t)|\, dt = 106.108754$$

The distance traveled by particle Q during the interval $A \leq t \leq B$ is 106.109 (or 106.108) meters.

3 : { 1 : interval
1 : definite integral
1 : distance

(d) From part (b), the position of particle P at time $t = 2.8$ is

$x_P(2.8) = \int_0^{2.8} v_P(t)\, dt \approx 40.75.$

$$x_Q(2.8) = x_Q(0) + \int_0^{2.8} v_Q(t)\, dt = -90 + 135.937653 = 45.937653$$

Therefore, at time $t = 2.8$, particles P and Q are approximately $45.937653 - 40.75 = 5.188$ (or 5.187) meters apart.

3 : { 1 : $\int_0^{2.8} v_Q(t)\, dt$
1 : position of particle Q
1 : answer

Student Performance

Part (a)

- Many students recognized the need to use the Mean Value Theorem.
- Errors in communicating the theorem hypotheses.
- Of those using the MVT, many did not present the relevant difference quotient.
- Errors in the difference quotient: reciprocal, arithmetic.
- Some students attempted to use the IVT.

Part(b)

- Many students presented the correct trapezoidal approximation with sufficient work. Some insufficient communication/supporting work.
- Some presented arithmetic errors.
- Some students used the average of left- and right-Riemann approximations.
- Most students understood the concept of adding area. Some had trouble using trapezoids.

Student Performance

Part (c)

- Most students presented an appropriate interval, with correct notation.
- Errors: listing only points, only one point of intersection.
- Many students understood the concept of distance traveled using a definite integral.
- Some decimal presentation errors and intermediate rounding errors.

Part(d)

- Many students presented the correct expression for the position of particle Q.
- Errors: no initial condition, incorrect values, no value, presentation errors.
- Most students recognized the need to find the difference in positions.

Background

(1) Units are not required in any part of this problem.

(2) Decimal and degree errors possible.

(3) Part (a): Three common student approaches
 (a) MVT argument: follow the scoring guidelines.
 (b) Local argument: follow the scoring guidelines.
 (c) IVT attempt: not a valid approach.

Part (a) 1: $v_P(2.8) - v_P(0.3) = 0$

Philosophy: $v_p(t)$ is equal at $t = 0.3$ and $t = 2.8$.

(1) $v_p(2.8) - v_p(0.3) = 0$ 1 - e

(2) $v_p(2.8) = v_p(0.3)$ 1 - e

(3) $\dfrac{55 - 55}{2.8 - 0.3} = \dfrac{55 - 55}{-2.5} = 0$ 1 - e

(4) $55 - 55$ 1 - e

(5) Velocity is the same at $t = 0.3$, $t = 2.8$ 1 - e

(6) $\dfrac{v_p(2.8) - v_p(0.3)}{2.8 - 0.3}$ 0 - 0

(7) Function is the same at $t = 0.3$, $t = 2.8$ 0 - 0

Part (a) 1: justification, using Mean Value Theorem

Eligibility: must earn the first point.

(1) Must declare v_p is continuous.

(2) Do not need to declare that v_p is differentiable.

(3) May use Rolle's Theorem instead of MVT.

(4) May relabel or *misspell* $v_p(t)$.

To earn this point:

(a) $v_p(t)$ is continuous.

(b) Connect $v_p'(t)$ to the first point.

Part (a) Examples

(1) Since $v(0.3) = 55 = v(2.8)$, and $v(t)$ is continuous, by Rolle's theorem, $v'(t) = 0$ for t in the interval. 1 - 1

(2) $f(.3) - f(2.8) = 0$, and f diff $\implies$ cont, $f'(x) = 0$ 1 - 1

(3) $f(.3) - f(2.8) = 55 - 55 = 0$
By MVT $f'(t) = 0$ at some point in interval 1 - 0

Part (b) 1: answer, using trapezoidal sum

Philosophy: point is earned for a correct trapezoidal sum.

(1) Evidence of correct values.

(2) A correct trapezoidal sum:
at least three products,
at least three terms,
a sum.

(3) Could earn the point with an implied sum.

(4) Could average a left and right Riemann sum.

(5) The symbols $\approx$ and $=$ are both OK.

Part (b) Examples

(1) $\frac{1}{2} \cdot 55 \cdot 0.3 + \frac{1}{2} \cdot 26 \cdot 1.4 + \frac{1}{2} \cdot 26 \cdot 1.1$ 1

(2) $8.25 + 18.2 + 14.3$ 0

(3) $\int_0^{2.8} v_p(t)\,dt \approx 40.75$ 0

(4) 40.75 0

Part (c) 1: interval

Philosophy: explicitly state the interval.

(1) Earned with open or closed intervals.

(2) Interval may be stated in exposition.

(3) With definite integral only: first point not earned.
The interval is not read from the limits of integration.

Part (c) Examples

(1)	$(1.866, 3.519)$	1 - e - e
(2)	t at $(1.866, 3.519)$	1 - e - e
(3)	$t = (1.866, 3.519)$	1 - e - e
(4)	$t = 1.866, \quad t = 3.519$	0 - e - e
(5)	$1.866, \quad 3.519$	0 - e - e
(6)	t from 1.86 to 3.51	0 - e - e

Part (c) 1: definite integral

Philosophy: state the appropriate definite integral.

(1) $\displaystyle\int_{t_0}^{t_1} v_Q(t)\,dt$ or $\displaystyle\int_{t_0}^{t_1} |v_Q(t)|\,dt \quad 0 \le t_0 < t_1 \le 4$

(2) A missing dt may earn the point.

(3) These expressions earn the second point.

$$\int_{1.87}^{3.52} |v_Q(t)|\,dt \;; \qquad \int_{2}^{3} |v_Q(t)|\,dt \;; \qquad \int_{0}^{4} v(t)\,dt$$

(4) Mishandling 60 and/or dt:

$$-60 + \int_{t_0}^{t_1} v(t)$$? - 1 - 0

$$\int_{t_0}^{t_1} v_Q(t) - 60\,dt$$? - 0 - 0

Part (c) 1: distance

(1) $\displaystyle\int_{1.866}^{3.519} |v_Q(t)|\, dt = 106.109$

(2) Bald answer not accepted.

(3) Decimal presentation error:

$[1.87, 3.52], \quad \displaystyle\int_{1.87}^{3.52} |v_Q(t)|\, dt = 106.11$ 0 - 1 - 1

(4) Decimal presentation and intermediate rounding errors:

$[1.87, 3.52], \quad \displaystyle\int_{1.87}^{3.52} |v_Q(t)|\, dt = 105.93$ 0 - 1 - 0

Part (d) 1: $\int_0^{2.8} v_Q(t)\,dt$

Philosophy: definite integral of $v_Q(t)$ from $t = 0$ to $t = 2.8$.

(1) $\displaystyle\int_0^{2.8} v_Q(t)\,dt$ or $\displaystyle\int_0^{2.8} |v_Q(t)|\,dt$

(2) Uses $v_Q(t)$ with the correct limits.

(3) Use of $v(t)$ does not earn first point.

(4) Missing dt may earn the first point.

$$-90 + \int_0^{2.8} v_Q(t)$$ 1 - e - e

$$\int_0^{2.8} v_Q(t) - 90$$ 0 - e - 0

Part (d) 1: position of particle Q

Philosophy: position of Q at $t = 2.8$

Simply: must be our answer.

(1) $-90 + \int_0^{2.8} v_Q(t)\,dt = 45.938$ 1 - 1 - e

(2) $-90 + \int_0^{2.8} v_Q(t)\,dt = 45.94$ 1 - 0 - e

Part (d) 1: answer

Philosophy: positive difference between $x_Q(2.8)$ and $x_P(2.8)$.

(1) Eligibility: must earn the first point (except for special case; copy error).

(2) Any value may be imported from part (b).

(3) The difference between $x_Q(2.8)$ and $x_P(2.8)$ may be implied.

Part (d) Examples

(1) $x_Q(2.8) = -90 + \int_0^{2.8} v_Q(t)\,dt = 45.938$

$x_P(2.8) = 40.75$

Distance $= 5.188$ 1 - 1 - 1

(2) $x_Q(2.8) = -90 + \int_0^{2.8} v_Q(t)\,dt = 45.94$

$45.94 - 40.75 = 5.19$ 1 - 0 - 1

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