TI in Focus: AP® Calculus

2019 AP® Calculus Exam: AB-2
Technology Solutions and Problem Extensions

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Outline

t (hours)	0	0.3	1.7	2.8	4
$v_P(t)$ (meters per hour)	0	55	–29	55	48

2. The velocity of a particle, P, moving along the x-axis is given by the differentiable function v_P, where $v_P(t)$ is measured in meters per hour and t is measured in hours. Selected values of $v_P(t)$ are shown in the table above. Particle P is at the origin at time $t = 0$.

(a) Justify why there must be at least one time t, for $0.3 \le t \le 2.8$, at which $v_P'(t)$, the acceleration of particle P, equals 0 meters per hour per hour.

(b) Use a trapezoidal sum with the three subintervals [0, 0.3], [0.3, 1.7], and [1.7, 2.8] to approximate the value of $\int_0^{2.8} v_P(t)\, dt$.

(c) A second particle, Q, also moves along the x-axis so that its velocity for $0 \leq t \leq 4$ is given by $v_Q(t) = 45\sqrt{t}\cos\left(0.063t^2\right)$ meters per hour. Find the time interval during which the velocity of particle Q is at least 60 meters per hour. Find the distance traveled by particle Q during the interval when the velocity of particle Q is at least 60 meters per hour.

(d) At time $t = 0$, particle Q is at position $x = -90$. Using the result from part (b) and the function v_Q from part (c), approximate the distance between particles P and Q at time $t = 2.8$.

(a) v_P is differentiable $\Rightarrow$ v_P is continuous on $0.3 \le t \le 2.8$.

$$\frac{v_P(2.8) - v_P(0.3)}{2.8 - 0.3} = \frac{55 - 55}{2.5} = 0$$

By the Mean Value Theorem, there is a value c, $0.3 < c < 2.8$, such that $v_P'(c) = 0$.

— OR —

v_P is differentiable $\Rightarrow$ v_P is continuous on $0.3 \le t \le 2.8$.

By the Extreme Value Theorem, v_p has a minimum on $[0.3, 2.8]$.

$v_P(0.3) = 55 > -29 = v_P(1.7)$ and $v_P(1.7) = -29 < 55 = v_P(2.8)$.

Thus, v_P has a minimum on the interval $(0.3, 2.8)$.

Because v_P is differentiable, $v_P'(t)$ must equal 0 at this minimum.

$2: \begin{cases} 1: v_P(2.8) - v_P(0.3) = 0 \\ 1: \text{justification, using} \\ \quad \text{Mean Value Theorem} \end{cases}$

— OR —

$2: \begin{cases} 1: v_P(0.3) > v_P(1.7) \\ \quad \text{and } v_P(1.7) < v_P(2.8) \\ 1: \text{justification, using} \\ \quad \text{Extreme Value Theorem} \end{cases}$

(b) $$\int_0^{2.8} v_P(t)\,dt \approx 0.3\left(\frac{v_P(0) + v_P(0.3)}{2}\right) + 1.4\left(\frac{v_P(0.3) + v_P(1.7)}{2}\right) + 1.1\left(\frac{v_P(1.7) + v_P(2.8)}{2}\right)$$

$$= 0.3\left(\frac{0 + 55}{2}\right) + 1.4\left(\frac{55 + (-29)}{2}\right) + 1.1\left(\frac{-29 + 55}{2}\right)$$

$$= 40.75$$

1 : answer, using trapezoidal sum

(c) $v_Q(t) = 60 \Rightarrow t = A = 1.866181$ or $t = B = 3.519174$

$v_Q(t) \geq 60$ for $A \leq t \leq B$

$$\int_A^B \left|v_Q(t)\right| dt = 106.108754$$

The distance traveled by particle Q during the interval $A \leq t \leq B$ is 106.109 (or 106.108) meters.

$$3 : \begin{cases} 1 : \text{interval} \\ 1 : \text{definite integral} \\ 1 : \text{distance} \end{cases}$$

(d) From part (b), the position of particle P at time $t = 2.8$ is

$x_P(2.8) = \int_0^{2.8} v_P(t)\, dt \approx 40.75.$

$$x_Q(2.8) = x_Q(0) + \int_0^{2.8} v_Q(t)\, dt = -90 + 135.937653 = 45.937653$$

Therefore, at time $t = 2.8$, particles P and Q are approximately $45.937653 - 40.75 = 5.188$ (or 5.187) meters apart.

$$3 : \begin{cases} 1 : \int_0^{2.8} v_Q(t)\, dt \\ 1 : \text{position of particle } Q \\ 1 : \text{answer} \end{cases}$$

Part (a)

Compute the appropriate difference quotient.

$$\frac{v_P(2.8) - v_P(0.3)}{2.8 - 0.3} = \frac{55 - 55}{2.5} = 0$$

v_P is differentiable (given). Therefore, v_P is continuous on $0.3 \leq t \leq 2.8$.

By the Mean Value Theorem, there is a value c, $0.3 < c < 2.8$, such that $v'_P(c) = 0$.

Notes

(a) Rolle's Theorem

(b) Use of the Extreme Value Theorem

Part (b)

Find the trapezoidal sum.

$$\int_0^{2.8} v_P(t)\,dt = 0.3\left[\frac{0+55}{2}\right] + 1.4\left[\frac{55+(-29)}{2}\right] + 1.1\left[\frac{-29+55}{2}\right]$$

Trapezoidal sum

$$= 8.25 + 18.2 + 14.3 = 40.75$$

Simplify

Technology Solution

1.1 | 1.2 | extension_ab2 | RAD

	A t	B velp	C	D
=				
1	0	0		
2	0.3	55		
3	1.7	-29		
4	2.8	55		
5	4	48		

1.1 | 1.2 | extension_ab2 | RAD

$$\sum_{i=1}^{3}\left(\frac{(t[i+1]-t[i])\cdot(velp[i]+velp[i+1])}{2}\right)$$

40.75

Example 1 Quick Acceleration

The velocity of a particle moving along the x-axis is given by a differentiable function v, where v is measured in meters per hour and t is measured in hours. Selected values of $v(t)$ are given in the table.

t	0	0.4	0.7	1.2	2	3	3.5	4
$v(t)$	0	14	-4	30	18	-10	48	50

(a) Show that there are at least two different times, $t_1 \neq t_2$, for $0 \leq t \leq 4$, such that the acceleration of the particle is 20 m/h^2.

(b) Use a left Riemann sum (L_5), and then a right Riemann sum (R_5), with subintervals as indicated in the table to approximate the value of $\displaystyle\int_0^3 v(t)\,dt$.

(c) Use a trapezoidal sum (T_5) with subintervals as indicated in the table to approximate the value of $\displaystyle\int_0^3 v(t)\,dt$.

(d) Verify that $T_5 = (L_5 + R_5)/2$.

Part (c)

Solve the equation $v_Q(t) = 60$ for $0 \le t \le 4$.

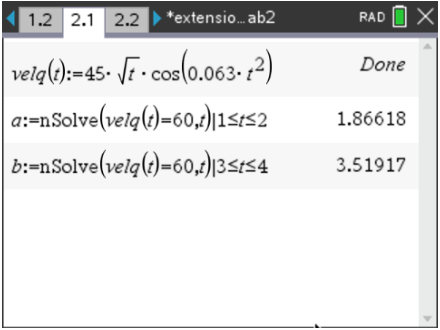
Using technology: $t = A = 1.866181$ and $t = B = 3.519174$

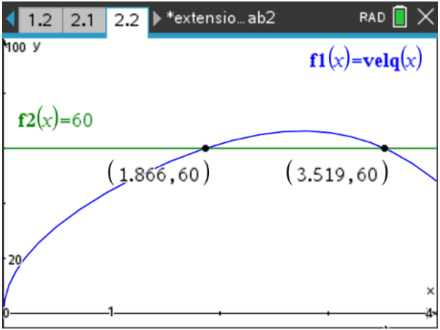
Therefore, $v_Q(t) \ge 60$ for $A \le t \le B$.

The distance traveled by particle Q over the interval $[A, B]$ is

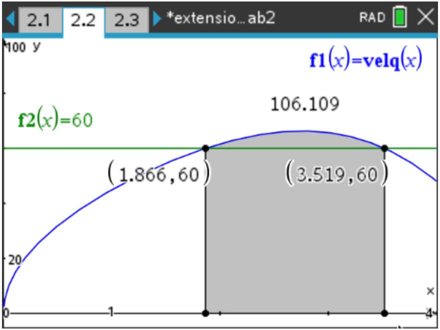
$$\int_A^B |v_Q(t)|\, dt = 106.108754$$

Technology Solution





Technology Solution



Example 2 Go The Distance

Suppose a particle Q moves along the x-axis so that its velocity for $0 \le t \le 9$ is given by $v_Q(t) = 45\sqrt{t}\cos(0.063t^2)$ meters per hour.

(a) Find the times t_1 and t_2, $4 \le t_1 < t_2 \le 9$, for which the velocity is 0. Find the distance traveled by the particle Q during the interval $[t_1, t_2]$.

(b) Find the absolute maximum velocity of particle Q over the interval $[0, 9]$ and the time at which this occurs, t_{max}. Justify your answer.

(c) Find the absolute minimum velocity of particle Q over the interval $[0, 9]$ and the time at which this occurs, t_{min}. Justify your answer.

(d) Find the distance traveled by the particle over the time interval $[t_{\text{max}}, t_{\text{min}}]$.

(e) Particle Q is at the origin at time $t = 0$. Find the maximum distance of particle Q from the origin over the time interval $[0, 9]$.

Part (d)

From part (b), position of particle P:

$$x_P(2.8) = \int_0^{2.8} v_P(t)\,dt \approx 40.75$$

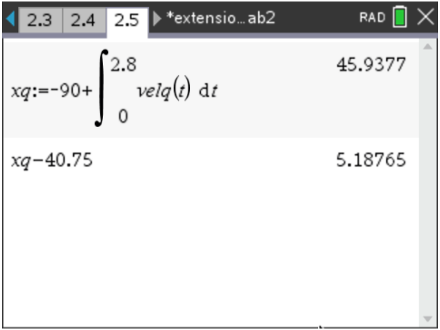
Position of particle Q:

$$x_Q(2.8) = -90 + \int_0^{2.8} v_Q(t)\,dt = -90 + 135.937653 = 45.937653$$

Distance between particles P and Q:

$$45.937653 - 40.75 = 5.188$$

Technology Solution



Example 3 Long Distance Friendship

Suppose the particle M moves along the x-axis so that its velocity for $0 \leq t \leq 9$ is given by $v_M(t) = -15t\ln(t+1)\sin(0.72t)$ meters per hour. Particle M is at the origin at time $t = 0$.

Find the maximum distance between the particle M and the particle Q (as described in Example 2).

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