

TI in Focus: AP[®] Calculus

2019 AP[®] Calculus Exam: AB-1/BC-1

Technology Solutions and Problem Extensions

Stephen Kokoska

Professor, Bloomsburg University

Former AP[®] Calculus Chief Reader

Outline

- (1) Free Response Question
- (2) Scoring Guidelines
- (3) Solutions in greater detail
- (4) Solutions using technology
- (5) Problem Extensions

1. Fish enter a lake at a rate modeled by the function E given by $E(t) = 20 + 15 \sin\left(\frac{\pi t}{6}\right)$. Fish leave the lake at a rate modeled by the function L given by $L(t) = 4 + 2^{0.1t^2}$. Both $E(t)$ and $L(t)$ are measured in fish per hour, and t is measured in hours since midnight ($t = 0$).
- (a) How many fish enter the lake over the 5-hour period from midnight ($t = 0$) to 5 A.M. ($t = 5$)? Give your answer to the nearest whole number.
- (b) What is the average number of fish that leave the lake per hour over the 5-hour period from midnight ($t = 0$) to 5 A.M. ($t = 5$)?
- (c) At what time t , for $0 \leq t \leq 8$, is the greatest number of fish in the lake? Justify your answer.
- (d) Is the rate of change in the number of fish in the lake increasing or decreasing at 5 A.M. ($t = 5$)? Explain your reasoning.

$$(a) \int_0^5 E(t) dt = 153.457690$$

To the nearest whole number, 153 fish enter the lake from midnight to 5 A.M.

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

$$(b) \frac{1}{5-0} \int_0^5 L(t) dt = 6.059038$$

The average number of fish that leave the lake per hour from midnight to 5 A.M. is 6.059 fish per hour.

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

- (c) The rate of change in the number of fish in the lake at time t is given by $E(t) - L(t)$.

3 : $\begin{cases} 1 : \text{sets } E(t) - L(t) = 0 \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

$$E(t) - L(t) = 0 \Rightarrow t = 6.20356$$

$E(t) - L(t) > 0$ for $0 \leq t < 6.20356$, and $E(t) - L(t) < 0$ for $6.20356 < t \leq 8$. Therefore, the greatest number of fish in the lake is at time $t = 6.204$ (or 6.203).

Let $A(t)$ be the change in the number of fish in the lake from midnight to t hours after midnight.

$$A(t) = \int_0^t (E(s) - L(s)) ds$$

$$A'(t) = E(t) - L(t) = 0 \Rightarrow t = C = 6.20356$$

| t | $A(t)$ |
|-----|-----------|
| 0 | 0 |
| C | 135.01492 |
| 8 | 80.91998 |

Therefore, the greatest number of fish in the lake is at time $t = 6.204$ (or 6.203).

(d) $E'(5) - L'(5) = -10.7228 < 0$

Because $E'(5) - L'(5) < 0$, the rate of change in the number of fish is decreasing at time $t = 5$.

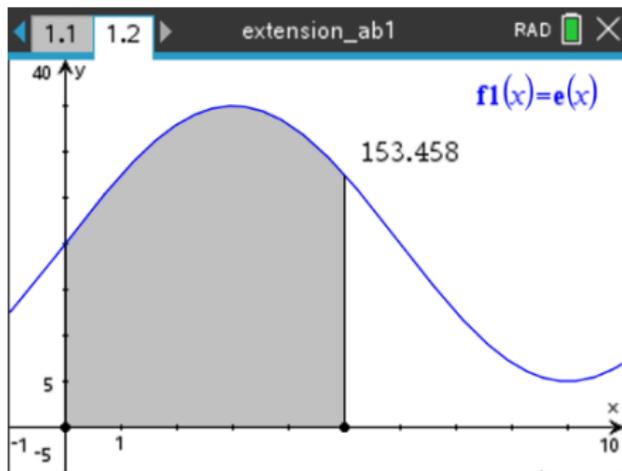
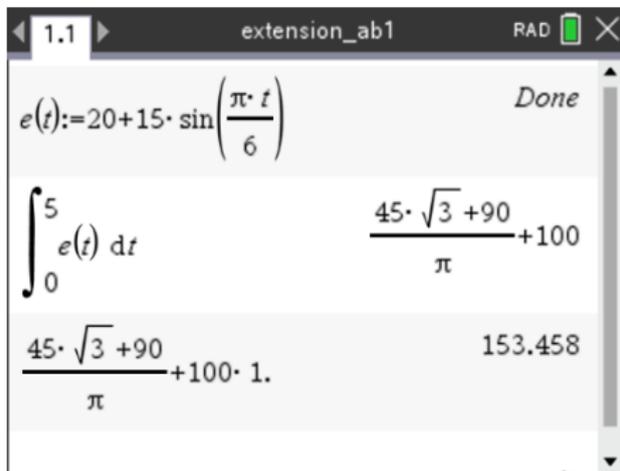
$$2 : \begin{cases} 1 : \text{considers } E'(5) \text{ and } L'(5) \\ 1 : \text{answer with explanation} \end{cases}$$

Part (a)

Solution

$$\int_0^5 E(t) dt = 153.458$$

To the nearest whole number, 153 fish enter the lake from midnight to 5 am.



Example 1 Lake Life

Fish enter a lake at a rate modeled by the function E . The value $E(t)$ is measured in fish per hours, and t is measured in hours since midnight.

- (a) If $E(t) = 20 + 15 \sin\left(\frac{\pi t}{6}\right)$, how many fish enter the lake over the 5-hour period from midnight to 5 am. Give your answer symbolically.
- (b) If $E(t) = 50e^{-t/20} \cos\left(\frac{t^2}{4}\right)$, how many fish enter the lake over the 5-hour period from midnight to 5 am. Give your answer to the nearest whole number.

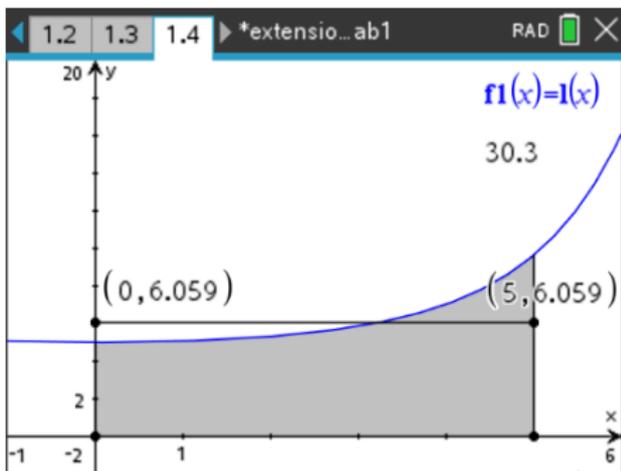
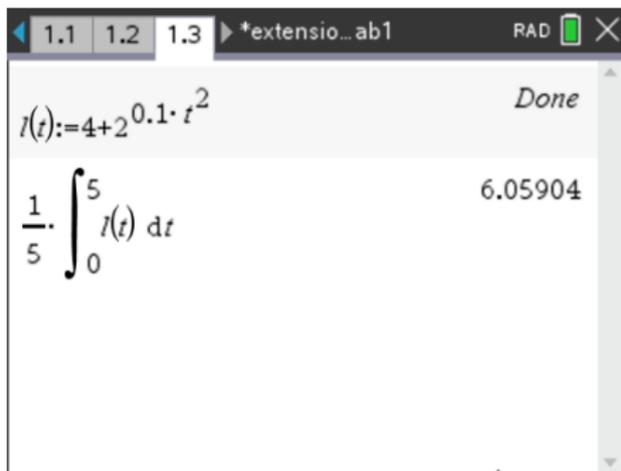
Find the time at which 75 fish have entered the lake.

Part (b)

Solution

$$\frac{1}{5-0} \int_0^5 L(t) dt = 6.059$$

The average number of fish that leave the lake per hour from midnight to 5 am is 6.059 fish per hour.



Example 2 Fish out of Water

Suppose fish enter a lake at a rate modeled by a function E and leave the lake at a rate modeled by L , as given in the Free Response Question. Both $E(t)$ and $L(t)$ are measured in fish per hour, and t is measured in hours since midnight ($t = 0$).

- Find the first time t hours after midnight such that the average number of fish that leave the lake over the period is 10.
- Suppose there are 2000 fish in the lake at midnight. Find the first time $t = t_a$ hours after midnight at which there are 2075 fish in the lake.
- Find the average number of fish in the lake over the time period from midnight ($t = 0$) to time $t = t_a$.

Part (c)

Solution

Let $A(t)$ be the change in the number of fish in the lake from midnight to t hours after midnight.

$$A(t) = \int_0^t [E(s) - L(s)] ds$$

$$A'(t) = E(t) - L(t) = 0 \implies t = C = 6.20356$$

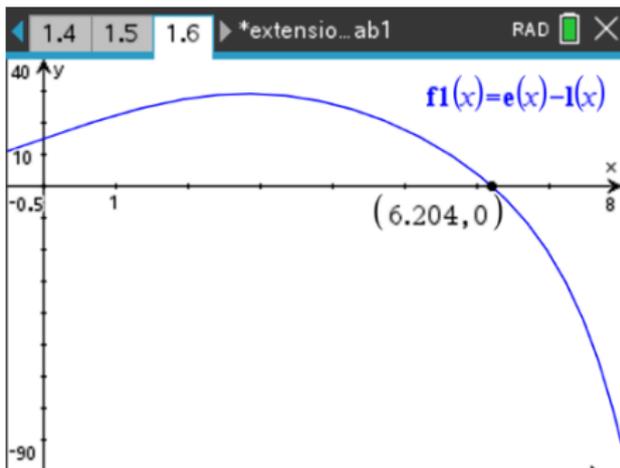
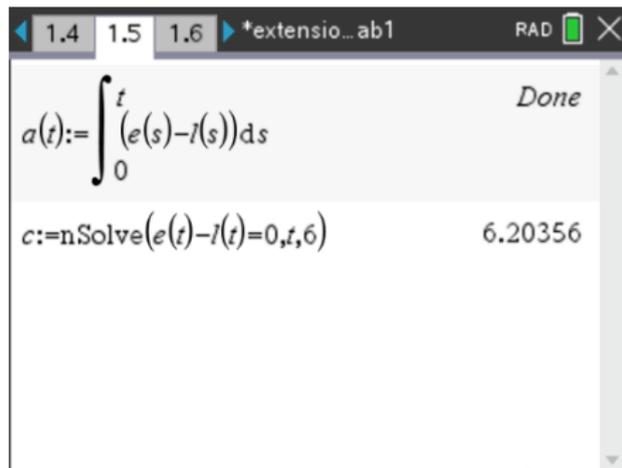
Create a table of values.

| t | $A(t)$ |
|-----|-----------|
| 0 | 0 |
| C | 135.01492 |
| 8 | 80.91998 |

The greatest number of fish in the lake is at time $t = 6.204$.

Part (c)

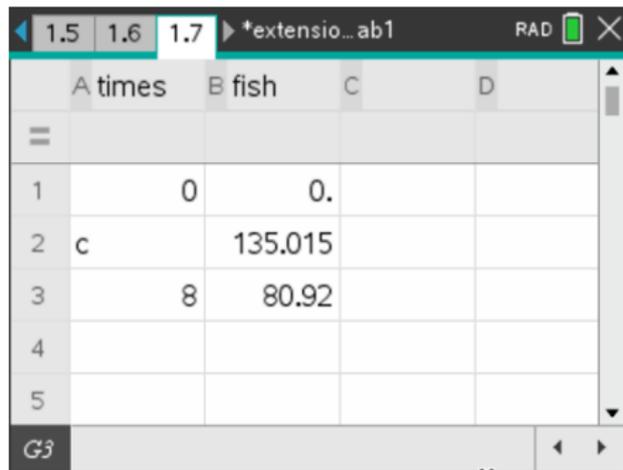
Technology Solution



There is one critical value in the interval $(0, 8)$.

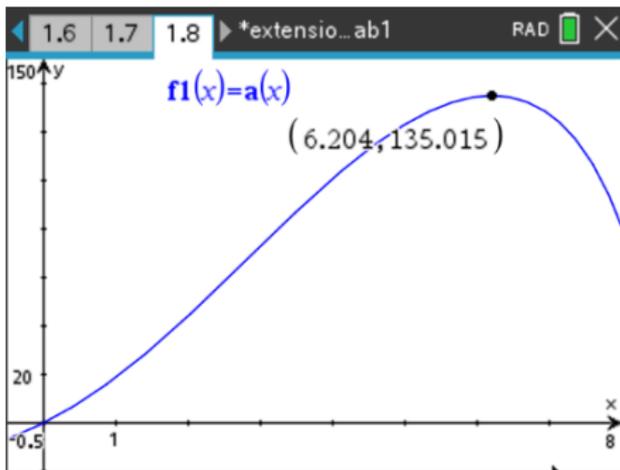
Part (c)

Consider a table of values.



The calculator screen displays a table with columns labeled A, B, C, and D. The data points are as follows:

| | A times | B fish | C | D |
|---|---------|---------|-------|---|
| 1 | | 0 | 0. | |
| 2 | c | 135.015 | | |
| 3 | | 8 | 80.92 | |
| 4 | | | | |
| 5 | | | | |



Example 3 Different Kettle of Fish

(a) Suppose fish enter a lake at a rate modeled by a function E and leave the lake at a rate modeled by L , as given in the Free Response Question. Both $E(t)$ and $L(t)$ are measured in fish per hour, and t is measured in hours since midnight ($t = 0$). Suppose there are 2000 fish in the lake at midnight. For t in the interval $0 \leq t \leq 8$, find the minimum number of fish in the lake.

(b) Suppose there are only 100 fish in the lake at midnight, $E(t) = \cos\left(\frac{t^2}{10}\right)$

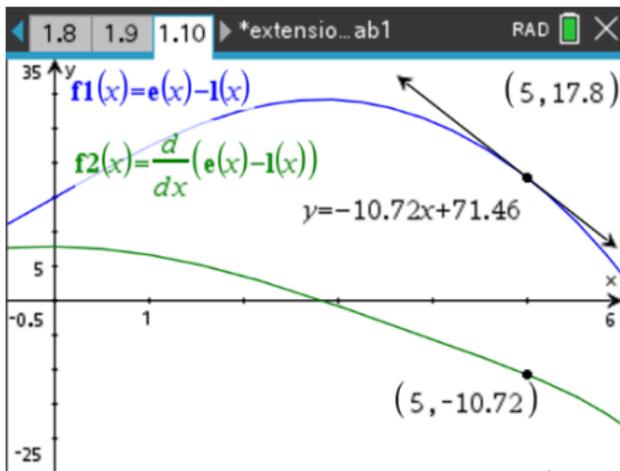
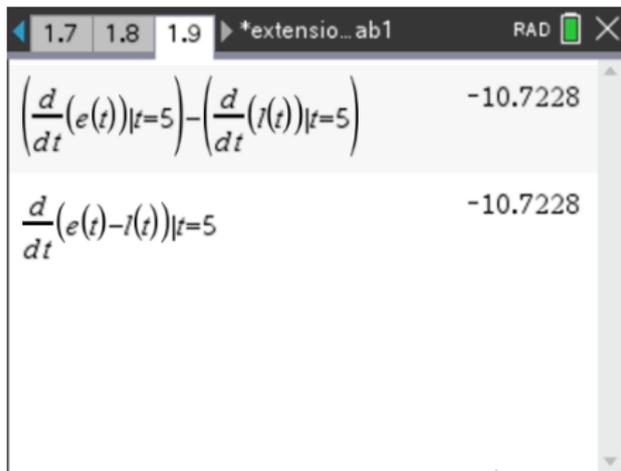
$$\text{and } L(t) = \arctan(t) \sin\left(\frac{t^2}{8}\right).$$

For t in the interval $0 \leq t \leq 10$, find the maximum number of fish in the lake, and the minimum number of fish in the lake.

Part (d)

Solution

Need to consider how the difference $E(t) - L(t)$ is changing.



Because $E'(5) - L'(5) < 0$, the rate of change in the number of fish is decreasing at time $t = 5$.

Example 4 A Day at the Beach

Cars enter a beach parking lot at a rate modeled by the function E given by $E(t) = 3 \ln(t + 3) \sin(t/4)$. Cars leave the parking lot at a rate modeled by the function L given by $L(t) = e^{-t^2/4}$. Both $E(t)$ and $L(t)$ are measured in cars per hour, and t is measured in hours since 6:00 am ($t = 0$). There are 25 cars in the parking lot at 6:00 am.

- How many cars enter the parking lot over the 6-hour period from 6:00 am to noon? Give your answer to the nearest whole number.
- What is the minimum and the maximum number of cars in the parking lot over the time interval $[0, 15]$?
- Let the function $C(t)$ be the total number of cars in the parking lot at time t , $0 \leq t \leq 15$. Write an expression for $C(t)$.
- For t in the interval $0 \leq t \leq 15$, find the time at which the rate of change of cars entering the parking lot is the same as the rate change of cars leaving the parking lot. Explain the meaning of this value in the context of the graph of the function C , and in the context of the problem.

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